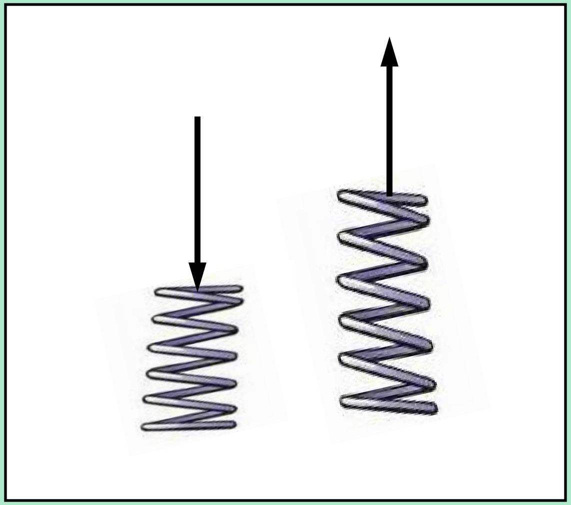
Stress and Strain

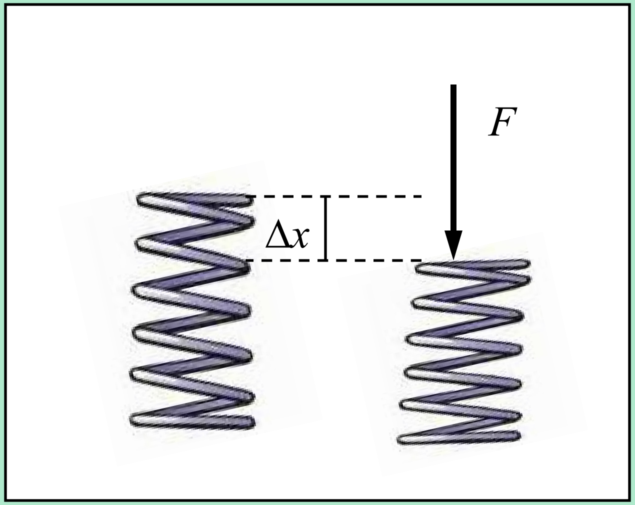
Solid Deformation

- Solids deform when they are subject to forces.
  - Compressed, stretched, bent, twisted
  - They can maintain or lose their shape



- The ratio of the force to the displacement is a constant.
  - Hooke's Law

$$F \propto x \qquad F = k\Delta x$$



Stress

- A force on a solid acts on an area.

- For compression or tension, the *normal stress* σ is the ratio of the force to the cross sectional area.
  - Measures pressure
  - SI unit pascal
  - Pa = N / m^2 = kg / m s^2

$$\sigma = \frac{F}{A}$$

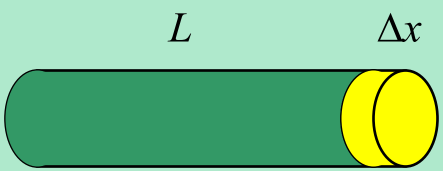
Stiletto Heels

- A 60 kg woman takes off a shoe with an area of 130 cm^2 and puts on a spiked heel with an area of 0.8 cm^2.
- What stress is placed on the floor?

- The force is due to weight, about 600 N.
- The stress with regular shoes is 600 N / 0.013 m^2 = 46 kN/m^2.
- The stress with spiked heels is 600 N/0.00008 m^2 = 7.5 MN/m^2.
- For comparison that is an increase of from about 7 psi to over 1000 psi!

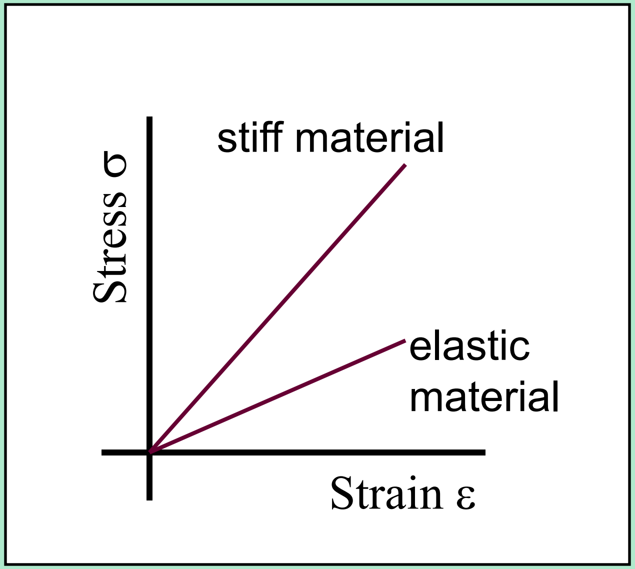
Strain

- Deformation is relative to the size of an object.
- The displacement compared to the length is the *strain* ε.
  - Measures a fractional change
  - Unitless quantity



$$\varepsilon = \frac{\Delta x}{L}$$

Young's Modulus

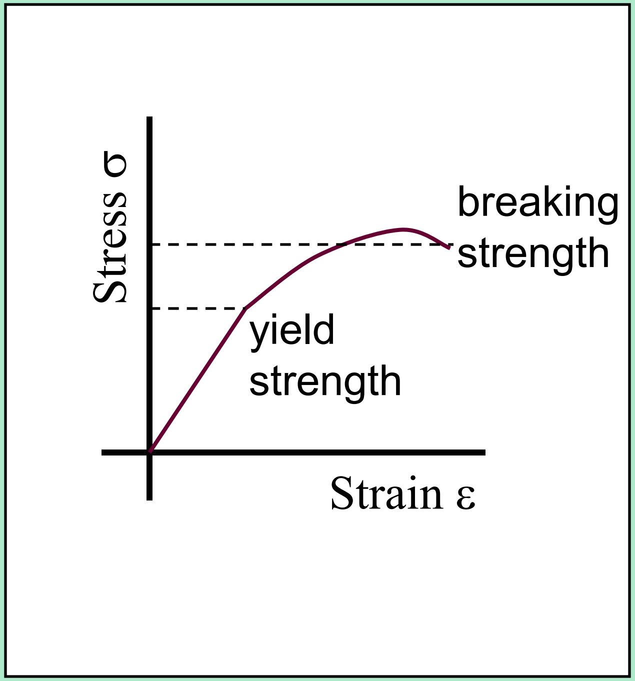


$$Y = \frac{\sigma}{\varepsilon} = \frac{F / A}{\Delta x / L} = \frac{FL}{A\Delta x}$$

$$F = (\frac{YA}{L})\Delta x = k\Delta x$$

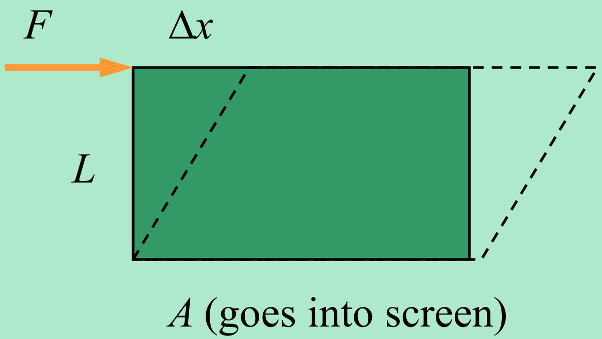
- A graph of stress versus strain is linear for small stresses.
- The slope of stress versus strain is a *modulus* that depends on the type of material.
- For normal stress this is Young's modulus *Y*.
  - Connects to Hooke's law

Inelastic Material



- The linear behavior of materials only lasts up to a certain strength – the *yield strength*.
- Materials can continue to deform but they won't restore their shape.
- For very high strain a material will break.

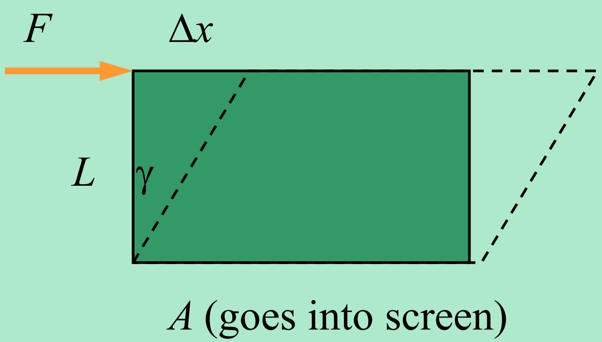
Shear Force



- Displacement of a solid can follow the surface of a solid.
- This is due a shear force.
- One can measure a shear stress σ_s and a shear strain ε_s.

$$\sigma_s = \frac{F}{A} \qquad \varepsilon_s = \frac{\Delta x}{L}$$

Shear Modulus



$$S = \frac{\sigma_s}{\varepsilon_s} = \frac{F/A}{\Delta x/L} = \frac{FL}{A\Delta x}$$

$$F = (\frac{SA}{L})\Delta x = k\Delta x$$

- Materials also have a modulus from shear forces.
- Shear modulus *S* also relates to Hooke's law.
- The angle $\gamma = \Delta x/L$ is sometimes used for shear.
  - Sine approximates angle

Twist a Leg

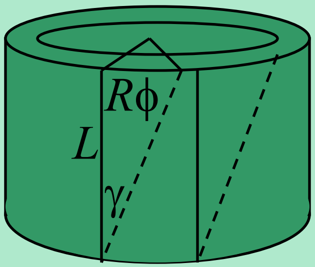
- One common fracture is a torsion fracture. A torque is applied to a bone causing a break.

  - The shear modulus of bone is 3.5 GPa.
  - The lower leg has a breaking angle of 3°.
  - It requires 100 Nm of torque.

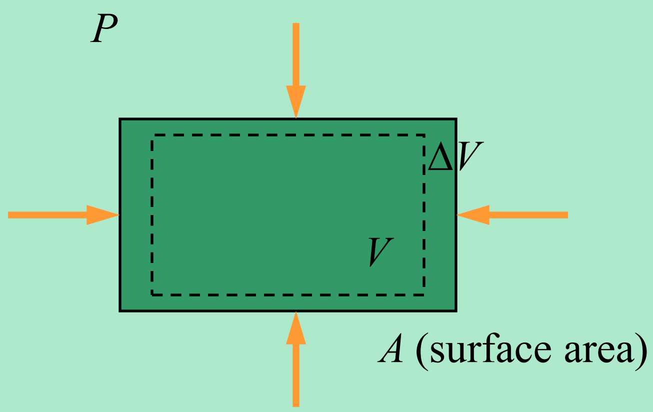
- Torque and angle apply.
  - Angle is $\Delta x/L = \tan\gamma$
  - Approximately $R\phi/L = \gamma$
- Sheer is related to torque.

$$S = \frac{\sigma_s}{\varepsilon_s} = \frac{FL}{A\Delta x} = \frac{F}{A\gamma}$$

$$S \propto \frac{\tau}{\gamma}$$



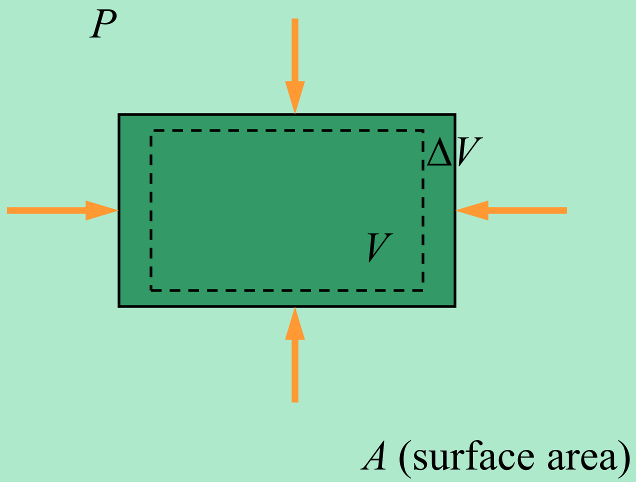
Volume Stress



- Pressure from all sides can change the volume of a solid.
- There is a volume stress and volume strain.

$$\sigma_V = \frac{F}{A} = P \qquad \varepsilon_V = \frac{\Delta V}{V}$$

Bulk Modulus

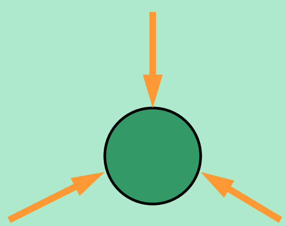


$$B = -\frac{\sigma_V}{\varepsilon_V} = -\frac{F / A}{\Delta V / V} = -\frac{\Delta P}{\Delta V / V}$$

- Pressure changes volume, not length.
- Bulk modulus *B* relates changes in pressure and volume.
- The negative sign represents the decrease in volume with increasing pressure.

Under Pressure

- Steel has a bulk modulus of B = 60 GPa. A sphere with a volume of 0.50 m^3 is constructed and lowered into the ocean where P = 20 MPa.
- How much does the volume change?



- Use the relation for bulk modulus.
  - $B = -(\Delta P) / (\Delta V/V)$
  - $\Delta V = -V \, \Delta P / B$
- Substitute values:
  - $(-0.50\ m^3)(2.0 \times 10^7\ Pa) / (6.0 \times 10^{10}\ Pa)$
  - $\Delta V = -1.6 \times 10^4\ m^3$

next