Fluid Mechanics

Fluid Statics

The word “statics” is derived from Greek word “statikos”= motionless

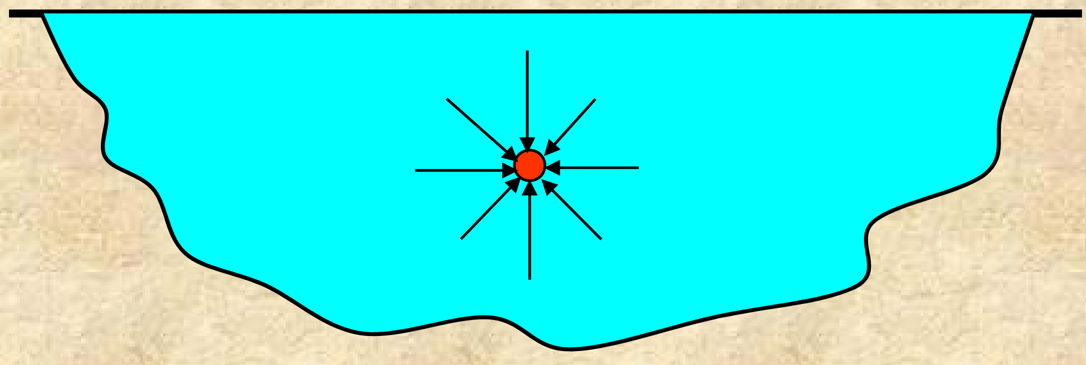
- For a fluid at rest or moving in such a manner that there is no relative motion between particles there are no shearing forces present:

Rigid body approximation

Definition of Pressure

Pressure is defined as the amount of force exerted on a unit area of a substance:

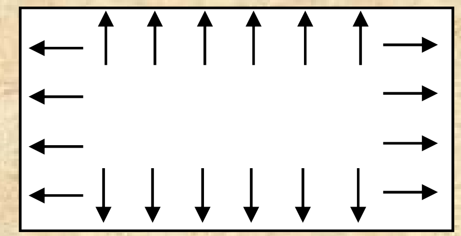
$$P = \frac{force}{area} = \frac{N}{m^2} = Pa$$



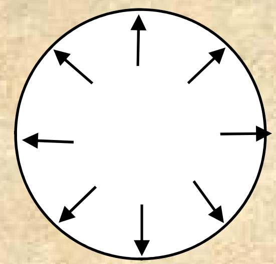
Pascal's Laws

- Pascals' laws:
  - Pressure acts uniformly in all directions on a small volume (point) of a fluid
  - In a fluid confined by solid boundaries, pressure acts perpendicular to the boundary – it is a <u>*normal*</u> force.

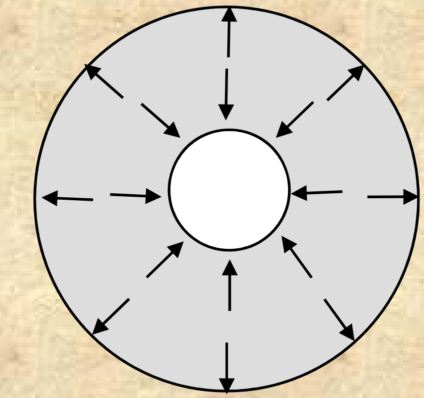
Direction of fluid pressure on boundaries



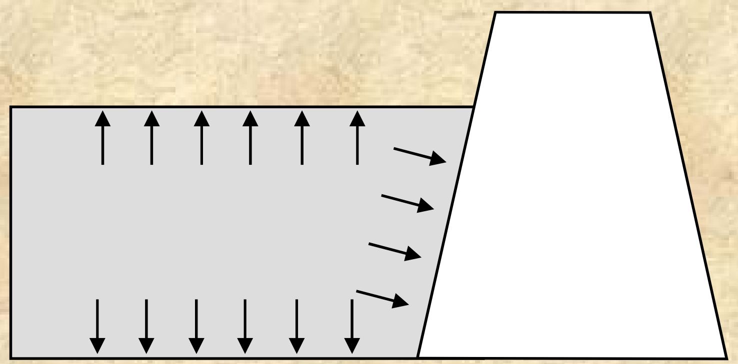
Furnace duct



Pipe or tube



Heat exchanger



Dam

Pressure is a *Normal Force*
(acts perpendicular to surfaces)
It is also called a *Surface Force*

Absolute and Gage Pressure

- Absolute pressure: The pressure of a fluid is expressed relative to that of vacuum (=0)

- Gage pressure: Pressure expressed as the difference between the pressure of the fluid and that of the surrounding atmosphere.

➢ Usual pressure gages record gage pressure. To calculate absolute pressure:

$$P_{abs} = P_{atm} + P_{gage}$$

Units for Pressure

Unit	Definition or Relationship
1 pascal (Pa)	1 kg m^{-1} s^{-2}
1 bar	1 x 10^5 Pa
1 atmosphere (atm)	101,325 Pa
1 torr	1 / 760 atm
760 mm Hg	1 atm
14.696 pounds per sq. in. (psi)	1 atm

Measurement of Pressure

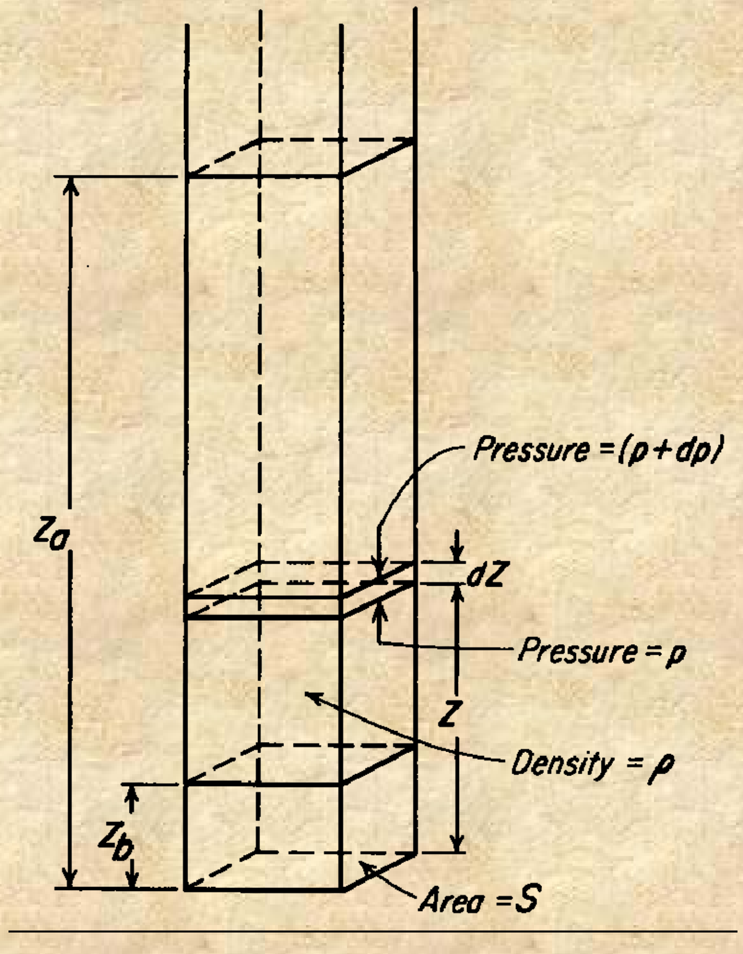
Mechanical and electronic pressure measuring devices:

- When a pressure acts on an elastic structure it will deform. This deformation can be related to the magnitude of the pressure.
  - Bourdon pressure gage
- Pressure transducers convert pressure into an electrical output
- Strain-gage pressure transducers are suitable for rapid changes in pressure and cover big ranges of pressure values

Pressure distribution for a fluid at rest

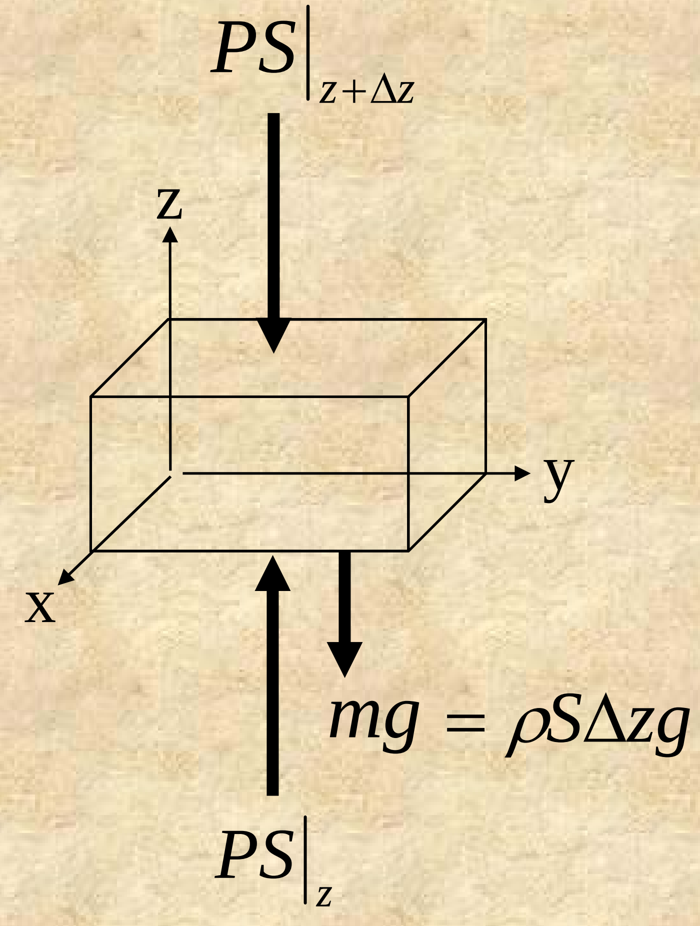
Let's determine the pressure distribution in a fluid at rest in which the only body force acting is due to gravity

∴ *The sum of the forces acting on the fluid must equal zero*



What are the z-direction forces?

Let P_z and $P_{z+\Delta z}$ denote the pressures at the base and top of the cube, where the elevations are z and z+Δz respectively.



Pressure distribution for a fluid at rest

A force balance in the *z direction* gives:

$$\sum F_z = 0 = PS\big|_z - PS\big|_{z+\Delta z} - \rho S \Delta z g$$

$$\frac{P_{z+\Delta z} - P_z}{\Delta z} = -\rho g$$

For an infinitesimal element ($\Delta z \rightarrow 0$)

$$\boxed{\frac{dP}{dz} = -\rho g}$$

Incompressible fluid

Liquids are incompressible i.e. their density is assumed to be constant:

$$P_2 - P_1 = -\rho g(z_2 - z_1)$$

When we have a liquid with a free surface the pressure P at any depth below the free surface is:

$$P = \rho g h + P_o$$

P_o is the pressure at the free surface (P_o=P_{atm})

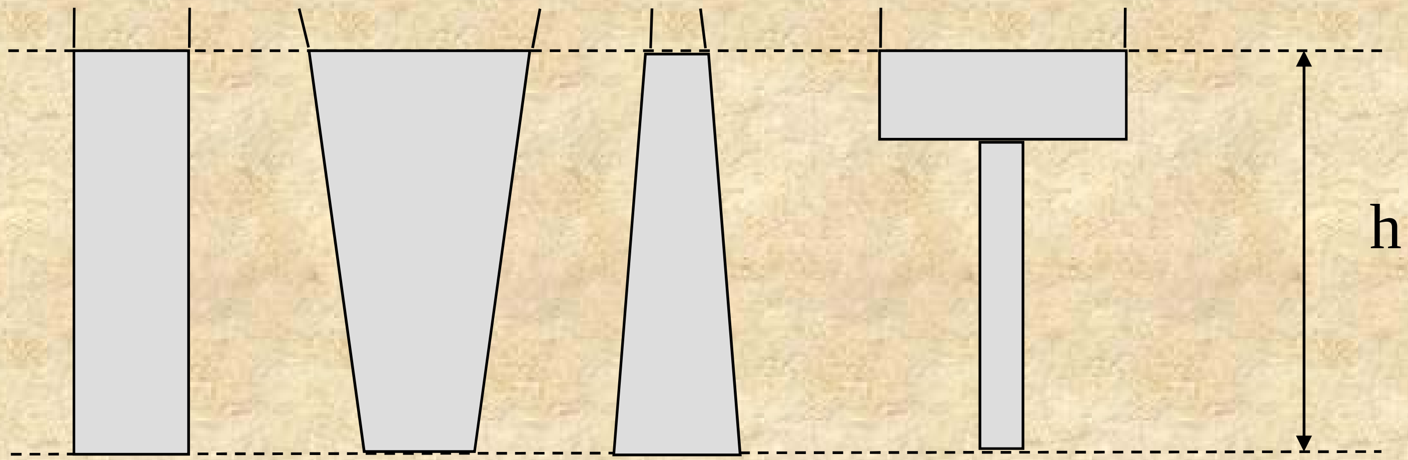
By using gage pressures we can simply write:

$$P = \rho g h$$

Pascal's principle (The hydrostatic paradox)

- The pressure in a homogeneous, incompressible fluid at rest depends on the depth of the fluid relative to some reference plane, and it is not influenced by the size or shape of the tank or container

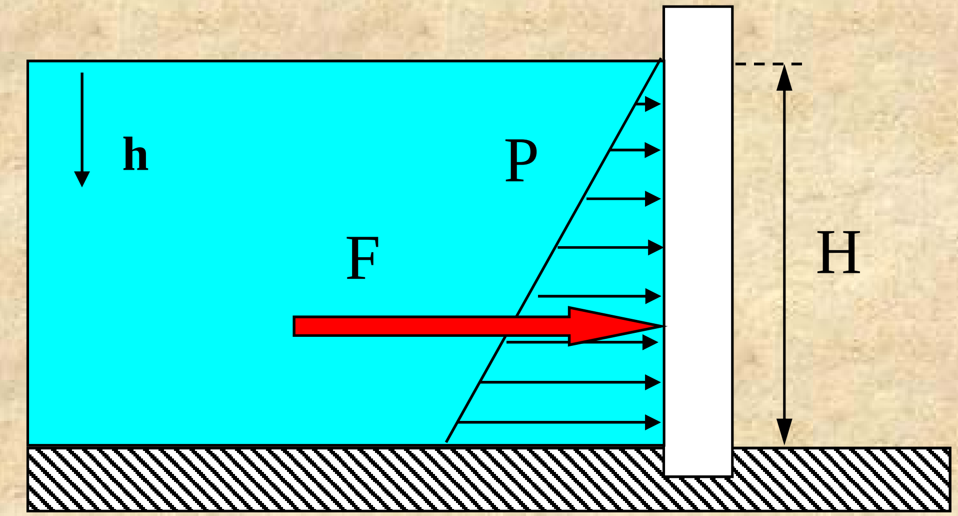
Fluid is the same in all containers



Pressure is the same at the bottom of all containers

Vertical plane surfaces

The lock gate of a canal is rectangular, 20 m wide and 10 m high. One side is exposed to the atmosphere and the other side to the water. What is the net force on the lock gate?



Vertical rectangular wall (wall width = W)

- Here the pressure varies linearly with depth: $P=\rho gh$

Vertical plane surfaces

- For an infinitesimal area dA the normal force due to the pressure is

$$dF = p\, dA$$

- Find resultant force acting on a finite surface by integration

$$F = \int P\, \mathrm{dA} = \int \rho g h\, d(Wh) = \rho g W \int h\, dh$$

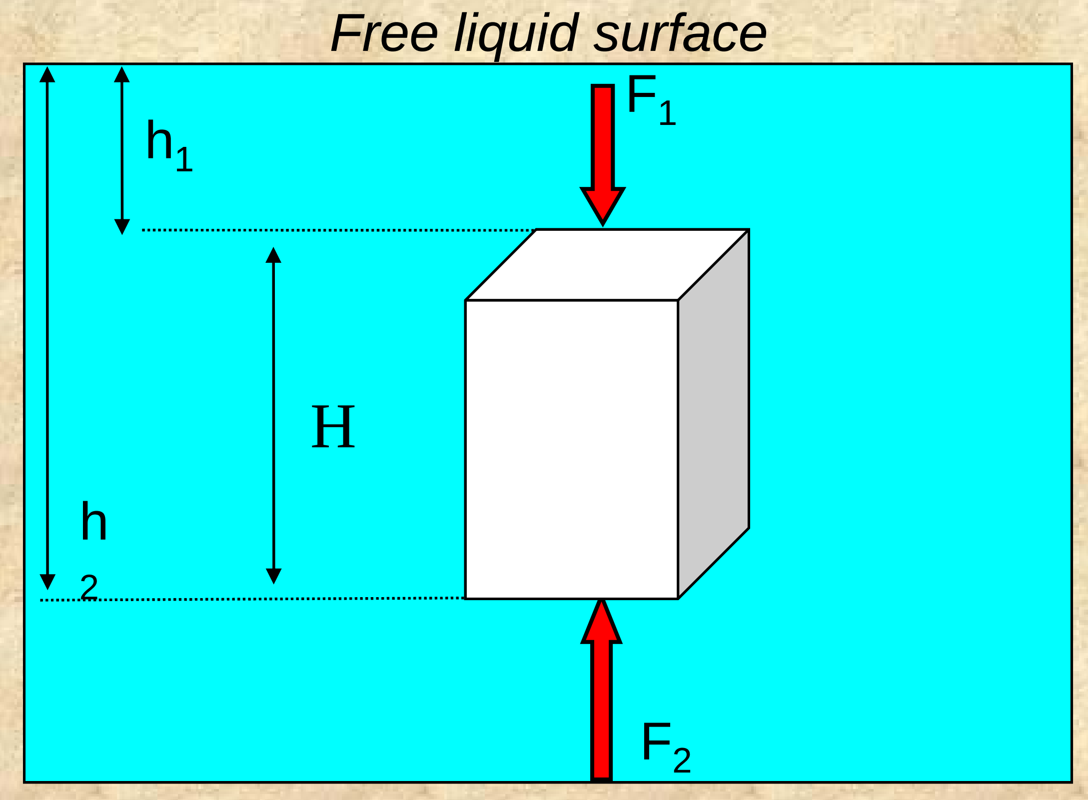
For vertical rectangular wall: $F = \frac{1}{2} \rho g W H^2$

Buoyancy

- A body immersed in a fluid experiences a vertical buoyant force equal to the weight of the fluid it displaces
- A floating body displaces its own weight in the fluid in which it floats

The upper surface of the body is subjected to a smaller force than the lower surface

∴ A net force is acting *upwards*



Buoyancy

The net force due to pressure in the vertical direction is:

$$F_B = F_2 - F_1 = (P_{bottom} - P_{top})(\Delta x \Delta y)$$

The pressure difference is:

$$P_{bottom} - P_{top} = \rho\, g\, (h_2 - h_1) = \rho\, g\, H$$

Combining:

$$F_B = \rho\, g\, H\, (\Delta x \Delta y)$$

Thus the buoyant force is:

$$\boxed{F_B = \rho\, g\, V}$$

Measurement of Pressure

The atmospheric pressure can be measured with a barometer.

$$p_{atm} = \rho g h + p_{vapor}$$

– For mercury barometers atmospheric pressure (101.33kPa) corresponds to h=760 mmHg (= 29.2 in)

– If water is used h = 10.33 m H_2O (= 34 ft)

Measurement of Pressure

Manometers are devices in which one or more columns of a liquid are used to determine the pressure difference between two points.

- U-tube manometer
- Inclined-tube manometer

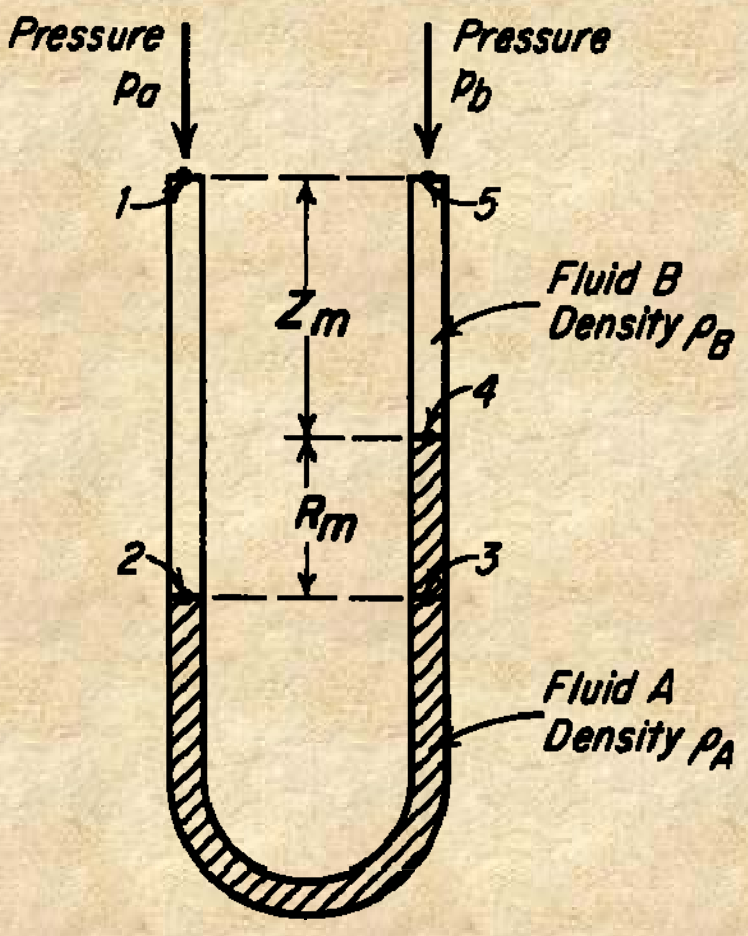
Measurement of Pressure Differences

Apply the basic equation of static fluids to both legs of manometer, realizing that P_2=P_3.

$$P_2 = P_a + \rho_b g(Z_m + R_m)$$

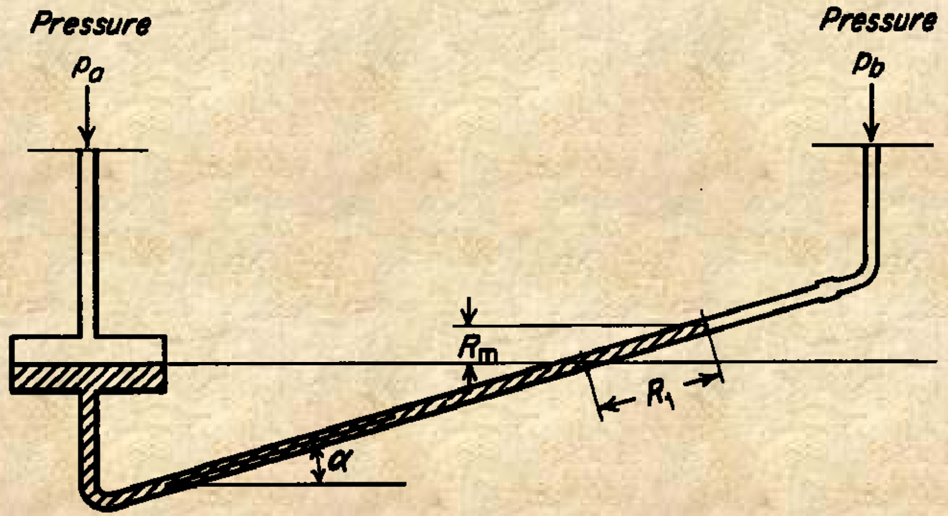
$$P_3 = P_b + \rho_b g(Z_m) + \rho_a g R_m$$

$$\boxed{P_a - P_b = gR_m(\rho_a - \rho_b)}$$



Inclined Manometer

- To measure small pressure differences need to magnify R_m some way.



$$P_a - P_b = gR_1(\rho_a - \rho_b)\sin\alpha$$

Compressible fluid

- Gases are compressible i.e. their density varies with temperature and pressure $\rho = P\,M/RT$
  - For small elevation changes (as in engineering applications, tanks, pipes etc) we can neglect the effect of elevation on pressure
  - In the general case start from:

$$\frac{dP}{dz} = -\rho g$$

$$for \;\; \mathrm{T} = T_o = \mathrm{const} \;\; :$$

$$P_2 = P_1 \exp\left[-\frac{g\;M\;(z_2 - z_1)}{RT_o}\right]$$

Compressible

Linear Temperature Gradient

$$T = T_0 - \alpha(z - z_0)$$

$$\int_{p_0}^{p} \frac{dp}{p} = -\frac{g}{R} \int_{z_0}^{z} \frac{dz}{T_0 - \alpha(z - z_0)}$$

$$p(z) = p_0 \left[\frac{T_0 - \alpha(z - z_0)}{T_0} \right]^{g/\alpha R}$$

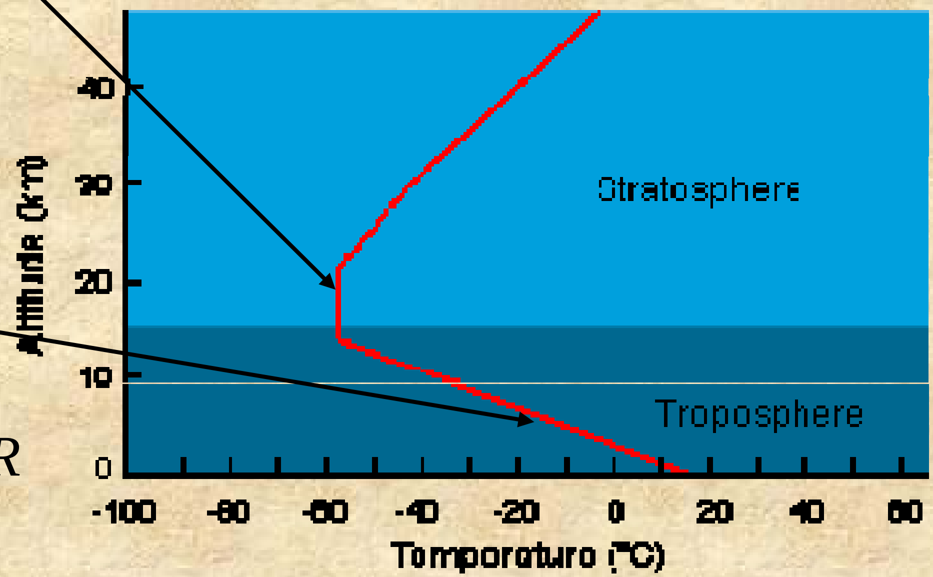
Atmospheric Equations

- Assume constant

$$p(z) = p_0 e^{-g(z-z_0)/RT_0}$$

- Assume linear

$$p(z) = p_0 \left[\frac{T_0 - \alpha(z - z_0)}{T_0} \right]^{g/\alpha R}$$



Temperature variation with altitude
for the U.S. standard atmosphere

Compressible Isentropic

$$\frac{P}{\rho^{\gamma}} = constant = \frac{P_1}{\rho_1^{\gamma}}$$

$$\frac{T}{T_1} = \left(\frac{P}{P_1}\right)^{\gamma-1/y}$$

$$\gamma = {C_p}/{C_v}$$

$$P_2 = P_1\left[1-\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{gM\Delta z}{RT_1}\right)\right]^{\gamma/\gamma-1}$$

$$T_2 = T_1\left[1-\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{gM\Delta z}{RT_1}\right)\right]$$

Application: bottom hole conditions in gas wells