BEAMS

Unit 2

Beam Shear

- Shear and Moment Diagrams
- Vertical shear: tendency for one part of a beam to move vertically with respect to an adjacent part

Beam Shear

- *Magnitude (V)* = sum of vertical forces on either side of the section
  - can be determined at any section along the length of the beam
- Upward forces (reactions) = positive
- Downward forces (loads) = negative

- **Vertical Shear** = reactions – loads
  (to the left of the section)

Beam Shear

- Why?
- necessary to know the maximum value of the shear
  - necessary to locate where the shear changes from positive to negative
    - where the shear passes through zero
- Use of shear diagrams give a graphical representation of vertical shear throughout the length of a beam

Beam Shear –

- Simple beam
  - Span = 20 feet
  - 2 concentrated loads
- Construct shear diagram

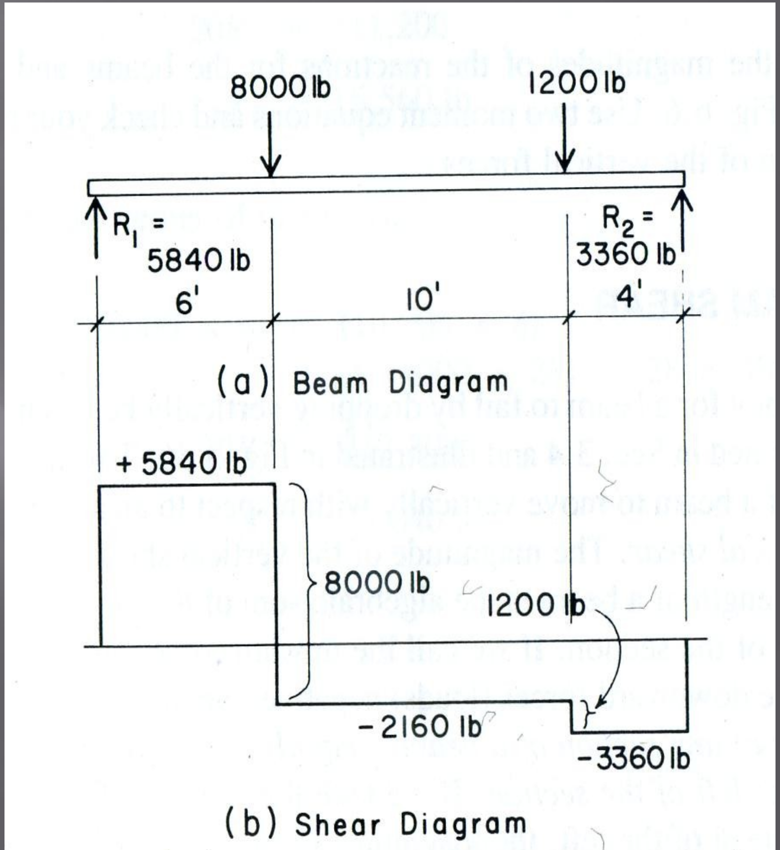


Figure 6.7

Beam Shear – Example 1

1) Determine the reactions

$$(1)(+\uparrow)\sum F_x = 0$$
$$(2)(+\uparrow)\sum F_y = 0 = R_1 - 8000^{lb.} - 1200^{lb.} + R_2$$
$$(3)(+\lrcorner)\sum M_1 = 0 = (8000^{lb.}\times 6') + (1200^{lb.}\times 16') - (R_2 \times 20')$$

Solving equation (3):

$$20' R_2 = 48{,}000^{lb.-ft.} + 19{,}200^{lb.-ft.}$$
$$R_2 = \frac{67{,}200^{lb.-ft.}}{20^{ft.}} = 3{,}360^{lb.}(\uparrow)$$

Solving equation (2):

$$R_1 = 8{,}000^{lb.} + 1{,}200^{lb.} - 3{,}360^{lb.}$$
$$R_1 = 5{,}840^{lb.}(\uparrow)$$

Figure 6.7a =>

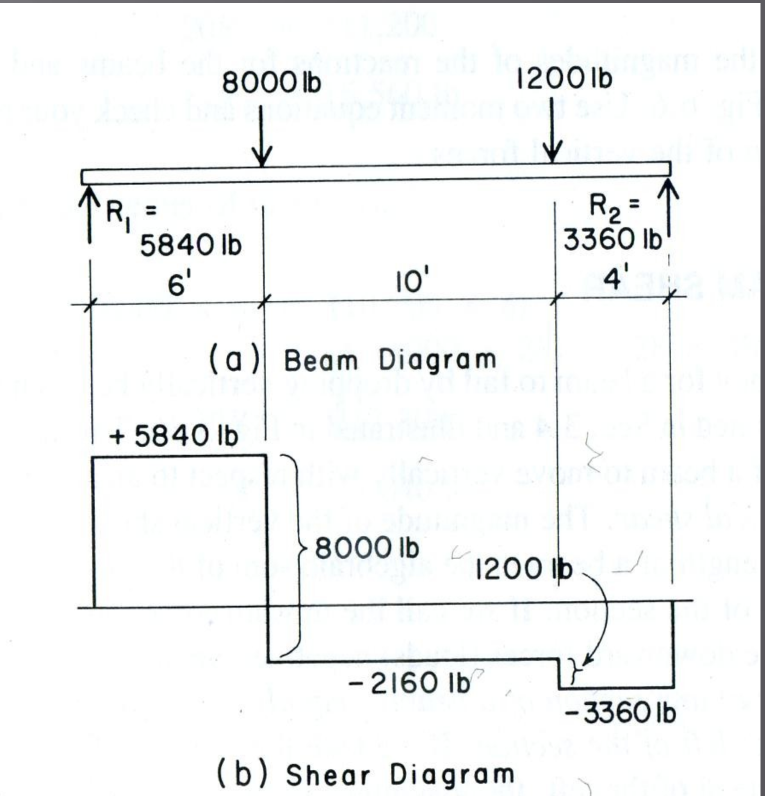


Figure 6.7

Beam Shear – Example 1 (pg. 64)

- Determine the shear at various points along the beam

$$V_{(x=1)} = 5{,}480 - 0 = +5{,}480^{lb.}$$

$$V_{(x=8)} = 5{,}480 - 8{,}000 = -2{,}160^{lb.}$$

$$V_{(x=18)} = 5{,}480 - 8{,}000 - 1{,}200 = -3{,}36$$

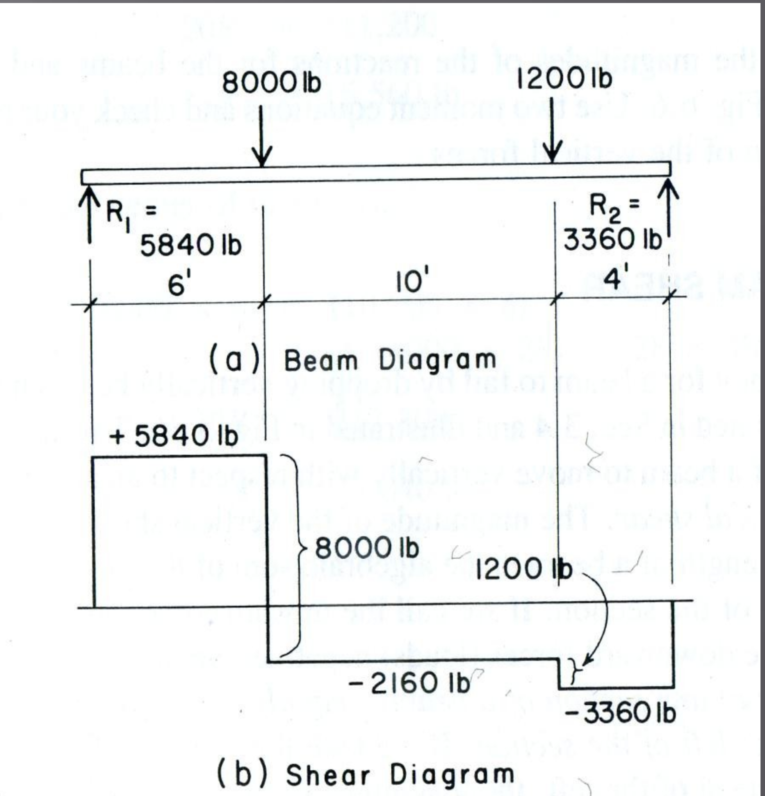


Figure 6.7

Beam Shear – Example 1

- Conclusions
  - max. vertical shear = 5,840 lb.
    - max. vertical shear occurs at greater reaction and equals the greater reaction (for simple spans)
  - shear changes sign under 8,000 lb. load
    - where max. bending occurs

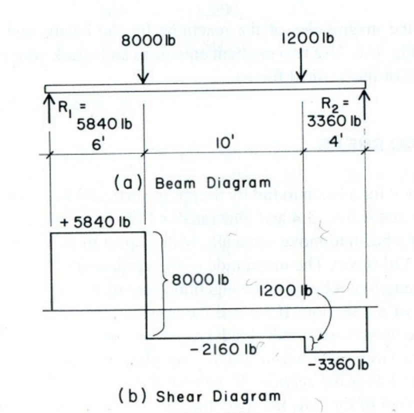


Figure 6.7

Beam Shear – Example 2

- Simple beam
  - Span = 20 feet
  - 1 concentrated load
  - 1 uniformly distr. load
- Construct shear diagram, designate maximum shear, locate where shear passes through zero

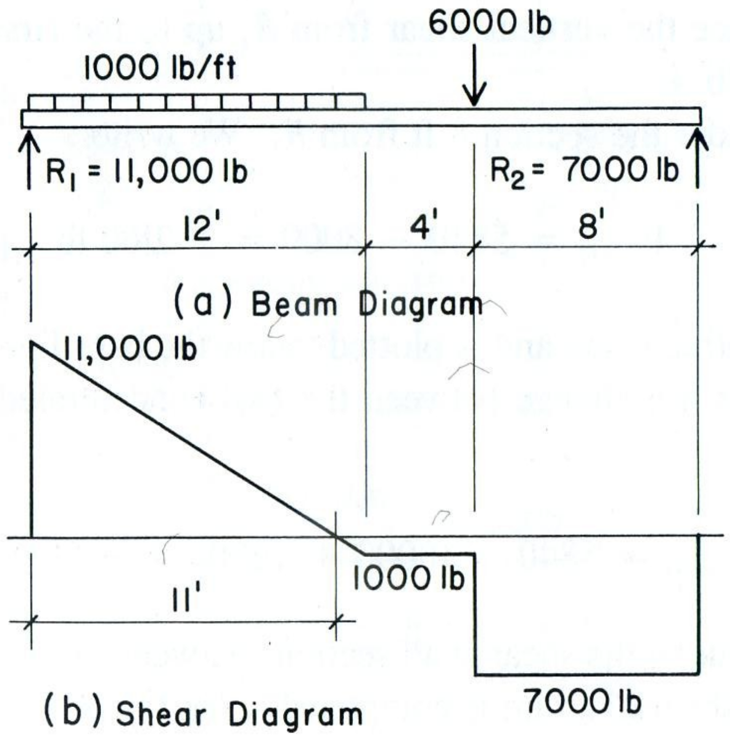


Figure 6.8

Beam Shear – Example 2

- Determine the reactions

$$(1)\left(+\uparrow\right)\sum F_x = 0$$

$$(2)\left(+\uparrow\right)\sum F_y = 0 = R_1 - (1{,}000^{lb/ft.} \times 12') - 6{,}000^{lb.} + R_2$$

$$(3)\left(+\lrcorner\right)\sum M_1 = 0 = [(1{,}000^{lb/ft.} \times 12')(6')] + (6000^{lb.} \times 16') - (R_2 \times 24')$$

Solving equation (3):

$$24' R_2 = 72{,}000^{lb.-ft.} + 96{,}000^{lb.-ft.}$$

$$R_2 = \frac{168{,}000^{lb.-ft.}}{24^{ft.}} = 7{,}000^{lb.}(\uparrow)$$

Solving equation (2):

$$R_1 = 12{,}000^{lb.} + 6{,}000^{lb.} - 7{,}000^{lb.}$$

$$R_1 = 11{,}000^{lb.}(\uparrow)$$

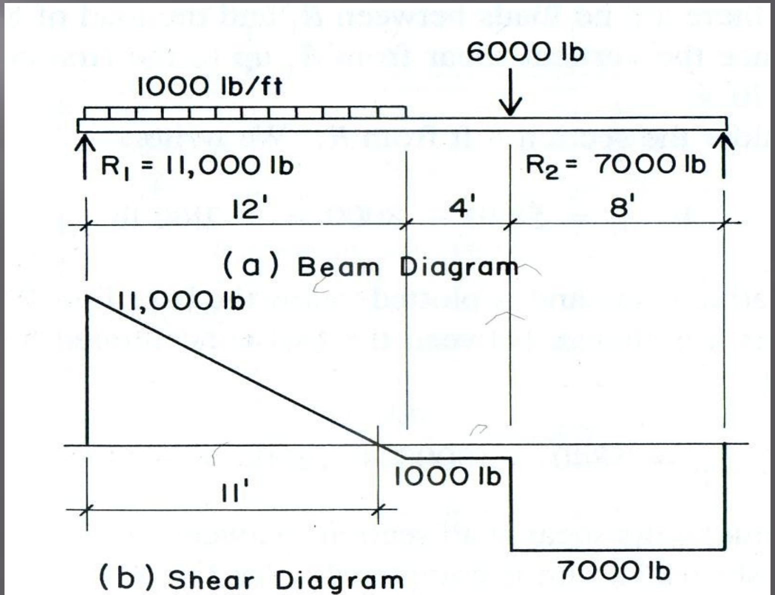


Figure 6.8

SHEAR AND MOMENT DIAGRAMS

Beam Shear – Example 2

- Determine the shear at various points along the beam

$$V_{(x=1)} = 11{,}000 - (1 \times 1{,}000) = 10{,}000^{lb.}$$

$$V_{(x=2)} = 11{,}000 - (2 \times 1{,}000) = 9{,}000^{lb.}$$

$$V_{(x=12)} = 11{,}000 - (12 \times 1{,}000) = -1{,}000^{lb.}$$

$$V_{(x=16-)} = 11{,}000 - (12 \times 1{,}000) = -1{,}000^{lb.}$$

$$V_{(x=16+)} = 11{,}000 - \left[(12 \times 1{,}000) + 6{,}000\right] = -7{,}$$

$$V_{(x=24)} = 11{,}000 - \left[(12 \times 1{,}000) + 6{,}000\right] = -7{,}0$$

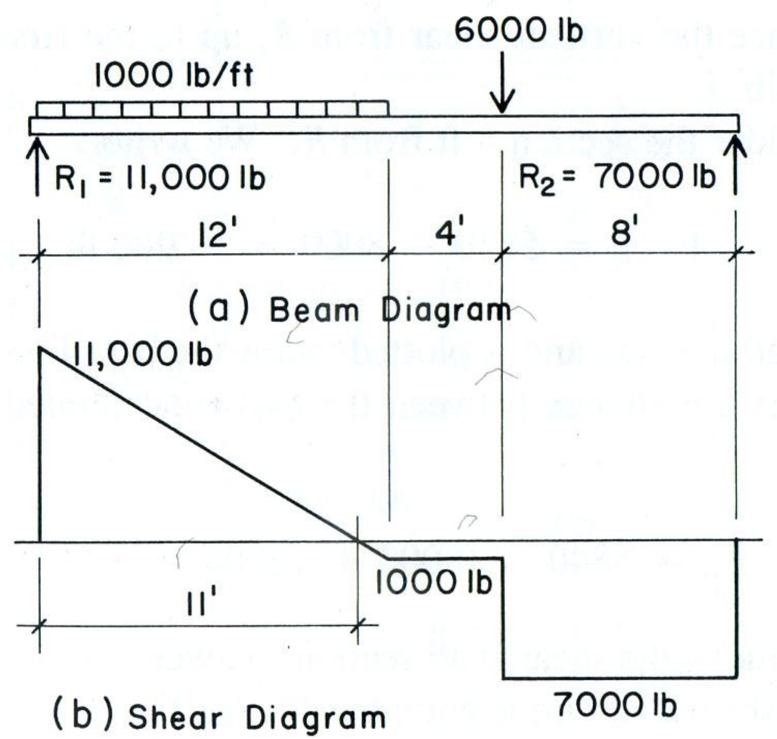


Figure 6.8

Beam Shear – Example 2

- Conclusions
  - max. vertical shear = 11,000 lb.
    - at left reaction
  - shear passes through zero at some point between the left end and the end of the distributed load
    - x = exact location from R_1
      - *at this location, V = 0*

$$V = 0 = 11{,}000 - (x \times 1{,}000)$$
$$x = 11\, feet$$

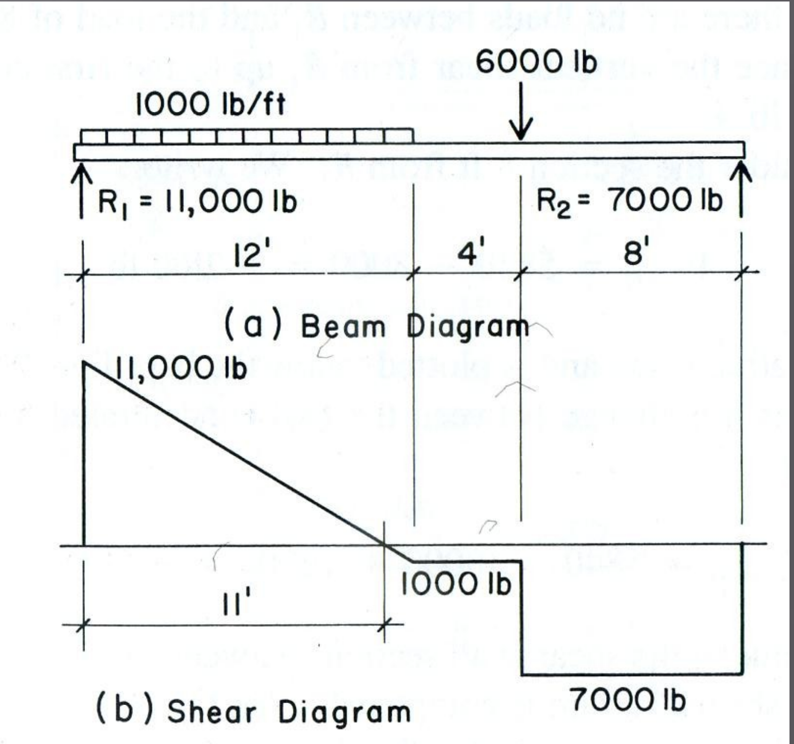


Figure 6.8

Beam Shear – Example 3

- Simple beam with overhanging ends
  - Span = 32 feet
  - 3 concentrated loads
  - 1 uniformly distr. load acting over the entire beam
- Construct shear diagram, designate maximum shear, locate where shear passes through zero

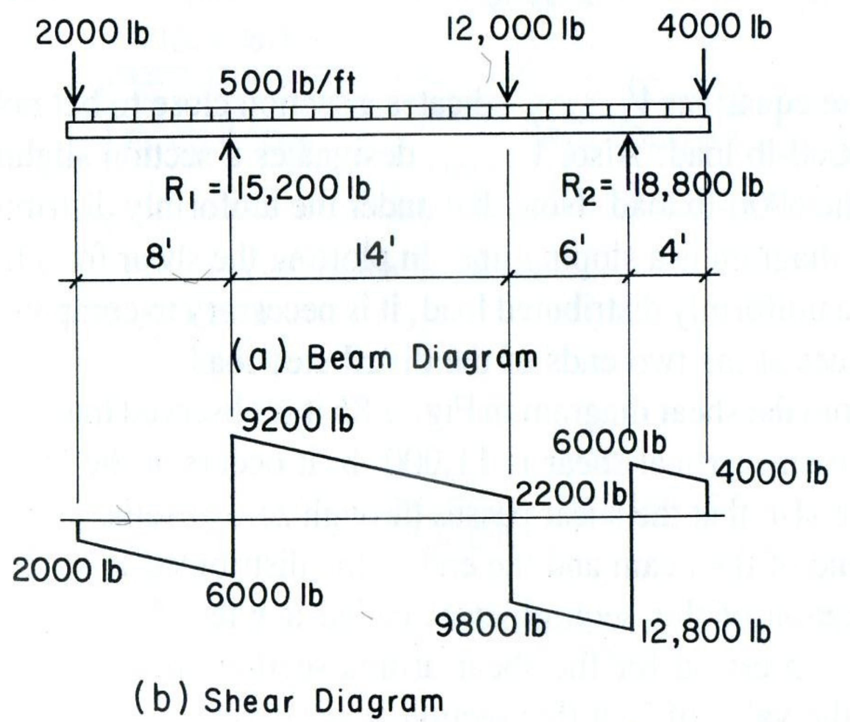


Figure 6.9

Beam Shear – Example 3

$$(1)\left(+\uparrow\right)\sum F_x = 0$$

$$(2)\left(+\uparrow\right)\sum F_y = 0 = -2{,}000^{lb.} + R_1 - 12{,}000^{lb.} - 4{,}000^{lb.} - (500^{lb/ft.} \times 32') + R_2$$

$$(3)\left(+\lrcorner\right)\sum M_1 = 0 = -(2{,}000^{lb.} \times 8') + (12{,}000^{lb.} \times 14') + (4{,}000^{lb.} \times 24') + \left[(500^{lb./ft.} \times 32')\left(\left(\tfrac{32}{2}\right) - 8\right)\right] - R_2(20')$$

$$(4)\left(+\lrcorner\right)\sum M_2 = 0 = (4{,}000^{lb.} \times 4') - (12{,}000^{lb.} \times 6') - (2{,}000^{lb.} \times 28') - \left[(500^{lb./ft.} \times 32')\left(\left(\tfrac{32}{2}\right) - 4\right)\right] + R_1(20')$$

$$20'R_2 = -16{,}000^{lb.-ft.} + 168{,}000^{lb.-ft.} + 96{,}000^{lb.-ft.} + 128{,}000^{lb.-ft.}$$

$$R_2 = \frac{376{,}000^{lb.-ft.}}{20^{ft.}} = 18{,}800^{lb.}(\uparrow)$$

$$20'R_1 = -16{,}000^{lb.-ft.} + 72{,}000^{lb.-ft.} + 56{,}000^{lb.-ft.} + 192{,}000^{lb.-ft.}$$

$$R_1 = \frac{304{,}000^{lb.-ft.}}{20'} = 15{,}200^{lb.}(\uparrow)$$

Beam Shear – Example 3

- Determine the shear at various points along the beam

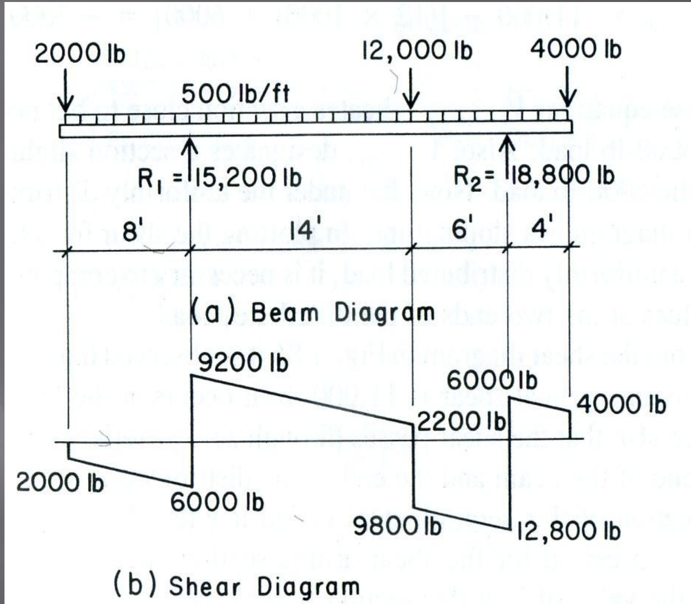


Figure 6.9

$$V_{(x=8-)} = 0 - 2{,}000^{lb.} - \left(500^{lb./ft.} \times 8'\right) = -6{,}000^{lb.}$$

$$V_{(x=8+)} = 15{,}200^{lb.} - 2{,}000^{lb.} - \left(500^{lb./ft.} \times 8'\right) = 9{,}200^{lb.}$$

$$V_{(x=22-)} = 15{,}200^{lb.} - 2{,}000^{lb.} - \left(500^{lb./ft.} \times 22'\right) = 2{,}200^{lb.}$$

$$V_{(x=22+)} = 15{,}200^{lb.} - 2{,}000^{lb.} - 12{,}000^{lb.} - \left(500^{lb./ft.} \times 22'\right) = -9{,}800^{lb.}$$

$$V_{(x=28-)} = 15{,}200^{lb.} - 2{,}000^{lb.} - 12{,}000^{lb.} - \left(500^{lb./ft.} \times 28'\right) = -12{,}800^{lb.}$$

$$V_{(x=28+)} = 15{,}200^{lb.} + 18{,}800^{lb.} - 2{,}000^{lb.} - 12{,}000^{lb.} - \left(500^{lb./ft.} \times 28'\right) = 6{,}000^{lb.}$$

$$V_{(x=32-)} = 15{,}200^{lb.} + 18{,}800^{lb.} - 2{,}000^{lb.} - 12{,}000^{lb.} - \left(500^{lb./ft.} \times 32'\right) = 4{,}000^{lb.}$$

Beam Shear – Example 3

- Conclusions
  - max. vertical shear = 12,800 lb.
    - disregard +/- notations
  - shear passes through zero at three points
    - R_1, R_2, and under the 12,000lb. load

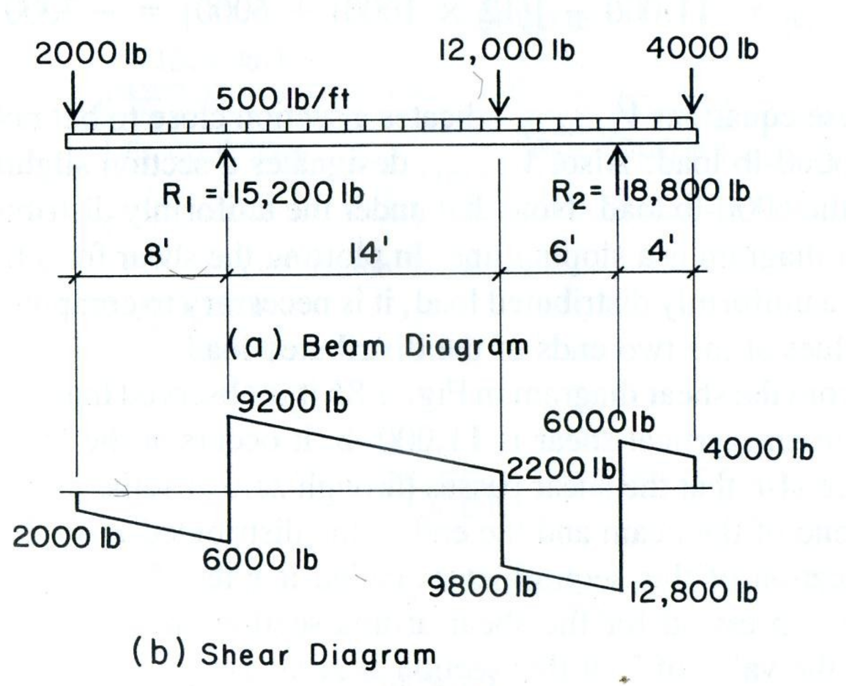


Figure 6.9

Bending Moment

- Bending moment: tendency of a beam to bend due to forces acting on it

- *Magnitude (M)* = sum of moments of forces on either side of the section
  - can be determined at any section along the length of the beam

- **Bending Moment** = moments of reactions – moments of loads
  - (to the left of the section)

Bending Moment

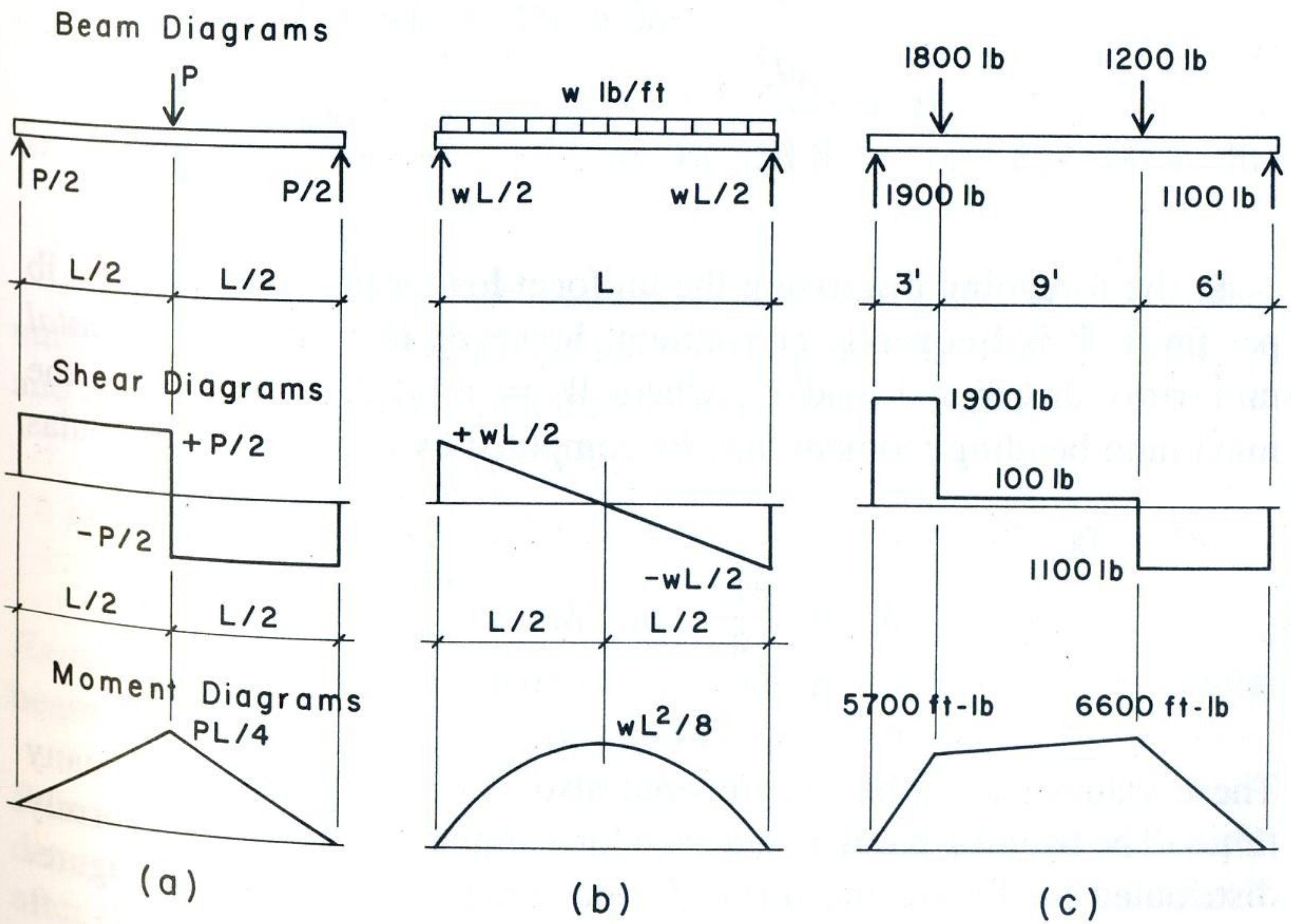


Figure 6.11 Diagrams of beam functions.

Bending Moment – Example 1

- Simple beam
  - span = 20 feet
  - 2 concentrated loads
  - shear diagram from earlier
- Construct moment diagram

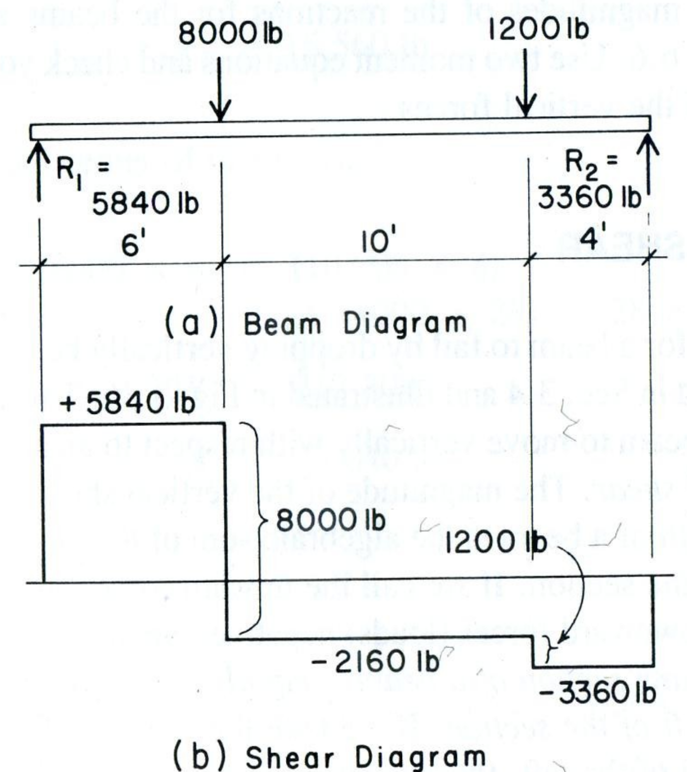


Figure 6.7

Bending Moment – Example 1

1) Compute moments at critical locations
   - under 8,000 lb. load & 1,200 lb. load

$$(+\circlearrowleft)M_{(x=6')} = (5{,}840^{lb.} \times 6') - 0 = 35{,}040^{lb.-ft.}$$

$$(+\circlearrowleft)M_{(x=16')} = (5{,}840^{lb.} \times 16') - (8{,}000^{lb.} \times 10') = 13{,}440^{lb.-ft.}$$

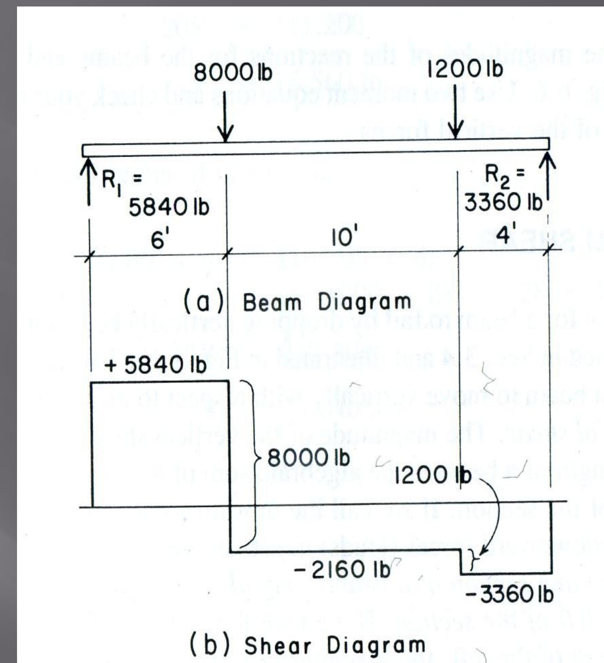
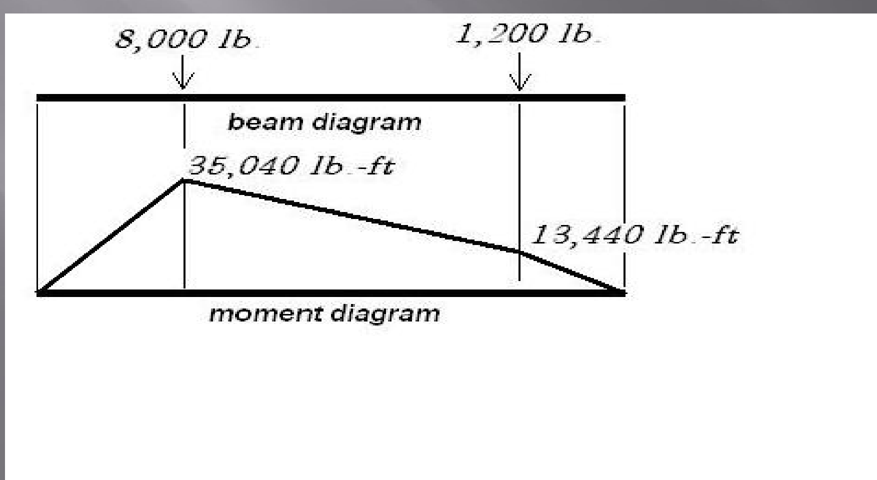


Figure 6.7



Bending Moment – Example 2

- Simple beam
  - Span = 20 feet
  - 1 concentrated load
  - 1 uniformly distr. Load
  - Shear diagram
- Construct moment diagram

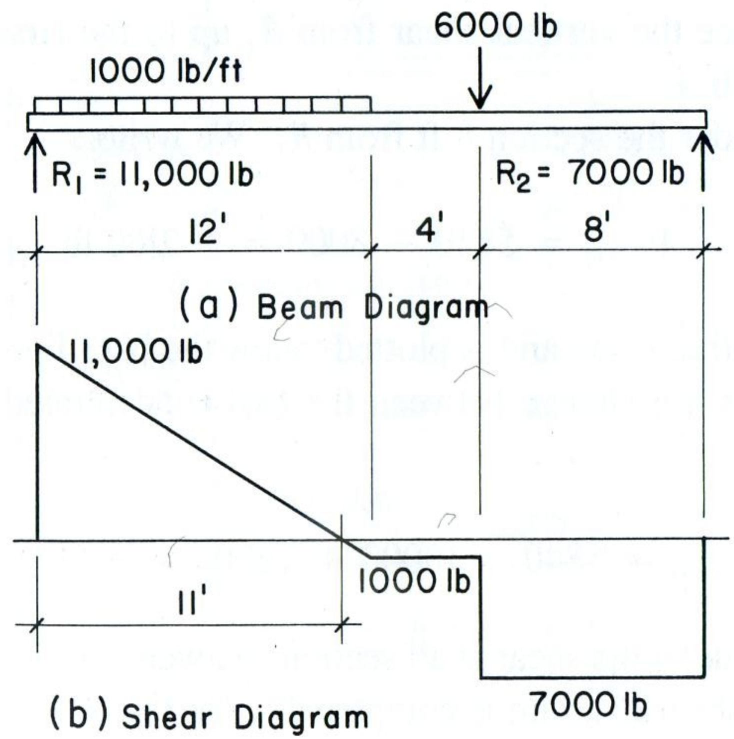


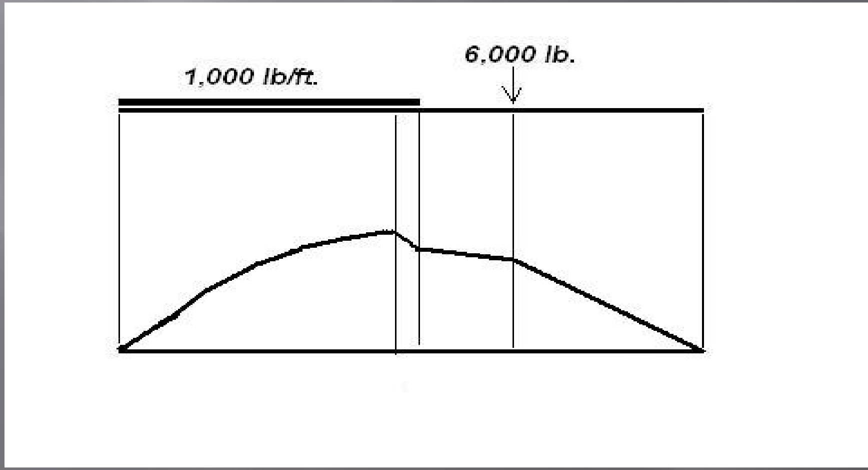
Figure 6.8

Bending Moment – Example 2

1) Compute moments at critical locations
   - When x = 11 ft. and under 6,000 lb. load

$$(+\circlearrowleft)M_{(x=11')} = (11{,}000^{lb.} \times 11') - \left[\left(1{,}000^{lb/ft} \times 11\right)\left(\tfrac{11}{2}\right)\right] = 60{,}500^{lb.-ft.}$$

$$(+\circlearrowleft)M_{(x=16')} = (11{,}000^{lb.} \times 16') - \left[\left(1{,}000^{lb/ft} \times 12\right)\left(\left(\tfrac{12}{2}\right) + 4\right)\right] = 56{,}000^{lb.-ft.}$$



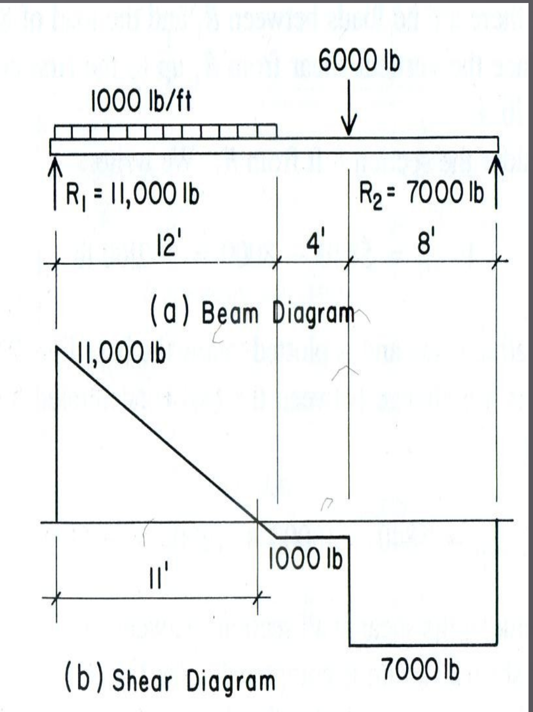


Figure 6.8

Negative Bending Moment

- Previously, simple beams subjected to positive bending moments only
  - moment diagrams on one side of the base line
    - concave upward (compression on top)
- Overhanging ends create negative moments
    - concave downward (compression on bottom)

Negative Bending Moment

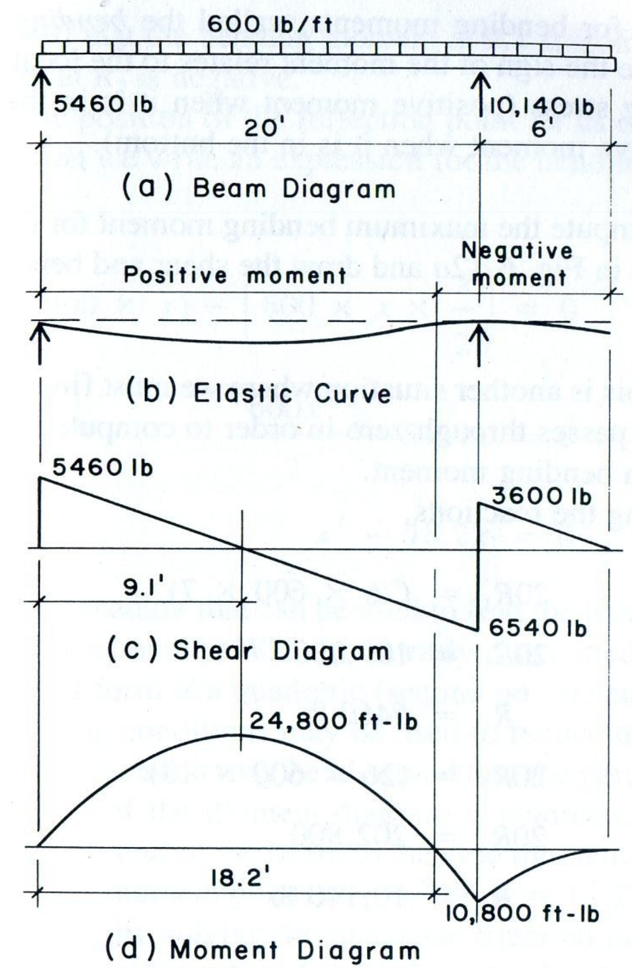


Figure 6.12 Diagrams for the overhanging beam.

- deflected shape has inflection point
  - bending moment = 0
- See example

Negative Bending Moment - Example

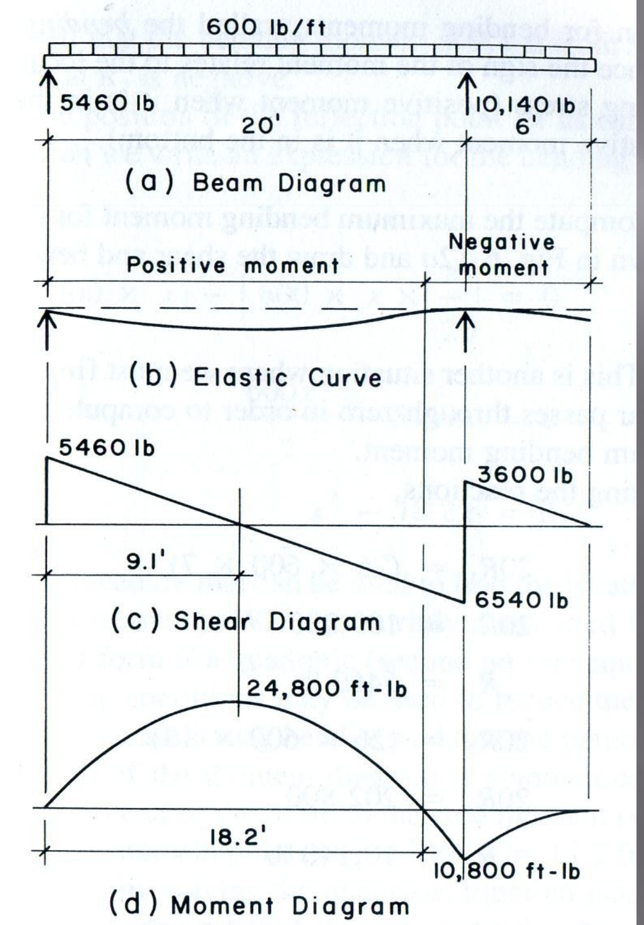


Figure 6.12 Diagrams for the overhanging beam.

- Simple beam with overhanging end on right side
  - Span = 20′
  - Overhang = 6′
  - Uniformly distributed load acting over entire span
- Construct the shear and moment diagram
- Figure 6.12

Negative Bending Moment - Example

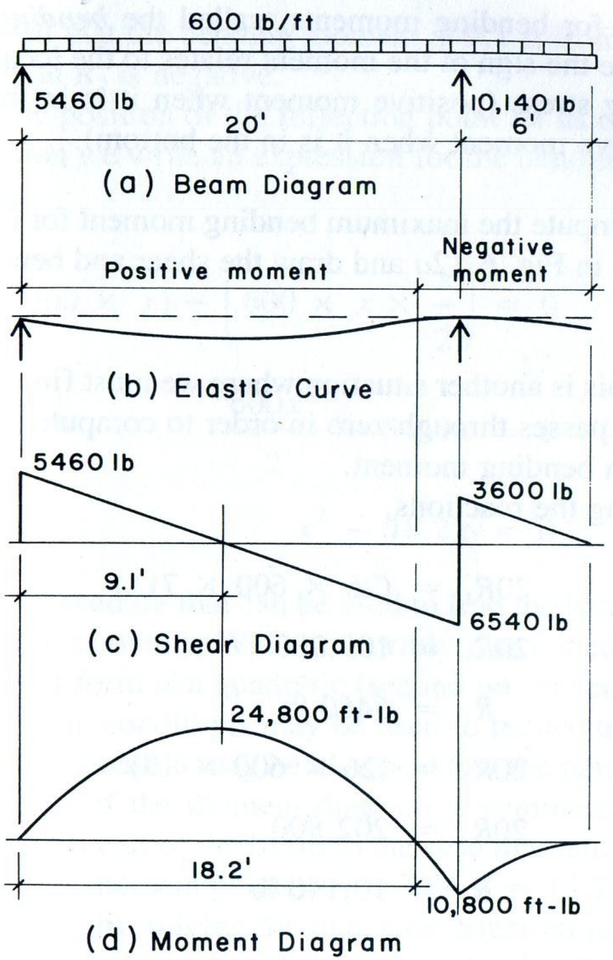


Figure 6.12 Diagrams for the overhanging beam.

1) Determine the reactions

$$(1)(+\uparrow)\sum F_x = 0$$

$$(2)(+\uparrow)\sum F_y = 0 = R_1 - (600^{lb/ft.} \times 26') + R_2$$

$$(3)(+\lrcorner)\sum M_1 = 0 = \left[(600^{lb/ft.} \times 26')(26/2)\right] - (R_2 \times 20')$$

- *Solving equation (3):*

$$20' R_2 = 202{,}800^{lb.-ft.}$$

$$R_2 = \frac{202{,}800^{lb.-ft.}}{20^{ft.}} = 10{,}140^{lb.}(\uparrow)$$

- *Solving equation (4):*

$$R_1 = 15{,}600^{lb.} - 10{,}140^{lb.}$$

$$R_1 = 5{,}460^{lb.}(\uparrow)$$

Negative Bending Moment - Example

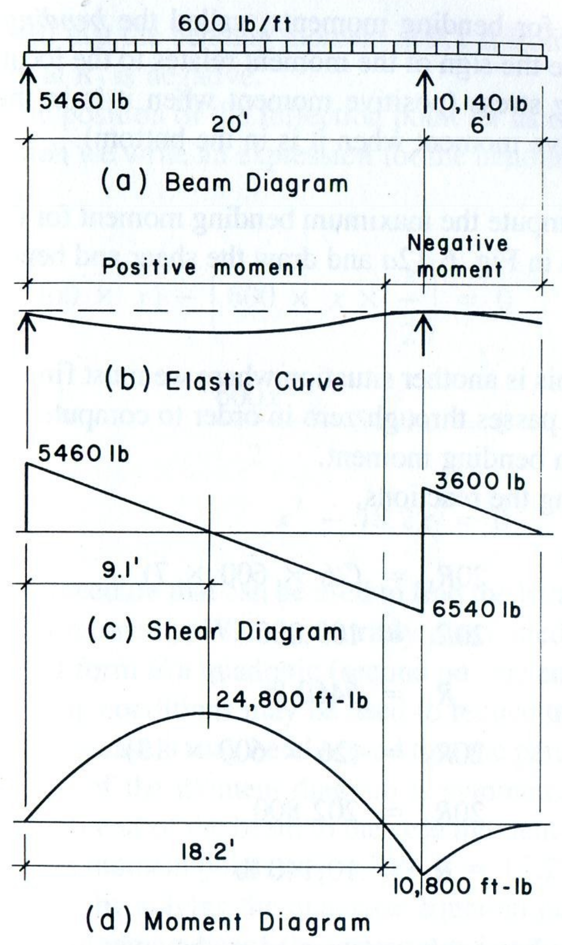


Figure 6.12 Diagrams for the overhanging beam.

2) Determine the shear at various points along the beam and draw the shear diagram

$$V_{(x=1)} = 5,460 - (1 \times 600) = 4,860^{lb.}$$

$$V_{(x=10)} = 5,460 - (10 \times 600) = -540^{lb.}$$

$$V_{(x=20-)} = 5,460 - (20 \times 600) = -6,540^{lb.}$$

$$V_{(x=20+)} = 5,460 + 10,140 - (20 \times 600) = 3,600^{lb.}$$

Negative Bending Moment - Example

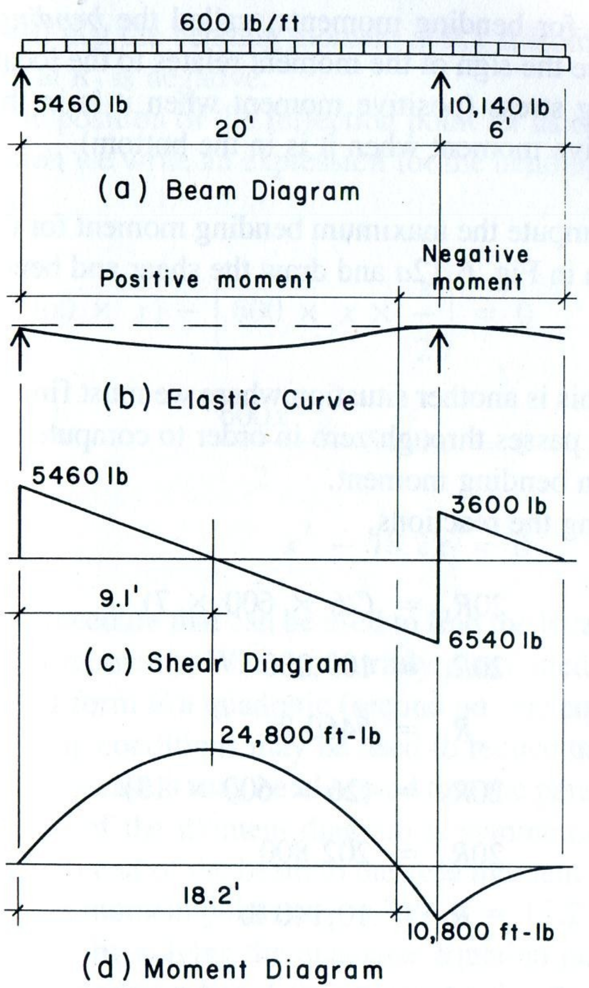


Figure 6.12 Diagrams for the overhanging beam.

3) Determine where the shear is at a maximum and where it crosses zero

- max shear occurs at the right reaction = 6,540 lb.

$$V = 0 = 5{,}460 - (x \times 600)$$

$$x = 9.1\,feet$$

Negative Bending Moment - Example

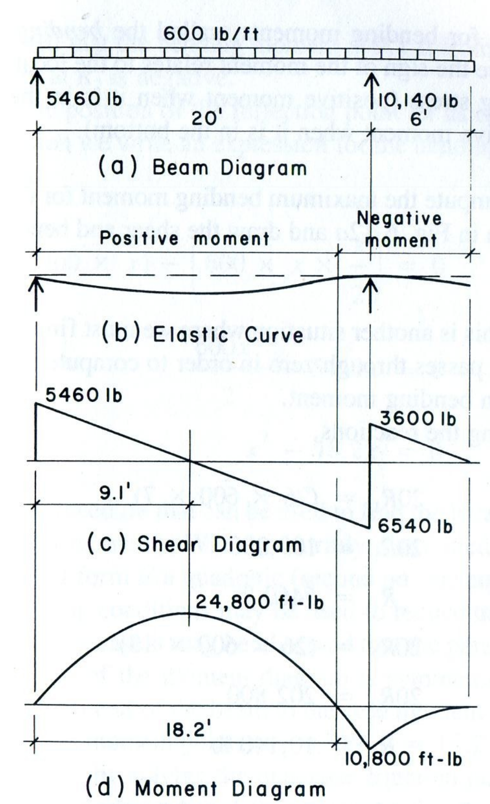


Figure 6.12 Diagrams for the overhanging beam.

4) Determine the moments that the critical shear points found in step 3) and draw the moment diagram

$$M_{(x=9.1)} = (5{,}460^{lb.} \times 9.1') - \left[\left(600^{lb/ft} \times 9.1'\right)\left(9.1/2\right)\right] = 24{,}843^{lb.-ft.}$$

$$M_{(x=20)} = (5{,}460^{lb.} \times 20') - \left[\left(600^{lb/ft} \times 20'\right)\left(20/2\right)\right] = -10{,}800^{lb.-ft.}$$

Negative Bending Moment - Example

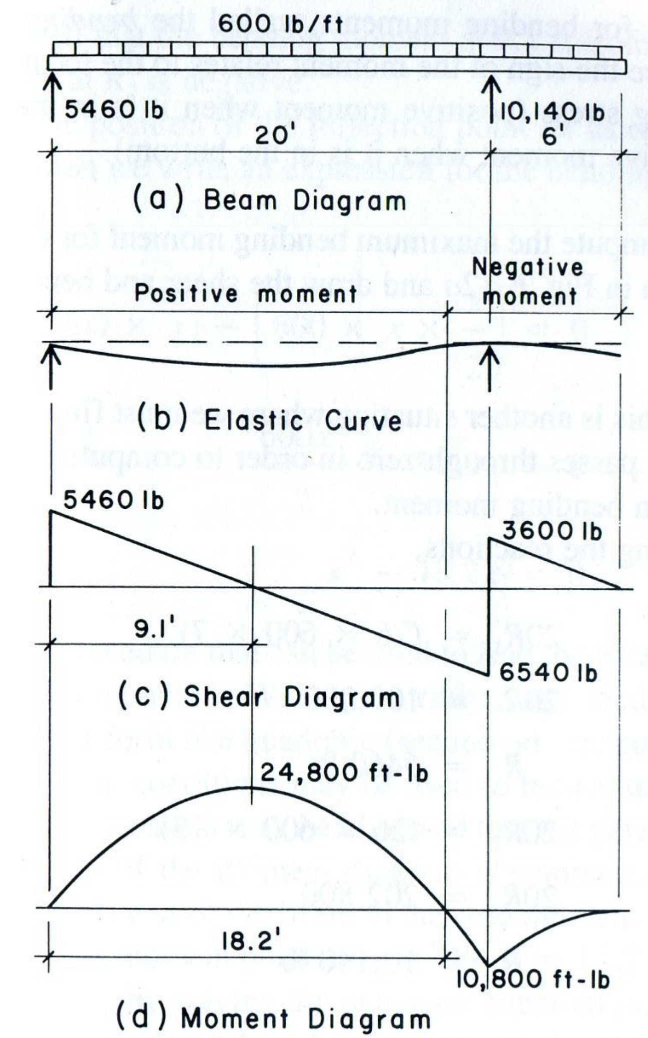


Figure 6.12 Diagrams for the overhanging beam.

4) Find the location of the inflection point (zero moment) and max. bending moment

$$M = 0 = (5{,}460 \times x) - \left[\left(600^{lb/ft} \times x\right)\left(x/2\right)\right]$$

$$M = 0 = 5{,}460x - 600\left(\frac{x^2}{2}\right)$$

$$M = 0 = 5{,}460x - 300x^2 = -300x^2 + 5{,}460x$$

$$x = \frac{-5{,}460 \pm \sqrt{(5{,}460)^2 - 4(-300)(0)}}{2(-300)}$$

$$x = \frac{-5460 \pm 5{,}460}{-600} = 0\,feet; 18.2\,feet$$

- since x cannot =0, then we use x=18.2′
- Max. bending moment =24,843 lb.-ft.

Rules of Thumb/Review

- shear is dependent on the loads and reactions
  - when a reaction occurs; the shear "jumps" up by the amount of the reaction
  - when a load occurs; the shear "jumps" down by the amount of the load
- point loads create straight lines on shear diagrams
- uniformly distributed loads create sloping lines of shear diagrams

Rules of Thumb/Review

- moment is dependent upon the shear diagram
  - the area under the shear diagram = change in the moment (i.e. $A_{\text{shear diagram}} = \Delta M$)
- straight lines on shear diagrams create sloping lines on moment diagrams
- sloping lines on shear diagrams create curves on moment diagrams
- positive shear = increasing slope
- negative shear = decreasing slope

Typical Loadings

- In beam design, only need to know:
  - reactions
  - max. shear
  - max. bending moment
  - max. deflection

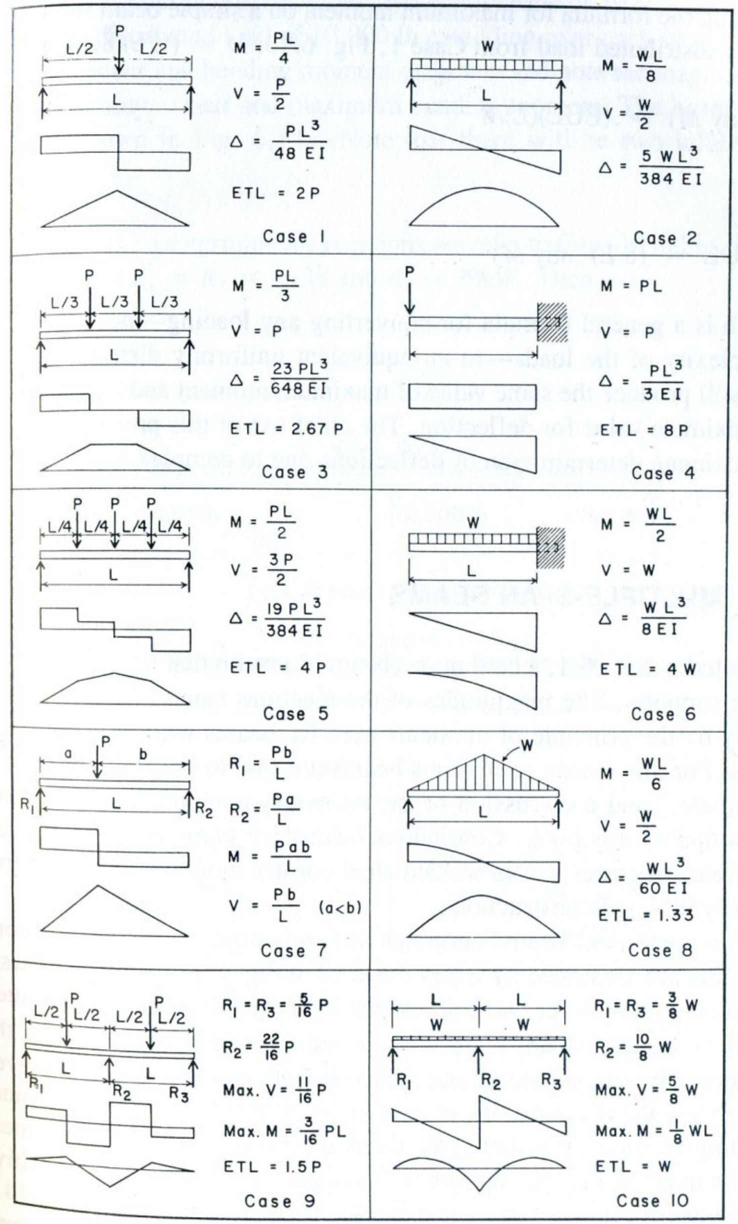


Figure 6.13 Values for typical beam loadings.