Thin and Thick Pressure Vessels

Unit 4

- Classify a pressure vessel as thin-walled or thick-walled
- Calculate the stress (hoop stress) in a sphere subject to an internal pressure
- Calculate the stress (hoop stress and longitudinal stress) in a cylinder subject to an internal pressure
- Determine the required wall thickness of a pressure vessel to safely resist a given internal pressure

Chapter Objectives

Thin-walled Pressure Vessels

- Terminology:
  - R_i , R_o, R_m, - Inside, outside and mean (average) radii
  - D_i , D_o, D_m, - Inside, outside and mean (average) diameters
  - t – wall thickness
- If $R_m / t \geq 10$, pressure vessel is considered thin-walled
  - In terms of the diameter; $D_m / t \geq 20$
- Derivation of formulas for stresses in thin-walled pressure vessels are based on the assumption that the stresses are constant throughout the wall of the vessel
- If $R_m / t \geq 10$, pressure vessel is considered thick-walled
  - Stresses in thick-walled pressure vessels are not constant throughout the wall of the vessel

Thin-walled Spheres

- Internal pressure in sphere acts perpendicular to the surface
  - Uniform over the interior surface
- Cut Free-Body Diagram through center of sphere
- Internal forces in walls appear on FBD
  - Since FBD was cut through center of sphere, these forces are horizontal
- For vertical equilibrium: $\sum F_Y = 0$
  - Vertical components of internal pressure are equal and opposite
  - Vertical components in opposite directions cancel each other
- For horizontal equilibrium: $\sum F_X = 0$
  - Internal force in wall must equal the resultant horizontal force due to internal pressure

Thin-walled Spheres

- Considering horizontal components of internal pressure
  - Resultant force $F_R = p\,A_P$
    - A_P = projected area of sphere on plane cut through the diameter = $\pi\,D_m^2 / 4$
  - Since $\sum F_X = 0$, internal force in wall = F_R
- Stress in wall: $\sigma = F/A = F_R / A_W$
  - A_W = Area of sphere wall
  - $A_W = \pi\,D_o^2 / 4 - \pi\,D_i^2 / 4 = \pi\,(D_o^2 - D_i^2) / 4$
- For a thin-walled sphere
  - $A_W \approx \pi\,D_m\,t$ --> the area of a strip of thickness = t and length = average circumference ($\pi\,D_m$)
- Stress in wall of sphere
  - $\boldsymbol{\sigma} = F_R / A_W = p\,A_P / A_W = p\,(\pi\,D_m^2 / 4) / \pi\,D_m\,t = \mathbf{p\,D_m / 4\,t}$

Thin-walled Cylinders

- Cylinders used as pressure vessels and for piping of fluids under pressure
- Two types of stresses
  - Longitudinal stress – along the long axis of the cylinder
  - Hoop stress (tangential stress) – around the circumference of the cylinder

- Longitudinal Stress
- Cut Free-Body Diagram through cylinder, perpendicular to longitudinal axis
- Longitudinal internal forces in walls appear on FBD
  - Forces are horizontal
- For horizontal equilibrium: $\sum F_X = 0$
  - Internal force in wall must equal the resultant horizontal force due to internal pressure

Longitudinal Stress in Thin-walled Cylinders

Longitudinal Stress in Thin-walled Cylinders

- If end of cylinder is closed, resultant force $F_R = p\ A = p\ \pi\ D_m^2 / 4$
  - Since $\sum F_X = 0$, longitudinal internal force in wall = F_R
- Stress in wall: $\sigma = F/A = F_R / A_W$
  - A_W = Area of sphere wall
  - $A_W = \pi\ D_o^2 / 4 - \pi\ D_i^2 / 4 = \pi\ (D_o^2 - D_i^2) / 4$
- For a thin-walled cylinder
  - $A_W \approx \pi\ D_m\ t$,
  - The area of a strip of thickness = t and length = average circumference ($\pi\ D_m$)
- Longitudinal stress in wall of cylinder
  - $\boldsymbol{\sigma} = F_R / A_W = p\ A_P / A_W = p\ (\pi\ D_m^2 / 4) / \pi\ D_m\ t = \mathbf{p\ D_m / 4\ t}$
  - Longitudinal stress is <u>same</u> as stress in a sphere

- Isolate a ring of length L from the cylinder
  - Cut a vertical section through ring, passing through its center
  - Draw a FBD of segment either side of section
- Similar to analysis of sphere, resultant force $F_R = p\ A_P$
  - A_P = projected area of ring = $D_m\ L$
- Stress in wall: $\sigma = F/A = F_R / A_W$
  - A_W = Cross-sectional area of cylinder wall = $2\ t\ L$
- Hoop stress in wall of cylinder
  - $\boldsymbol{\sigma} = F_R / A_W = p\ A_P / A_W = p\ D_m L / 2\ t\ L = \mathbf{p\ D_m / 2\ t}$
  - Hoop stress is twice the magnitude of longitudinal stress
  - Hoop stress in the cylinder is also twice the stress in a sphere of the same diameter carrying the same pressure

Hoop Stress in Thin-walled Cylinders