

# *Lagrange Interpolation*

# Lagrange Interpolation

- 1st-order Lagrange polynomial

$$f_1(x) = L_0 f(x_0) + L_1(x) f(x_1) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

- Second-order Lagrange polynomial

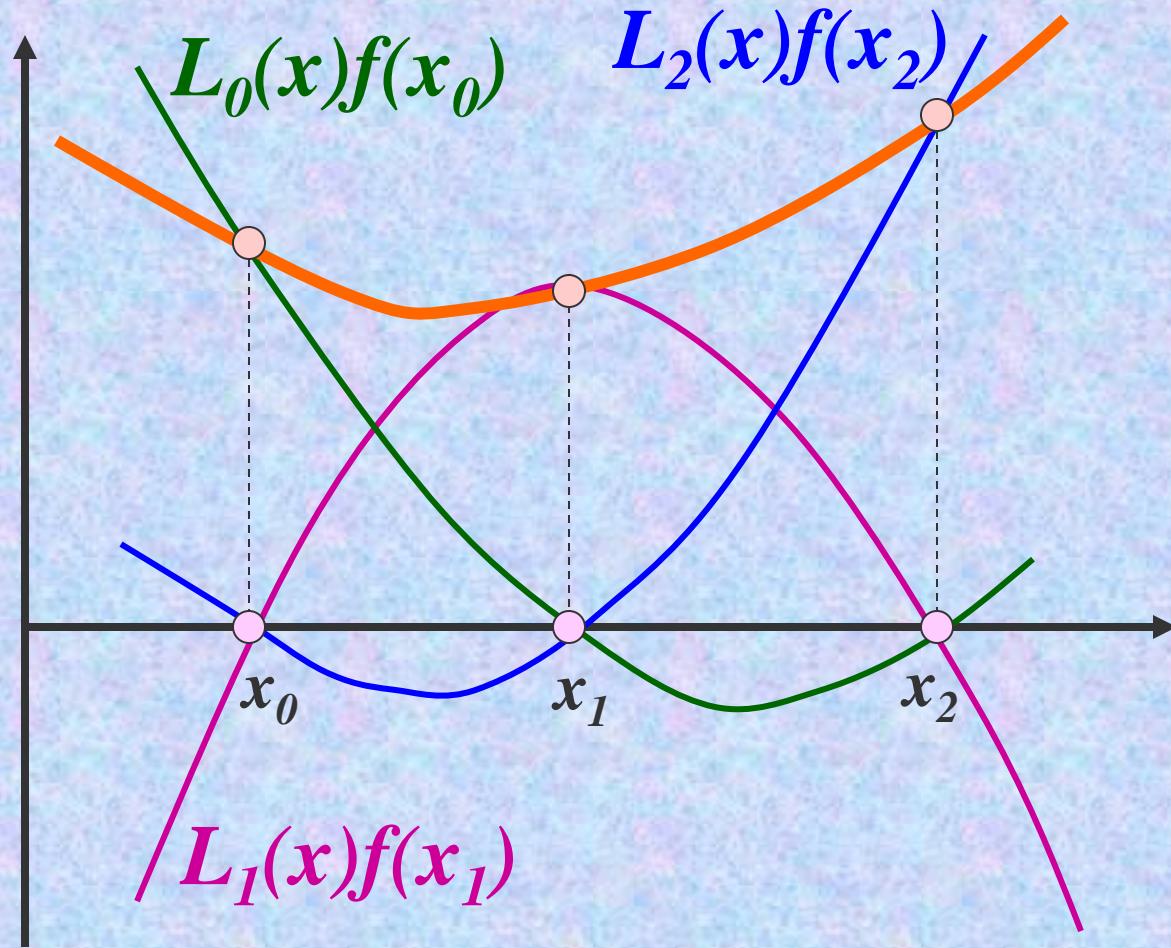
$$\begin{aligned} f_2(x) &= \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} f(x_0) \\ &\quad + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} f(x_1) \\ &\quad + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} f(x_2) \end{aligned}$$

# Lagrange Interpolation

- Third-order Lagrange polynomial

$$\begin{aligned}f_3(x) &= \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} f(x_0) \\&+ \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} f(x_1) \\&+ \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} f(x_2) \\&+ \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} f(x_3)\end{aligned}$$

# Lagrange Interpolation



# *General Three-Point Formula*

- Lagrange interpolation polynomial for unequally spaced data

$$\begin{aligned}f(x) &= L_{i-1}(x)f(x_{i-1}) + L_i(x)f(x_i) + L_{i+1}(x)f(x_{i+1}) \\&= f(x_{i-1}) \frac{(x - x_i)(x - x_{i+1})}{(x_{i-1} - x_i)(x_{i-1} - x_{i+1})} + f(x_i) \frac{(x - x_{i-1})(x - x_{i+1})}{(x_i - x_{i-1})(x_i - x_{i+1})} \\&\quad + f(x_{i+1}) \frac{(x - x_{i-1})(x - x_i)}{(x_{i+1} - x_{i-1})(x_{i+1} - x_i)}\end{aligned}$$

- First derivative

$$\begin{aligned}f'(x) &= f(x_{i-1}) \frac{2x - x_i - x_{i+1}}{(x_{i-1} - x_i)(x_{i-1} - x_{i+1})} + f(x_i) \frac{2x - x_{i-1} - x_{i+1}}{(x_i - x_{i-1})(x_i - x_{i+1})} \\&\quad + f(x_{i+1}) \frac{2x - x_{i-1} - x_i}{(x_{i+1} - x_{i-1})(x_{i+1} - x_i)}\end{aligned}$$

# *Second Derivative*

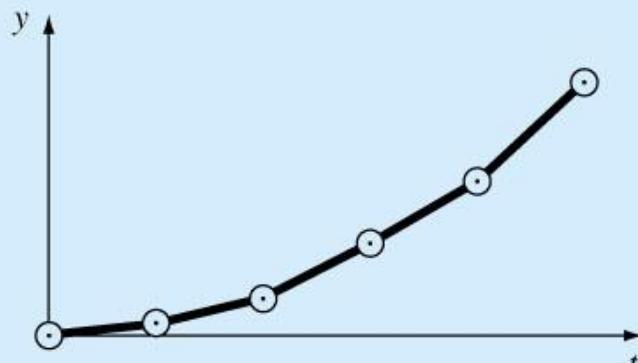
- First Derivative for unequally spaced data

$$f'(x) = f(x_{i-1}) \frac{2x - x_i - x_{i+1}}{(x_{i-1} - x_i)(x_{i-1} - x_{i+1})} + f(x_i) \frac{2x - x_{i-1} - x_{i+1}}{(x_i - x_{i-1})(x_i - x_{i+1})}$$
$$+ f(x_{i+1}) \frac{2x - x_{i-1} - x_i}{(x_{i+1} - x_{i-1})(x_{i+1} - x_i)}$$

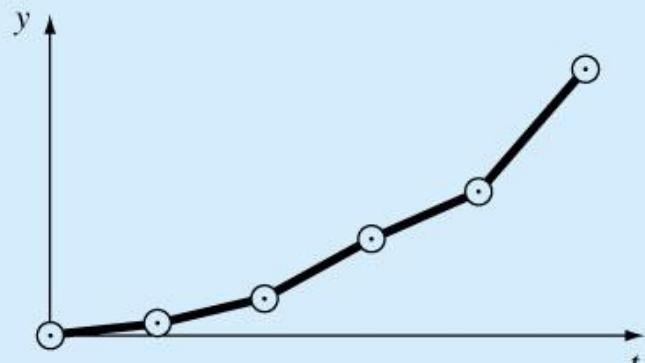
- Second Derivative for unequally spaced data

$$f''(x) = f(x_{i-1}) \frac{2}{(x_{i-1} - x_i)(x_{i-1} - x_{i+1})} + f(x_i) \frac{2}{(x_i - x_{i-1})(x_i - x_{i+1})}$$
$$+ f(x_{i+1}) \frac{2}{(x_{i+1} - x_{i-1})(x_{i+1} - x_i)}$$

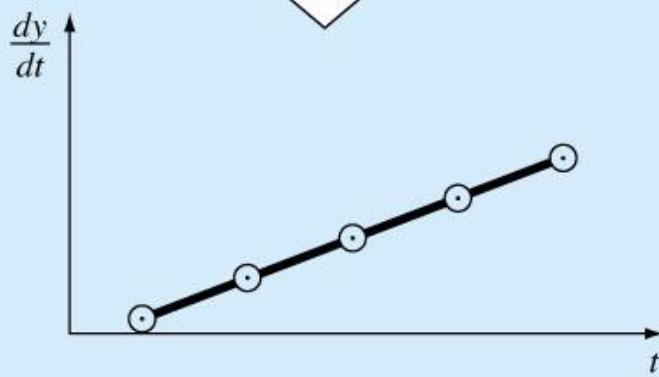
# *Differentiation of Noisy Data*



(a)



(b)



(c)



(d)

Differentiate