Ways to express Bernoulli equation

- conservation of energy (no friction loss)

Energy per unit volume:

$$p + \gamma z + \frac{1}{2}\rho V^2 = \text{constant (along streamline)}$$

Energy per unit mass:

$$\frac{p}{\rho} + gz + \frac{1}{2}V^2 = \text{constant (along streamline)}$$

Energy per unit weight:

$$\frac{p}{\gamma} + z + \frac{V^2}{2g} = \text{constant (along streamline)}$$

Civil Engineers often use the "energy per unit weight" form:

$$\frac{p}{\gamma} + z + \frac{V^2}{2g} = \text{constant (along streamline)}$$

$\dfrac{p}{\gamma} + z + \dfrac{V^2}{2g}$ **is often referred to as total head**

$\dfrac{p}{\gamma}$ **is often referred to as pressure head**

z **is often referred to as elevation (or potential) head**

$\dfrac{V^2}{2g}$ **is often referred to as velocity head**

Mechanical engineers often use the "energy per unit volume" form:

$$p + \gamma z + \frac{1}{2}\rho V^2 = \text{constant (along streamline)}$$

$p + \gamma z + \frac{1}{2}\rho V^2$ **is often referred to as total pressure**

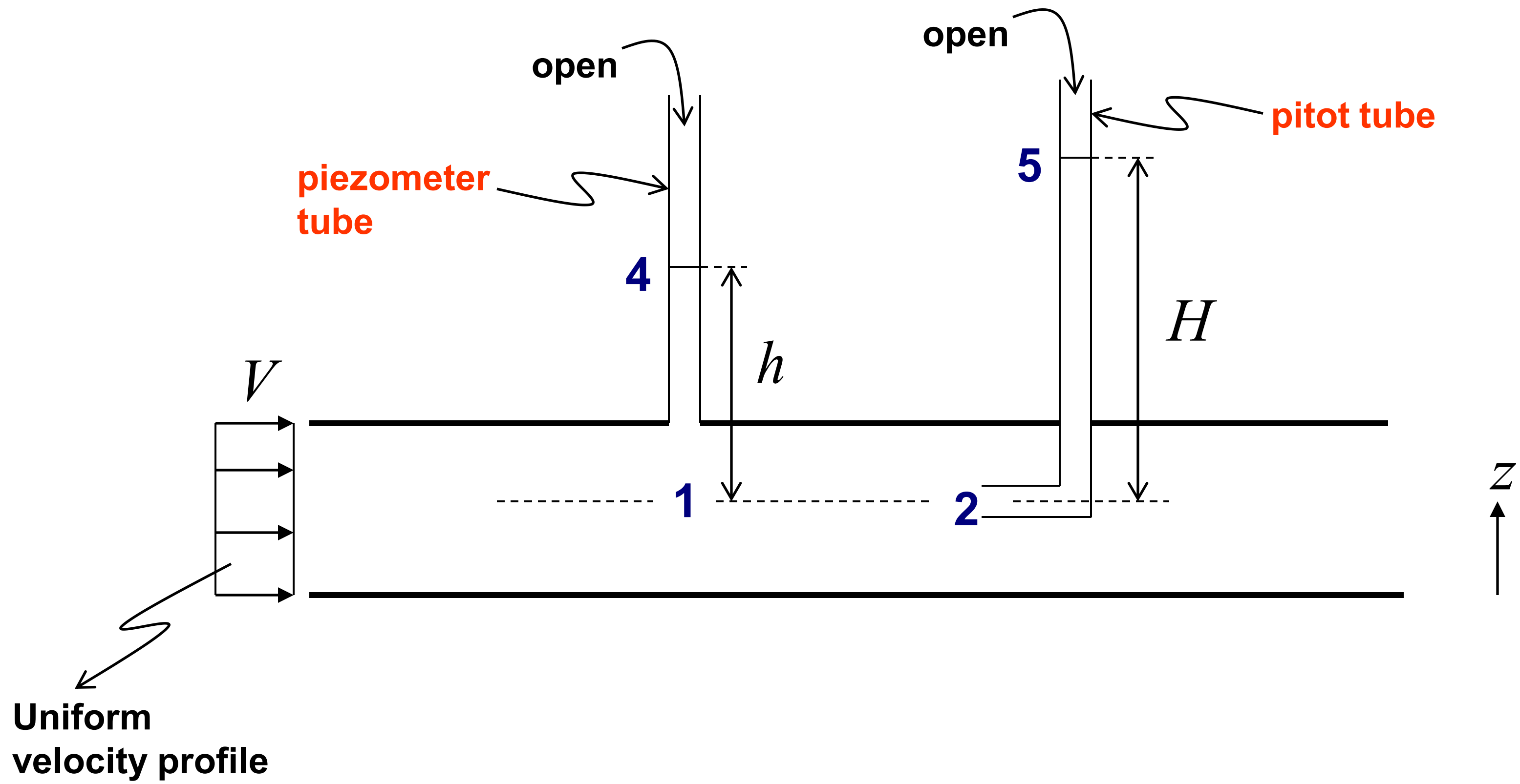
p **is often referred to as static pressure**

γz **is often referred to as hydrostatic pressure**

$\frac{1}{2}\rho V^2$ **is often referred to as dynamic pressure**

Pressure measurements (static, dynamic and stagnation pressure)

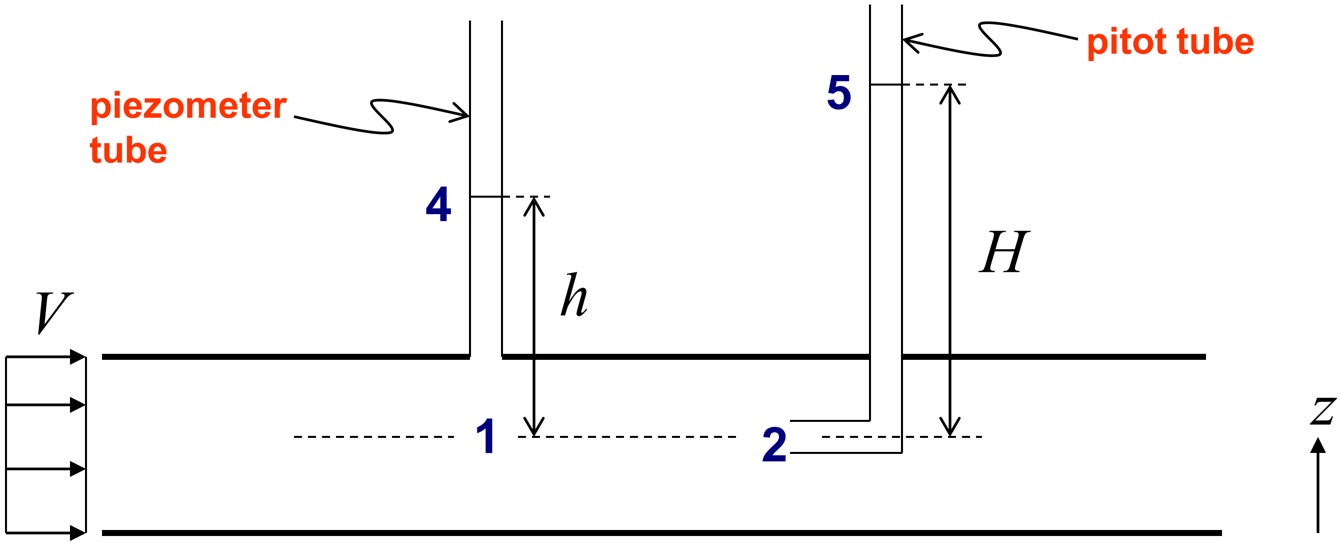
Consider the following closed channel flow (neglect friction):



Velocity at point 1 is the velocity of the flow: $V_1 = V$

Point 2 is at the entrance of the pitot tube where velocity is zero

Pressure measurements (static pressure)



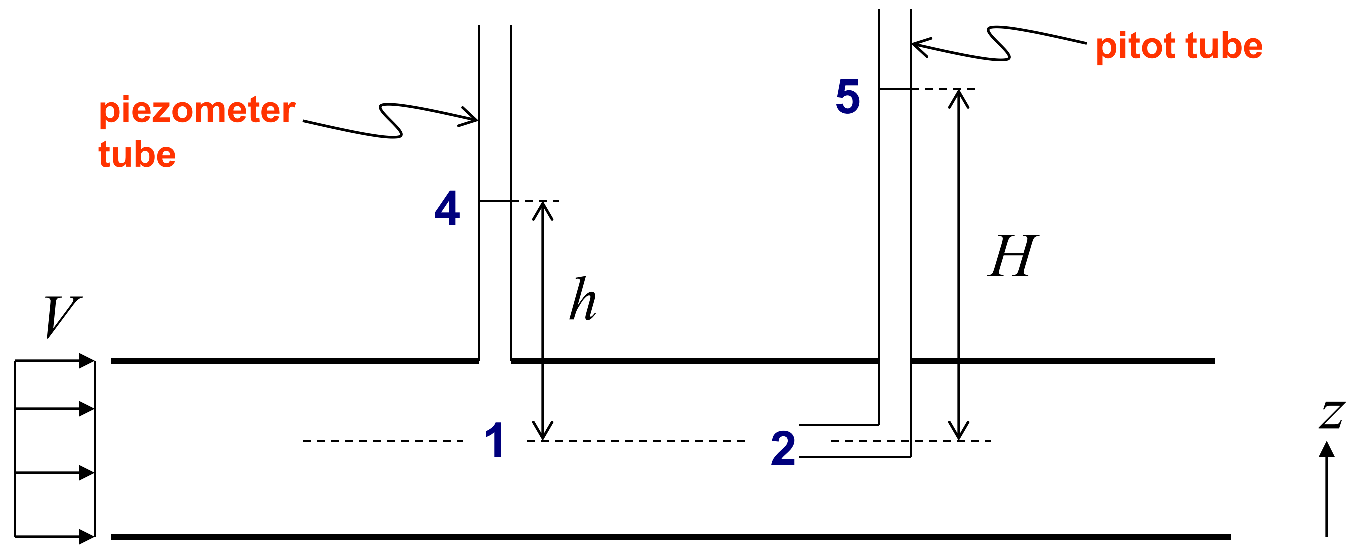
To measure static pressure say at point 1 we use piezometer tube along with $p + \gamma z = constant$ **across straight streamlines between pts. 1 and 4:**

$$p_1 = p_4 + \gamma h$$

$$p_1 = p_{atm} + \gamma h$$

$$\boxed{(p_1)_{gage} = 0 + \gamma h}$$

Pressure measurements (dynamic pressure)

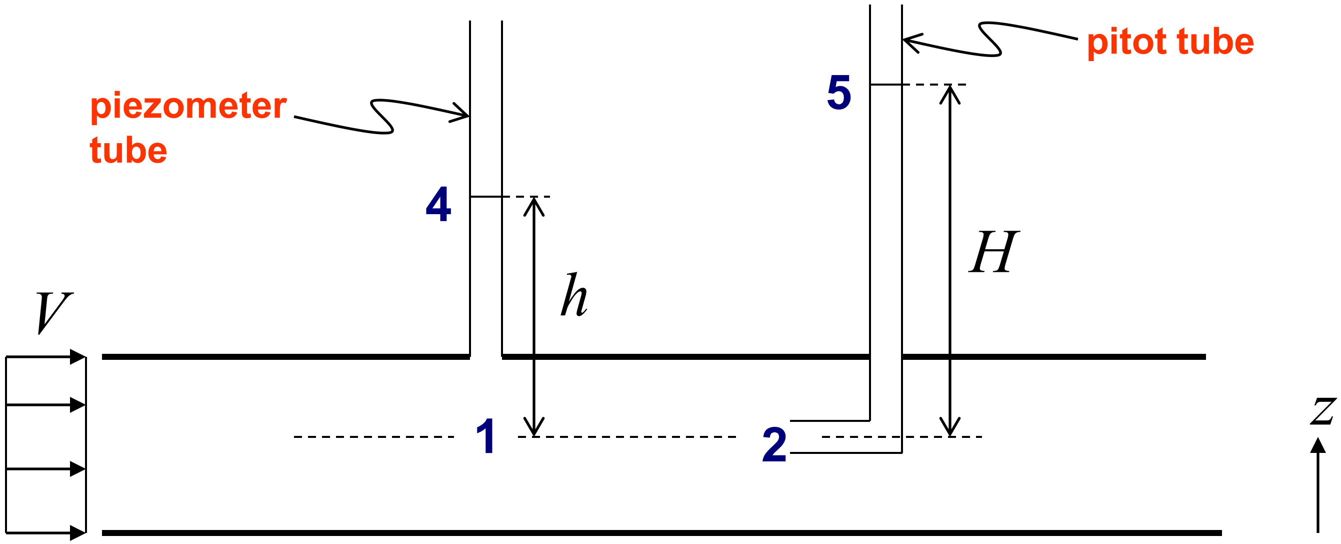


To measure dynamic pressure say at point 1 we use pitot tube along with Bernoulli equation from point 1 to point 5:

$$p_1 + \gamma z_1 + \frac{1}{2}\rho V_1^2 = p_5 + \gamma z_5 + \frac{1}{2}\rho V_5^2$$

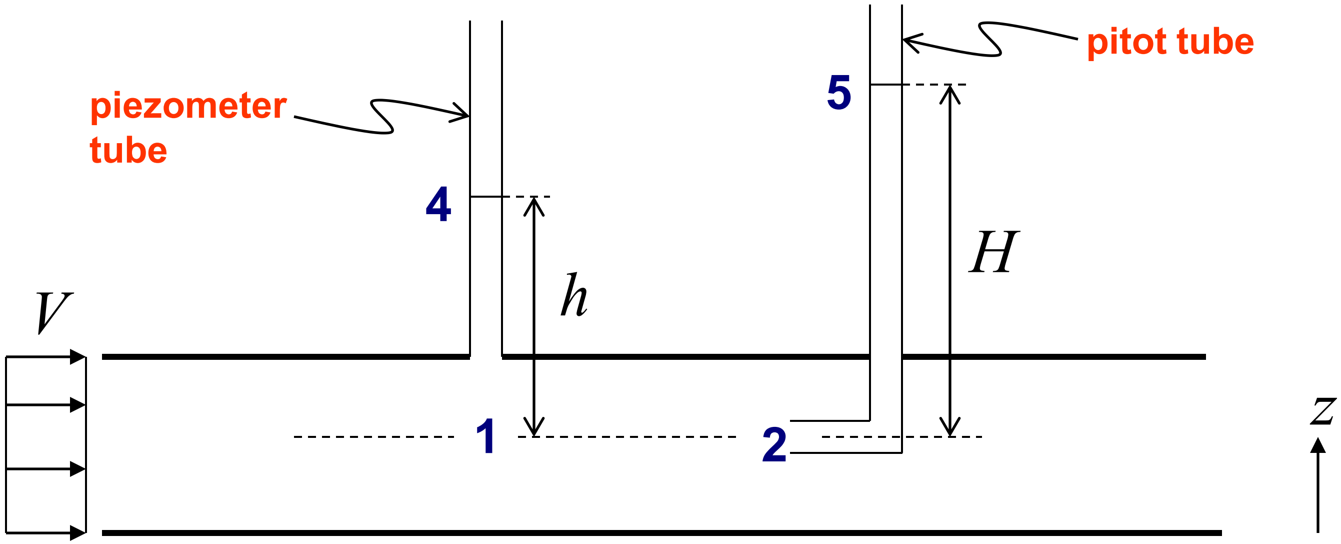
$$\text{dynamic pressure at pt.1} = \frac{1}{2}\rho V_1^2 = p_5 + \gamma z_5 + \frac{1}{2}\rho V_5^2 - \gamma z_1 - p_1$$

Pressure measurements (dynamic pressure)



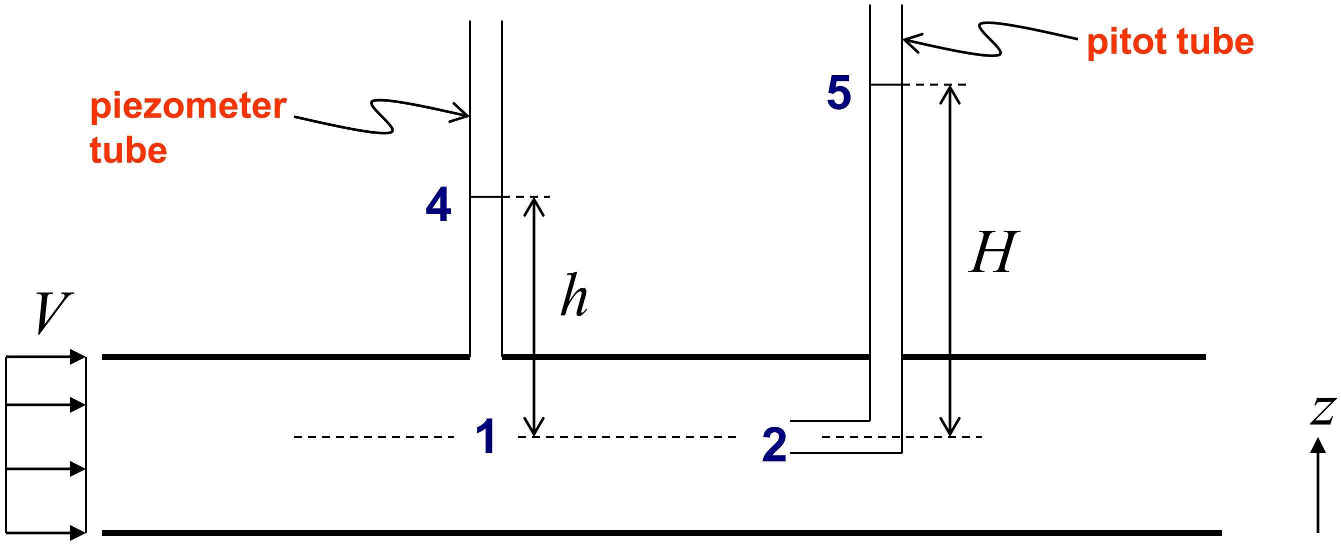
$$\text{dynamic pressure at pt.1} = \frac{1}{2}\rho V_1^2 = p_5 + \gamma z_5 + \frac{1}{2}\rho V_5^2 - \gamma z_1 - p_1$$

Pressure measurements (dynamic pressure)



$$\text{dynamic pressure at pt.1} = \frac{1}{2}\rho V_1^2 = \underbrace{p_5}_{p_{atm}=0} + \gamma z_5 + \frac{1}{2}\rho \underbrace{V_5^2}_{0} - \gamma z_1 - \underbrace{p_1}_{\gamma h}$$

Pressure measurements (dynamic pressure)

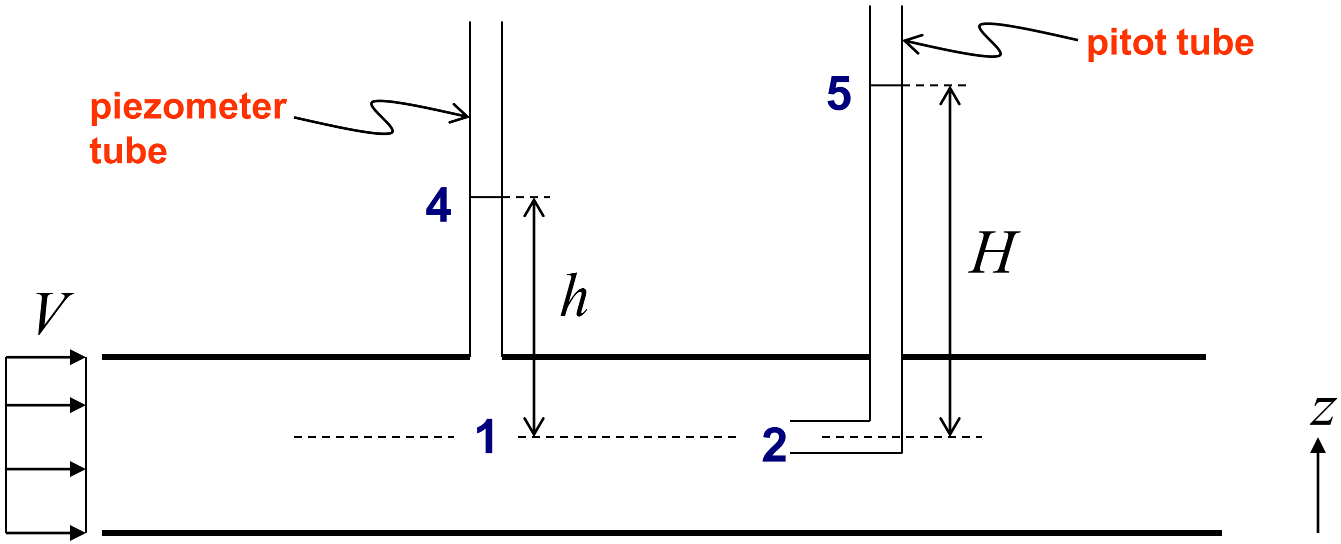


$$\text{dynamic pressure at pt.1} = \frac{1}{2}\rho V_1^2 = \underbrace{p_5}_{p_{atm}=0} + \gamma z_5 + \underbrace{\frac{1}{2}\rho V_5^2}_{0} - \gamma z_1 - \underbrace{p_1}_{\gamma h}$$

$$\boxed{\frac{1}{2}\rho V_1^2 = \gamma\overbrace{(z_5 - z_1)}^{=H} - \gamma h = \gamma(H-h)}$$

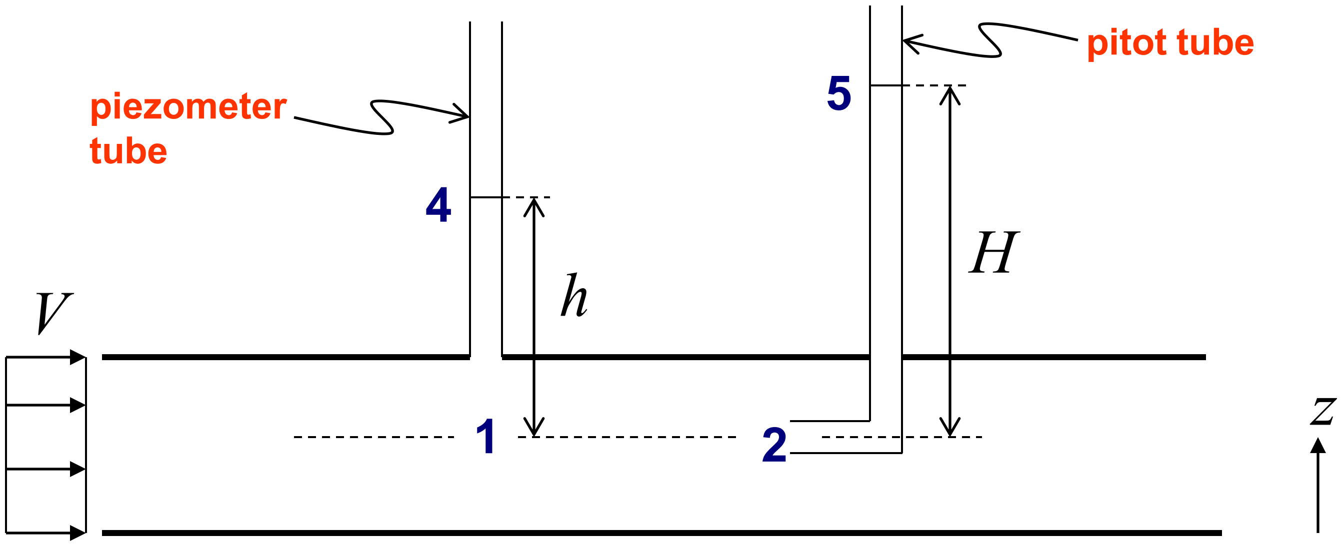
$$V_1 = V$$

Pressure measurements (stagnation pressure (pressure at pt. 2))



Stagnation pressure is pressure where velocity is zero (at entrance of pitot tube (pt. 2))

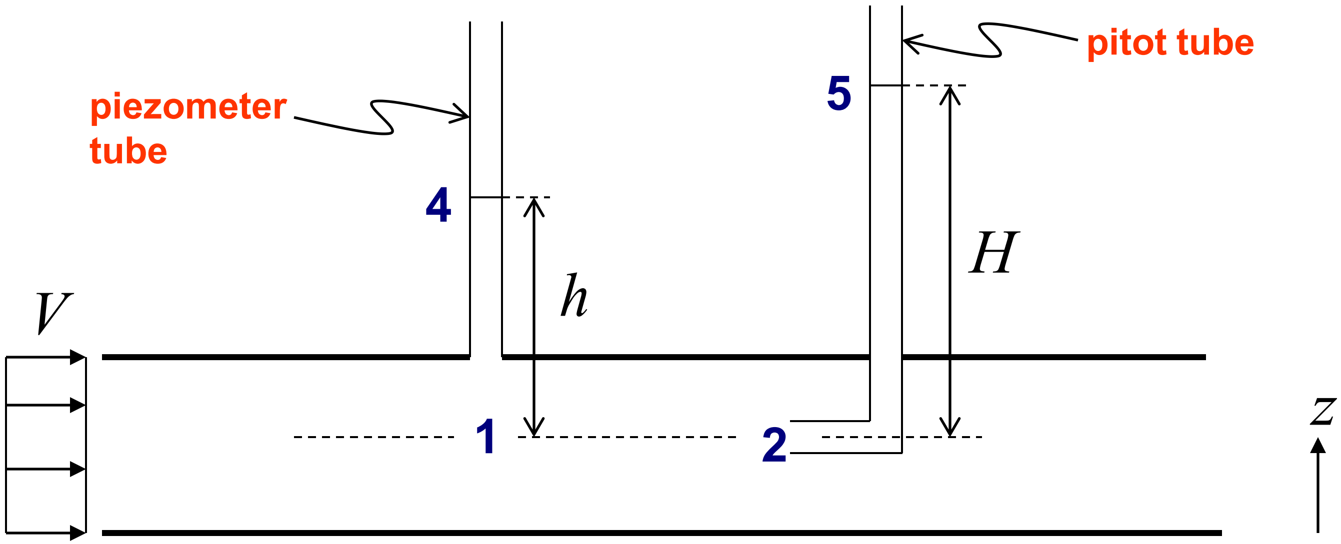
Pressure measurements (stagnation pressure (pressure at pt. 2))



Stagnation pressure is pressure where velocity is zero (at entrance of pitot tube (pt. 2))

Bernoulli from pt. 1 to pt. 2: $p_1 + \gamma z_1 + \frac{1}{2}\rho V_1^2 = p_2 + \gamma z_2 + \frac{1}{2}\rho V_2^2 \;\; ; \;\; z_1 = z_2$

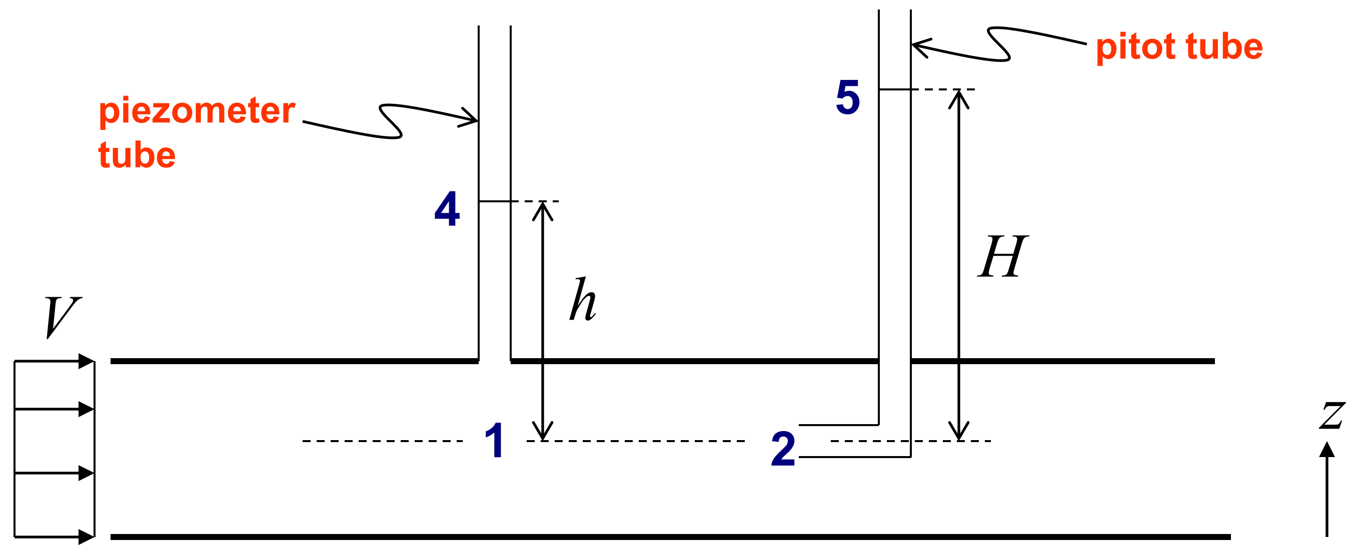
Pressure measurements (stagnation pressure (pressure at pt. 2))



Stagnation pressure is pressure where velocity is zero (at entrance of pitot tube (pt. 2))

Bernoulli from pt. 1 to pt. 2: $$p_1 + \cancel{\gamma z_1} + \frac{1}{2}\rho V_1^2 = p_2 + \cancel{\gamma z_2} + \frac{1}{2}\rho \cancelto{0}{V_2^2} \;\; ; \;\; z_1 = z_2$$

Pressure measurements (stagnation pressure (pressure at pt. 2))



Stagnation pressure is pressure where velocity is zero (at entrance of pitot tube (pt. 2))

Bernoulli from pt. 1 to pt. 2: $$p_1 + \cancel{\gamma z_1} + \frac{1}{2}\rho V_1^2 = p_2 + \cancel{\gamma z_2} + \frac{1}{2}\rho \cancelto{0}{V_2^2} \quad ; \quad z_1 = z_2$$

Stagnation pressure at pt. 2 is: $$\boxed{p_2 = p_1 + \frac{1}{2}\rho V_1^2} \qquad V_1 = V$$

Pressure measurements

$$p_2 = p_1 + \frac{1}{2}\rho V^2$$

Stagnation pressure   Static pressure

Note that
$$V = \sqrt{\frac{2(p_2 - p_1)}{\rho}}$$

Airplanes use pitot-static tubes (a combination of piezometer and pitot tubes) to measure p_2 and p_1 and compute airplane speed using previous equation

Energy Grade Line (EGL) and Hydraulic Grade Line (HGL)

Graphical interpretations of the energy along a pipeline may be obtained through the EGL and HGL:

$$EGL = \frac{p}{\gamma} + \frac{V^2}{2g} + z$$

$$HGL = \frac{p}{\gamma} + z$$

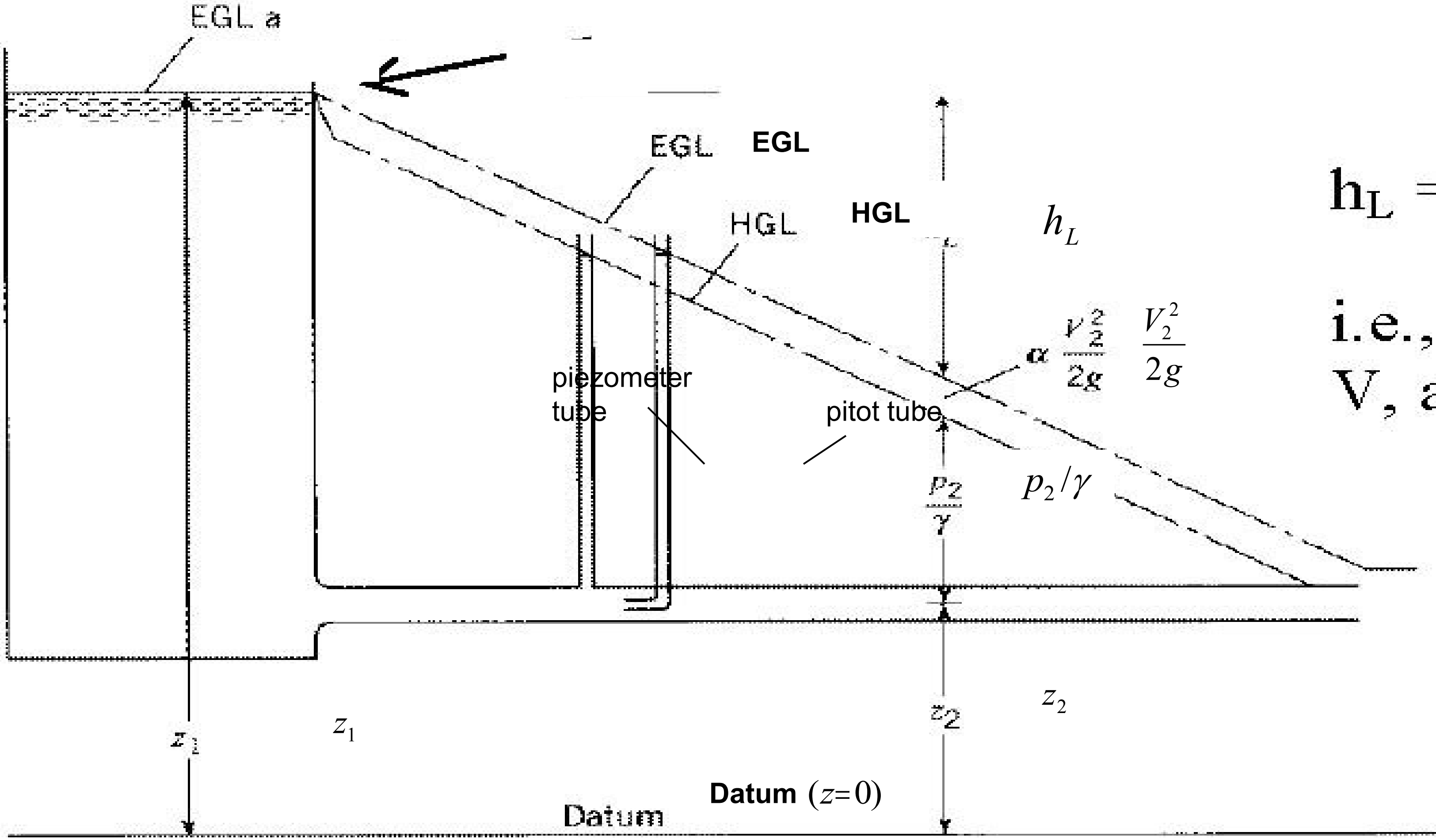
EGL and HGL may be obtained via a pitot tube and a piezometer tube, respectively

In our discussion we will be taking atmospheric pressure equal to zero, thus we will be working with gage pressures

Energy Grade Line (EGL) and Hydraulic Grade Line (HGL)

$$EGL = \frac{p}{\gamma} + \frac{V^2}{2g} + z \qquad HGL = \frac{p}{\gamma} + z \qquad h_L = h_f$$

- head loss, say, due to friction



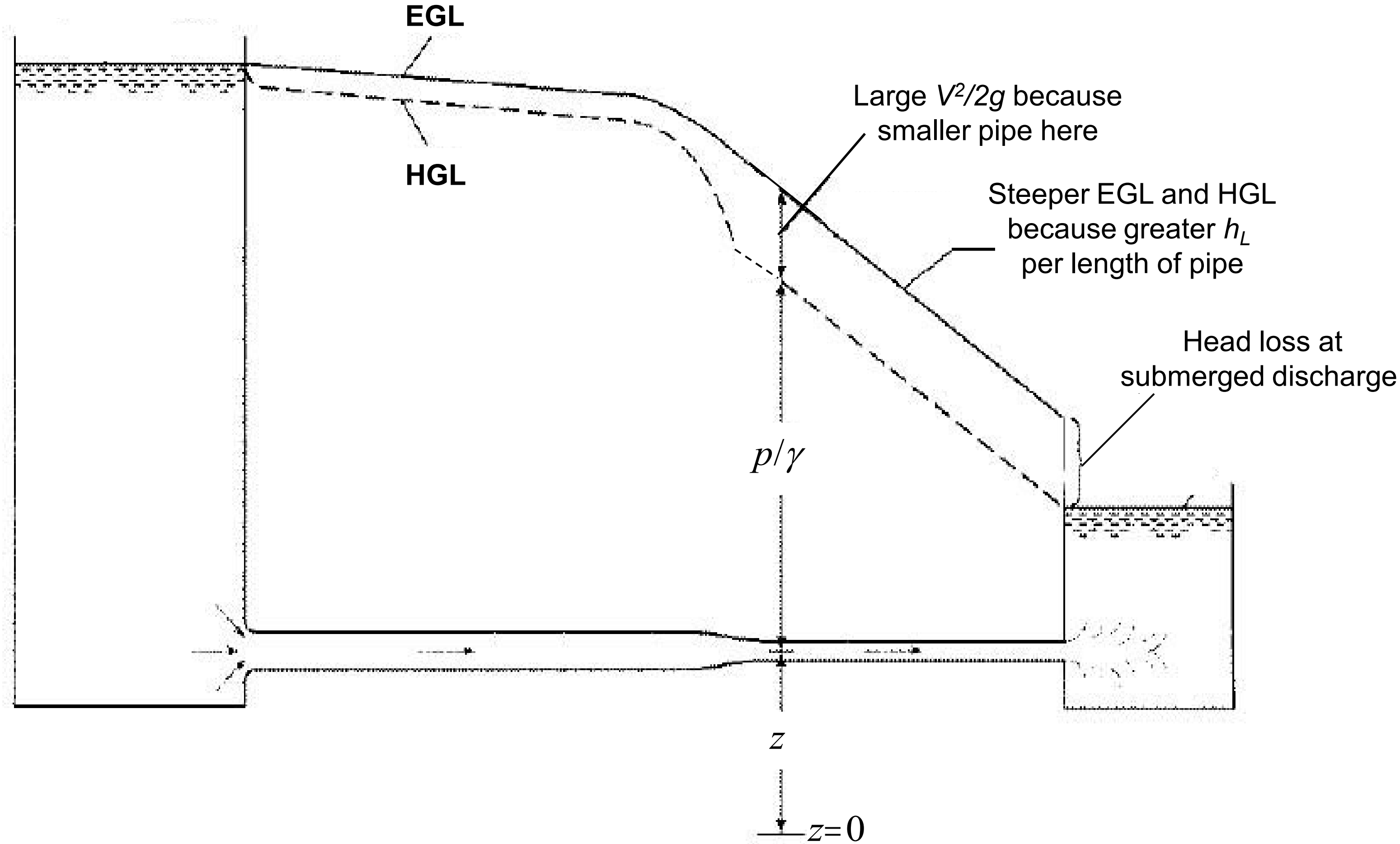
$$h_L = f \frac{L}{D} \frac{V}{2}$$

i.e., linear

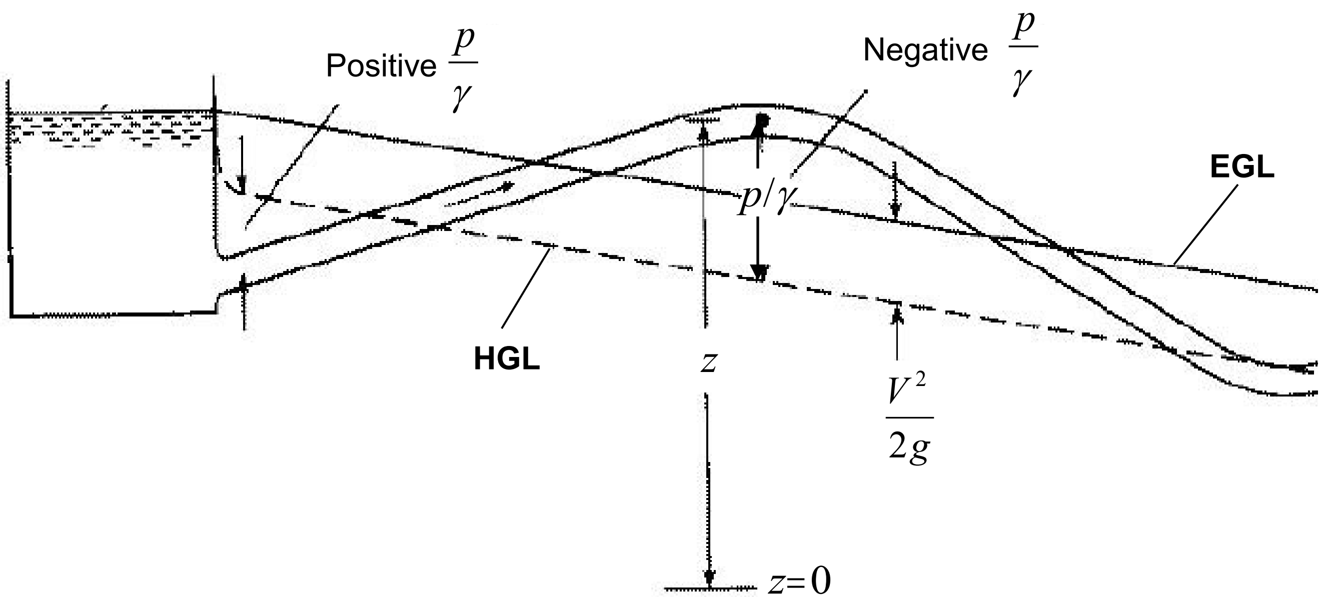
V, and f c

Energy Grade Line (EGL) and Hydraulic Grade Line (HGL)

$$EGL = \frac{p}{\gamma} + \frac{V^2}{2g} + z \qquad HGL = \frac{p}{\gamma} + z \qquad h_L = h_f$$



Energy Grade Line (EGL) and Hydraulic Grade Line (HGL)



$$EGL = \frac{p}{\gamma} + \frac{V^2}{2g} + z$$

$$HGL = \frac{p}{\gamma} + z$$

$$h_L = h_f$$

If $HGL < z$ then $\frac{P}{\gamma} < 0$ and cavitation may be possible

Energy Grade Line (EGL) and Hydraulic Grade Line (HGL)

Helpful hints when drawing HGL and EGL:

1. *EGL = HGL +* $V^2/2g$*,* *EGL = HGL for V=0*

2. If $p=0$, then $HGL=z$

3. A change in pipe diameter leads to a change in V ($V^2/2g$) due to continuity and thus a change in distance between HGL and EGL

4. A change in head loss (h_L) leads to a change in slope of EGL and HGL

5. If $HGL < z$ then $\frac{P}{\gamma} < 0$ and cavitation may be possible

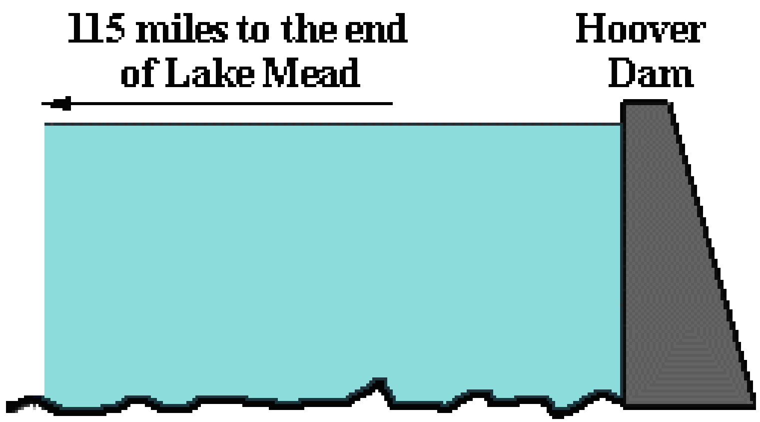
Helpful hints when drawing HGL and EGL (cont.):

6. A sudden head loss due to a turbine leads to a sudden drop in EGL and HGL

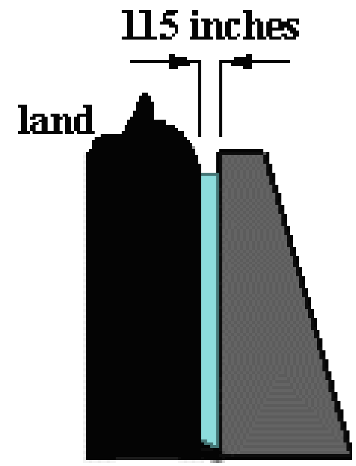
7. A sudden head gain due to a pump leads to a sudden rise in EGL and HGL

8. A sudden head loss due to a submerged discharge leads to a sudden drop in EGL

Hydrostatic Paradox



Hoover Dam and Lake Mead



Hoover Dam and Lake Mudd

At Lake Mudd and Lake Mead, the depth is ~600 ft.

At Lake Mead, the horizontal thrust near the base of the dam is ~18 tons per square foot.

Here is the paradox: **in both cases, the horizontal thrust on the dam is the SAME**

Hydrostatic Paradox

The reason for this paradox is that the pressure depends only on the depth of the water, not on its horizontal extent:

$$p + \gamma z = const$$