Inverse Fourier Transform

Fourier Transform and Its Inverse

- Writing the exponential function in (4) as a product of exponential functions, we have

(5)
$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(v) e^{-iwv}\, dv \right] e^{iwx}\, dw.$$

The expression in brackets is a function of *w*, is denoted by $\hat{f}(w)$ and is called the **Fourier transform** of *f*; writing *v* = *x*, we have

(6)
$$\hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx}\, dx.$$

With this, (5) becomes

(7) $$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(w) e^{iwx} \, dw$$

and is called the **inverse Fourier transform** of $\hat{f}(w)$

- Another notation for the Fourier transform is

  so that $$\hat{f} = \mathscr{F}(f), \qquad f = \mathscr{F}^{-1}(\hat{f}).$$

- The process of obtaining the Fourier transform

  $\mathscr{F}(f) = \hat{f}$

  from a given f is also called the **Fourier transform** or the *Fourier transform method.*

Table of Fourier Transforms

	$f(x)$	$\hat{f}(\omega)$
1.	$\dfrac{1}{x^2+a^2} \quad (a>0)$	$\dfrac{\pi}{a} e^{-a\lvert\omega\rvert}$
2.	$H(x)e^{-ax} \quad (\operatorname{Re} a > 0)$	$\dfrac{1}{a+i\omega}$
3.	$H(-x)e^{ax} \quad (\operatorname{Re} a > 0)$	$\dfrac{1}{a-i\omega}$
4.	$e^{-a\lvert x\rvert} \quad (a>0)$	$\dfrac{2a}{\omega^2+a^2}$
5.	e^{-x^2}	$\sqrt{\pi}e^{-\omega^2/4}$
6.	$\dfrac{1}{2a\sqrt{\pi}} e^{-x^2/(2a)^2} \quad (a>0)$	$e^{-a^2\omega^2}$
7.	$\dfrac{1}{\sqrt{\lvert x\rvert}}$	$\sqrt{\dfrac{2\pi}{\lvert\omega\rvert}}$
8.	$e^{-a\lvert x\rvert/\sqrt{2}} \sin\left(\dfrac{a}{\sqrt{2}}\lvert x\rvert + \dfrac{\pi}{4}\right) \quad (a>0)$	$\dfrac{2a^3}{\omega^4+a^4}$
9.	$H(x+a) - H(x-a)$	$\dfrac{2\sin\omega a}{\omega}$