Example 1 - Superheat Rankine Cycle

2

boiler

Q_{in}

3

W_{out}

Turbine

W_{in}

pump

condenser

1

4

Q_{out}

T
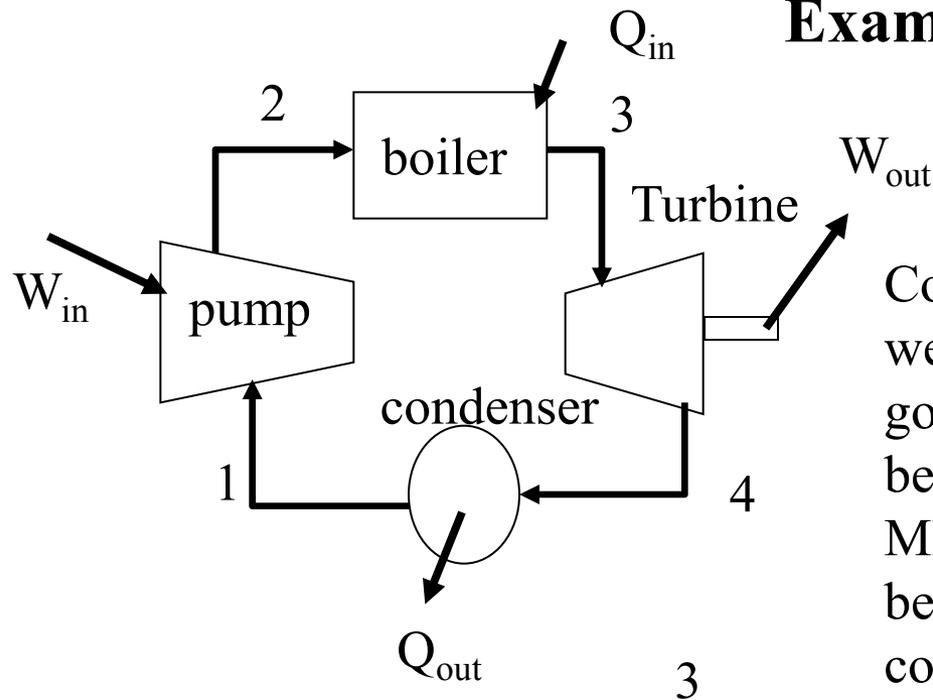
3

2

1

4

s

Consider the same Rankine power cycle as we analyzed before. But this time we are going to superheat the steam in the boiler before allowing it to enter the turbine at 6 MPa. The steam exits from the turbine will be 100% saturated vapor as shown. After condensing, saturated liquid enters the pump at a pressure of 0.1 MPa. Determine (a) the rate of heat transfer into the boiler per unit mass, (b) the net power generation per unit mass. (c) the thermal efficiency,
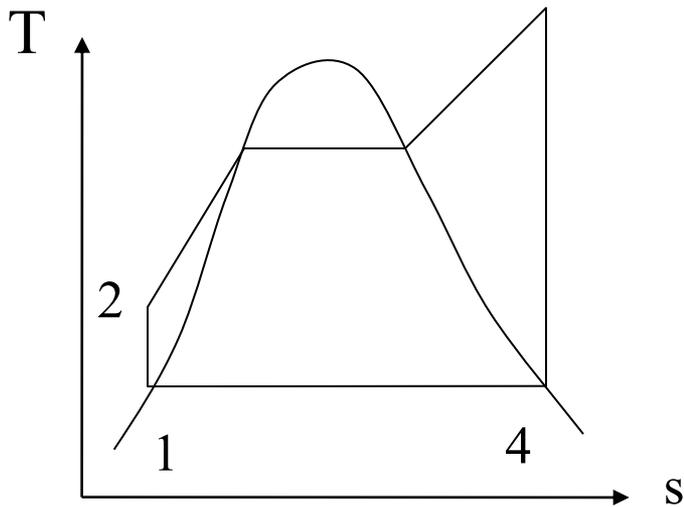
Solution – Superheat Rankine Cycle Example

(4) Let us analyze state 4 first, when the steam exits from the turbine

$P_4 = 0.1(MPa)$, From saturated steam table ~~C-2~~ A-5

$s_4 = s_g = 7.3602(kJ/kgK), h_4 = h_g = 2675.5(kJ/kgK)$ A-5

(1) Now look at the state 1 when the steam enters the pump, again use ~~C-2~~

$s_1 = s_f = 1.3029(kJ/kgK), h_1 = h_f = 417.4(kJ/kgK)$

$v_1 = v_f = 0.001043(m^3/kg)$

(2) From 1-2, the pump compressed the saturared liquid into compressed liquid

The process is isentropic, s=constant, therefore, from the Tds equation

$$Tds = dh - vdP, \ ds = 0, \ dh = vdP, \ int egrate \ h_2 - h_1 = \int_1^2 vdP$$

Since the substance is compressed liquid, v=constant

$$h_2 - h_1 = \int_1^2 vdP = v_1(P_2 - P_1) = (0.001043)(6000 - 100) = 6.15(kJ/kg) = W_{pump}$$

$$h_2 = h_1 + W_{pump} = 417.4 + 6.15 = 423.6(kJ/kg)$$

Solution(cont.)

(3) Finally, since the turbine is going through an isentropic expansion

$s_3 = s_4 = 7.36(kJ / kgK), P_3 = 6 MPa$

We can determine the thermodynamic properties of this superheated vapor using superheated table C~~-3~~ A-6 through interpolation

$T_3 = 674°C, h_3 = 3832.9(kJ / kg)$

(a) The rate of heat transfer into the boiler

$q_{in} = \dot{m}(h_3 - h_2) = (1)(3832.9 - 423.6) = 3409.3(kW)$ per kg of steam

(b) The net power generation

$\dot{W}_{net} = \dot{W}_{turbine} - \dot{W}_{pump} = \dot{m}[(h_3 - h_4) - (h_2 - h_1)] = 1157.4 - 6.15$

$= 1151.2(kW)$ net power generation per kg of steam

(c) Thermal efficiency

$\eta = \dot{W}_{net} / q_{in} = 1151.2 / 3409.3 = 33.8\%$

Discussion

• Without going through (tedious) calculation, can you roughly estimate the thermal efficiency of the first Rankine system (without superheating) ? Is this estimated value close to the calculated one (35.3%).

•By increasing the condensing pressure from 0.01 MPa to 0.1 MPa, how will the thermal efficiency of the system will vary due to this change?

• Can you estimate the change (order of magnitude only) without going through calculation?

• Based on our calculation, the thermal efficiency actually decreases from 35.3% to 33.8%. Is this value consistent with your estimation? Why or why not?

Discussion (cont.)

• If we assume the system is operated under a Carnot cycle, then $\eta=1-T_L/T_H$, where $T_L=45.8°C$, $T_H=275.6 °C$, both from table A-5. (How?). Therefore, $\eta=0.418$. The Rankine efficiency should be less than that.

• Increase pressure to 0.1 MPa, the condensing temperature increases to 99.6 °C. Therefore, the efficiency based on Carnot cycle should decrease to 0.321.

• The percentage change is (0.418-0.321)/0.418=23.2%, a significant drop in thermal efficiency.

• However, the real change of the thermal efficiency is very small. Explain why?

Example 2 - Super-Reheat Rankine Cycle
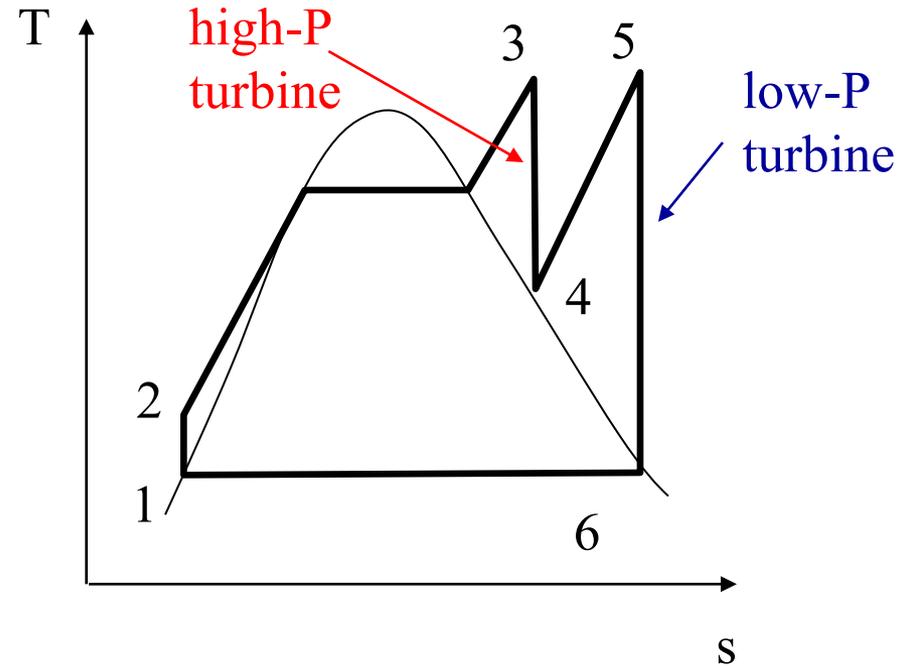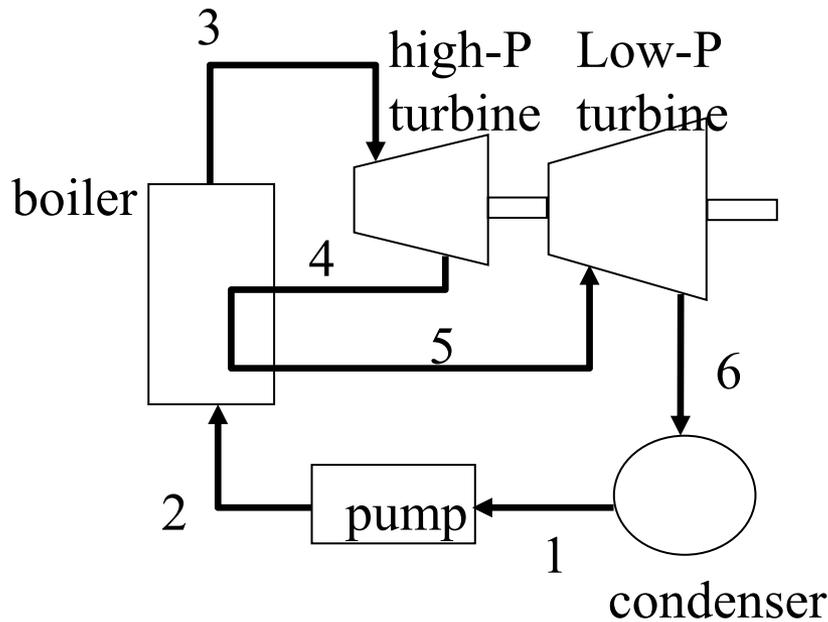
We are going to add a low pressure turbine (5-6) to the system we just analyzed. Before going into the L-P turbine, the exit steam from the first turbine (3-4) is reheated in the boiler at a constant pressure. Assume both 4 & 6 are at 100% saturated vapor state and the vapor exiting from the H-P turbine (state 4) expands to a lower pressure of 2 MPa (P_4=2 MPa) before it is being reheated at a constant pressure to the state 5. Recalculate (a) the thermal efficiency of the system.

Solution

States 1, 2 are unchanged

(1) $s_1 = s_f = 1.3029(kJ/kgK), h_1 = h_f = 417.4(kJ/kgK)$

$v_1 = v_f = 0.001043(m^3/kg)$

(2) $h_2 - h_1 = \int_1^2 vdP = v_1(P_2 - P_1) = (0.001043)(6000 - 100) = 6.15(kJ/kg) = W_{pump}$

$h_2 = h_1 + W_{pump} = 417.4 + 6.15 = 423.6(kJ/kg)$

State 6 is the same as the state 4 from the previous cycle

(6) From table ~~C-2~~ A-5, when $P_6 = 0.1$ MPa

$s_6 = s_g = 7.3602(kJ/kgK), h_6 = h_g = 2675.5(kJ/kgK)$

(5) From 5-6, the vapor is undergoing an isentropic expansion

$s_5 = s_6 = 7.3602(kJ/kgK),$

$P_5 = P_4 = 2MPa,$ (const. pressure heat addition from 4 to 5)

From table ~~C-3~~ A-6, $T_5 = 475.4°C, h_5 = 3413.4(kJ/kg)$

(4) $P_4 = 2MPa,$ saturation table C-2

$s_4 = s_g = 6.3417(kJ/kgK), h_4 = h_g = 2799.5(kJ/kgK)$

Solution (cont.)

(3) From 3-4, vapor is going through an isentropic expansion in H-P turbine

$s_3 = s_4 = 6.3417(kJ/kgK)$, $P_3 = 6MPa$

From superheated table C-3, ~~C-3~~ A-6, $T_3 = 351.8°C$, $h_3 = 3047.8(kJ/kg)$

The turbine work is the sum of both turbines

$W_{turbine} = W_{3\text{-}4} + W_{5\text{-}6} = (h_3 - h_4) + (h_5 - h_6) = (3047.8\text{-}2799.5) + (3413.4\text{-}2675.5)$

$= 248.3 + 737.9 = 986.2(kJ/kg)$

The heat transfer into the boiler is the sum of the primary heat and reheat

$q_{boiler} = q_{2\text{-}3} + q_{4\text{-}5} = (h_3 - h_2) + (h_5 - h_4) = (3047.8\text{-}423.6) + (3413.4\text{-}2799.5)$

$= 2624.2 + 613.9 = 3238.1$

$W_{pump} = h_2 - h_1 = 6.15(kJ/kg)$

$\eta = W_{net}/q_{in} = (W_{turbine} - W_{pump})/q_{in} = (986.2 - 6.15)/3238.1 = 0.303 = 30.3\%$

Question: Why the thermal efficiency decreases when we reheat the steam?