Deflection of Beams

Unit 5

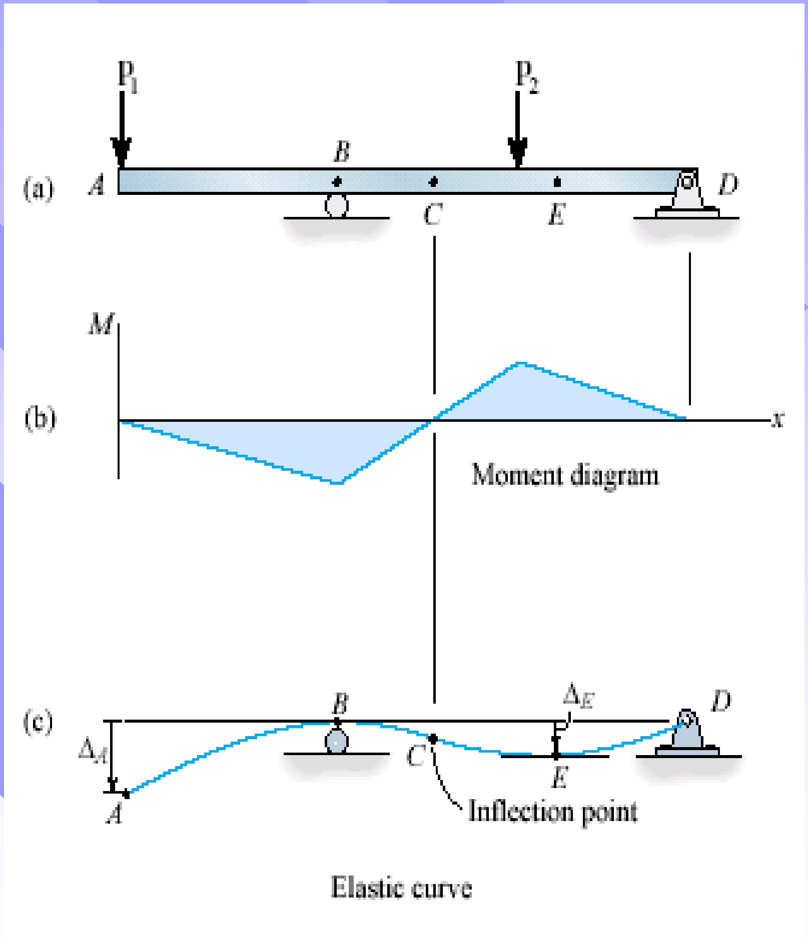
Deflections

Introduction

- Calculation of deflections is an important part of structural analysis
- Excessive beam deflection can be seen as a mode of failure.
  - Extensive glass breakage in tall buildings can be attributed to excessive deflections
  - Large deflections in buildings are unsightly (and unnerving) and can cause cracks in ceilings and walls.
  - Deflections are limited to prevent undesirable vibrations

Beam Deflection

- Bending changes the initially straight longitudinal axis of the beam into a curve that is called the **Deflection Curve** or **Elastic Curve**



Beam Deflection

- Consider a cantilever beam with a concentrated load acting upward at the free end.
- Under the action of this load the axis of the beam deforms into a curve
- The deflection Δ is the displacement in the y direction on any point on the axis of the beam

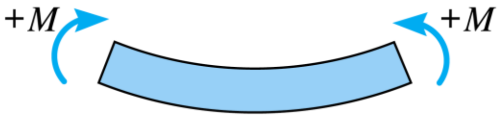
Beam Deflection

- Because the y axis is positive upward, the deflections are also positive when upward.
  - Traditional symbols for displacement in the x, y, and z directions are u, v, and w respectively.

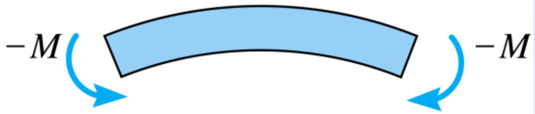
Beam Deflection

- To determine the deflection curve:
  - Draw shear and moment diagram for the beam
  - Directly under the moment diagram draw a line for the beam and label all supports
  - At the supports displacement is zero
  - Where the moment is negative, the deflection curve is concave downward.
  - Where the moment is positive the deflection curve is concave upward
  - Where the two curve meet is the Inflection Point

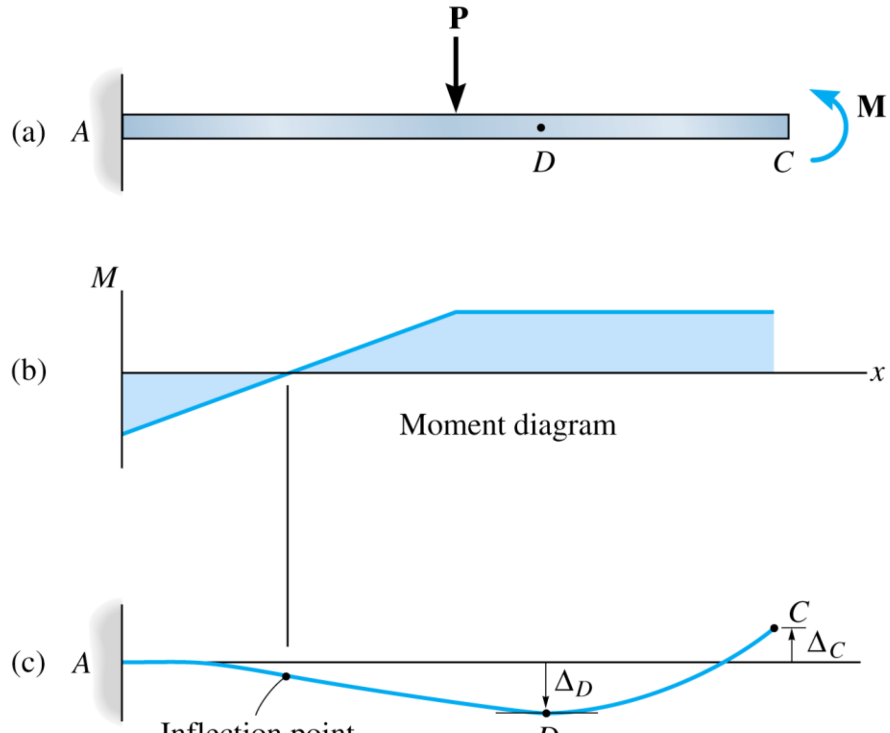
Beam Deflection



Positive internal moment
concave upwards
(a)

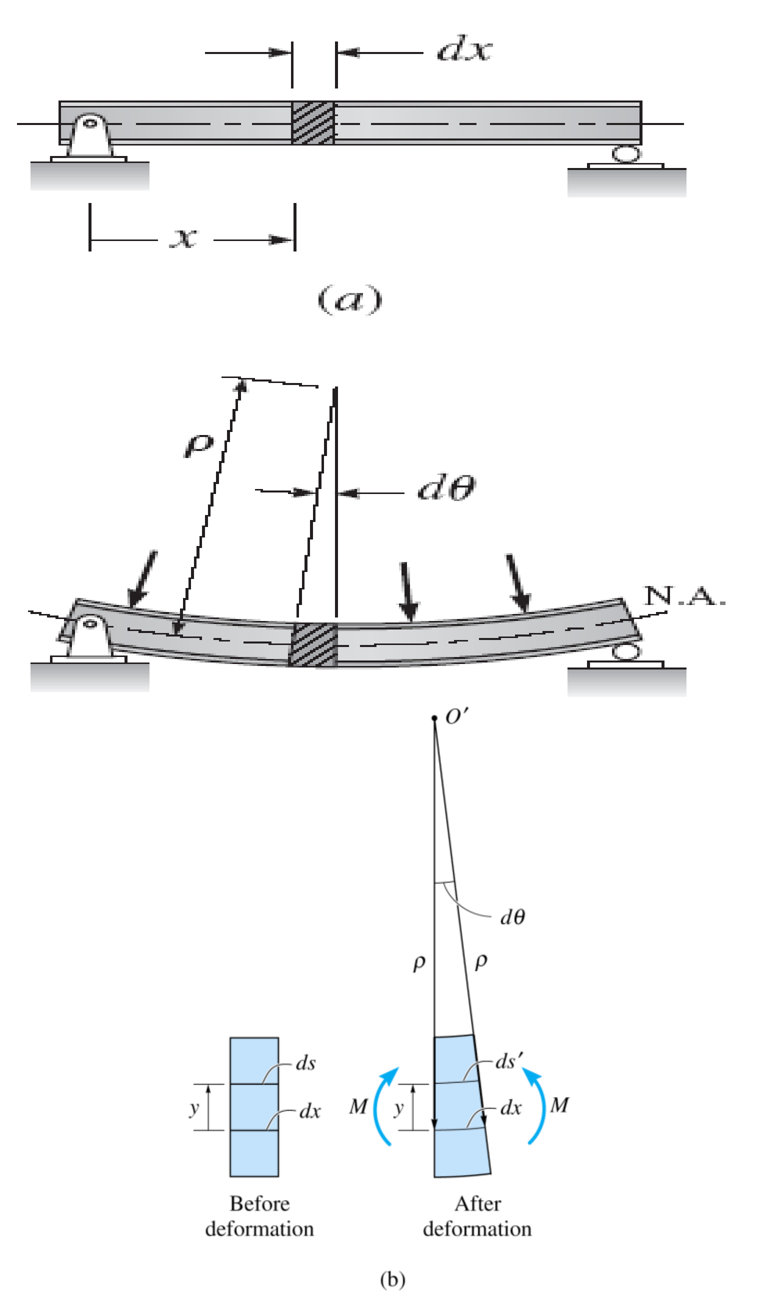


Negative internal moment
concave downwards
(b)



Elastic-Beam Theory

- Consider a differential element of a beam subjected to pure bending.
- The radius of curvature ρ is measured from the center of curvature to the neutral axis
- Since the NA is unstretched, the dx=ρdθ



Elastic-Beam Theory

- The fibers below the NA are lengthened
- The unit strain in these fibers is:

$$\varepsilon = \frac{(ds' - ds)}{ds} = \frac{(\rho - y)d\theta - \rho d\theta}{\rho d\theta} \quad \text{or}$$

$$\frac{1}{\rho} = -\frac{\varepsilon}{y}$$

Elastic-Beam Theory

- Below the NA the strain is positive and above the NA the strain is negative for positive bending moments.
- Applying Hooke's law and the Flexure formula, we obtain:

$$\frac{1}{\rho} = \frac{M}{EI}$$

- The Moment curvature equation

Elastic-Beam Theory

- The product *EI* is referred to as the flexural rigidity.
- Since $dx = \rho d\theta$, then

$$d\theta = \frac{M}{EI}dx \quad (Slope)$$

- In most calculus books

$$\frac{1}{\rho} = \frac{d^2v/dx^2}{\left[1+(dv/dx)^2\right]^{\frac{3}{2}}}$$

$$\frac{M}{EI} = \frac{d^2v/dx^2}{\left[1+(dv/dx)^2\right]^{\frac{3}{2}}} \qquad (exact \quad solution)$$

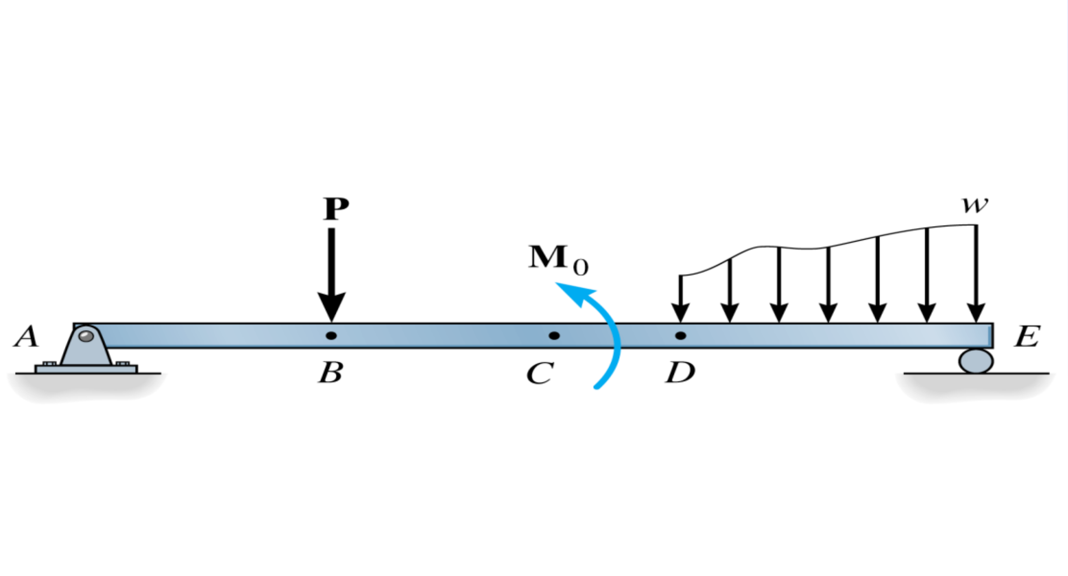
$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

The Double Integration Method

- Once M is expressed as a function of position x, then successive integrations of the previous equations will yield the beams slope and the equation of the elastic curve, respectively.
- Wherever there is a discontinuity in the loading on a beam or where there is a support, there will be a discontinuity.

Consider a beam with several applied loads.

  - The beam has four intervals, AB, BC, CD, DE
  - Four separate functions for Shear and Moment



The Double Integration Method

Relate Moments to Deflections

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

$$\theta(x) = \frac{dv}{dx} = \int \frac{M(x)}{EI(x)} dx$$

Integration Constants

Use Boundary Conditions to Evaluate Integration Constants

$$v(x) = \iint \frac{M(x)}{EI(x)} dx^2$$

Moment-Area Theorems

- The moment-area theorems procedure can be summarized as:
- If A and B are two points on the deflection curve of a beam, EI is constant and B is a point of zero slope, then the Mohr's theorems state that:
- (1) Slope at A = 1/EI x area of B.M. diagram between A and B
- (2) Deflection at A relative to B = 1/EI x first moment of area of B.M diagram between A and B about A.