UNIT IV

STABILITY ANALYSIS

Numerical Integration methods

- Modified Euler's method
- Runge-Kutta method

MODIFIED EULER'S METHOD

- Using first derivative of the initial point next point is obtained
- $$X_1^{\,p} = X_0 + \frac{dX}{dt}\Delta t$$ the step $$t_1 = t_0 + \Delta t$$
- Using this x_1^p dx/dt at $x_1^p = f(t_{1,}\ x_1^p)$

MODIFIED EULER'S METHOD

- Corrected value is

$$X_1^P = X_0 + \left(\frac{\left(\frac{dx}{dt}\right)_{X_0} + \left(\frac{dx}{dt}\right)_{X_1^p}}{2} \right) \Delta t$$

$$X_{i+1}^c = X_i + \left(\frac{\left(\frac{dx}{dt}\right)_{X_i} + \left(\frac{dx}{dt}\right)_{X_{i-1}^p}}{2} \right) \Delta t$$

Numerical Solution of the swing equation

- Input power p_m=constant
- At steady state p_e=p_m,

$$\delta_0 = \sin^{-1}\left(\frac{p_m}{p_{1\max}}\right)$$

$$p_{1\max} = \frac{|E'||V|}{X_1}$$

- At synchronous speed

$$\Delta\omega_0 = 0$$

$$p_{2\max} = \frac{|E'||V|}{X_2}$$

The swing equation

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = p_a = \left(p_m\right) - p_e$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H}\left(p_m - p_{2\max}\sin\delta\right) = \frac{\pi f_0}{H}p_a$$

$$\frac{d\delta}{dt} = \Delta\omega$$

$$\frac{d\Delta\omega}{dt} = \frac{\pi f_0}{H}p_a = \frac{d^2\delta}{dt^2}$$

Applying Modified Eulers method to above equation

$$t_1 = t_0 + \Delta t$$

$$\delta_{i+1}^{p} = \delta_i + \left(\frac{d\delta}{dt}\right)_{\Delta\omega_i} \Delta t$$

$$\Delta\omega_{i+1}^{p} = \Delta\omega_i + \left(\frac{d\Delta\omega}{dt}\right)_{\delta_i} \Delta t$$

- The derivatives at the end of interval

$$\left(\frac{d\delta}{dt}\right)_{\Delta\omega_{i+1}^{p}} = \Delta\omega_{i+1}^{p}$$

$$\left(\frac{d\Delta\omega}{dt}\right)_{\delta_{i+1}^{p}} = \left(\frac{\pi f_0}{H} p_a\right)_{\delta_{i+1}^{p}}$$

The corrected value

$$\delta_{i+1}^{c} = \delta_i + \left(\frac{\left(\frac{d\delta}{dt}\right)_{\Delta\omega_i} + \left(\frac{d\delta}{dt}\right)_{\Delta\omega_{i+1}^{p}}}{2} \right) \Delta t$$

$$\Delta\omega_{i+1}^{c} = \Delta\omega_i + \left(\frac{\left(\frac{d\Delta\omega}{dt}\right)_{\delta_i} + \left(\frac{d\Delta\omega}{dt}\right)_{\delta_{i+1}^{p}}}{2} \right) \Delta t$$

Runge-Kutta Method

- Obtain a load flow solution for pretransient conditions
- Calculate the generator internal voltages behind transient reactance.
- Assume the occurrence of a fault and calculate the reduced admittance matrix
- Initialize time count K=0,J=0

Runge-Kutta Method

- Determine the eight constants

$$K_1^k = f_1(\delta^k, \omega^k)\Delta t$$

$$l_1^k = f_2(\delta^k, \omega^k)\Delta t$$

$$K_2^k = f_1(\delta^k + \frac{K_1^k}{2}, \omega^k + \frac{l_1^k}{2})\Delta t$$

$$l_2^k = f_2(\delta^k + \frac{K_1^k}{2}, \omega^k + \frac{l_1^k}{2})\Delta t$$

$$K_3^k = f_1(\delta^k + \frac{K_2^k}{2}, \omega^k + \frac{l_2^k}{2})\Delta t$$

$$l_3^k = f_2(\delta^k + \frac{K_2^k}{2}, \omega^k + \frac{l_2^k}{2})\Delta t$$

$$K_4^k = f_1(\delta^k + \frac{K_3^k}{2}, \omega^k + \frac{l_3^k}{2})\Delta t$$

$$l_4^k = f_2(\delta^k + \frac{K_3^k}{2}, \omega^k + \frac{l_3^k}{2})\Delta t$$

$$\Delta\delta^k = \frac{\left(K_1^k + 2K_2^k + 2K_3^k + K_4^k\right)}{6}$$

$$\Delta\omega^k = \frac{\left(l_1^k + 2l_2^k + 2l_3^k + l_4^k\right)}{6}$$

Compute the change in state vector

$$\Delta\delta^k = \frac{\left(K_1^k + 2K_2^k + 2K_3^k + K_4^k\right)}{6}$$

$$\Delta\omega^k = \frac{\left(l_1^k + 2l_2^k + 2l_3^k + l_4^k\right)}{6}$$

Evaluate the new state vector

$$\delta^{k+1} = \delta^k + \Delta\delta^k$$

$$\omega^{k+1} = \omega^k + \Delta\omega^k$$

Evaluate the internal voltage behind transient reactance using the relation

$$E_p^{k+1} = \left|E_p^k\right| \cos \delta_p^{k+1} + j \left|E_p^k\right| \sin \delta_p^{k+1}$$

Check if $t<t_c$ yes K=K+1

Check if j=0,yes modify the network data and obtain the new reduced admittance matrix and set j=j+1

set K=K+1

Check if K<Kmax, yes start from finding 8 constants

Thank you