

UNIT 4

MECHANICAL SYSTEM DESIGN

- ▣ *Chapter 7-System Evaluation*
- ▣ *Chapter 8-Calculus Method for Optimization*

CHAPTER-7

System Evaluation

Feasibility Assessment

- ▣ Before a new plant or business is set up a throughout survey is taken that may be referred as product analysis.
- ▣ Survey is analysis of products, market, economy, volume and size of production, Capital etc.
- ▣ Product analysis determine marketability and future demand of product.

Types of Feasibility

- ▣ *Economic Feasibility*: Benefits obtained by product must be larger than efforts. Ratio of benefit to cost must be greater than 1. nancial
- ▣ *Financial Feasibility*: A design may be physical realizable and of great economic worth but may not be financial viable. Only those projects are financial viable which are in reach of sponsor. Man govt. projects are example of financial infeasibility.

Planning Horizon

- ▣ Planning is necessarily connected to future.
- ▣ Planning horizon indicates the time for which a plan is fixed.

Time Value of money

- ▣ Money is borrowed for a time and it is considered that the returned value of money will be greater. This difference is called interest.
- ▣ If an amount P is borrowed for a period in n year at an annual interest i , then the amount to be paid is

$$F = P(1 + ni)$$

Time Value of money

- ▣ P=present worth
- ▣ F=future worth
- ▣ Usually P,F,I and n are related as $F = P(1 + i)^n$
- ▣
is Single Payment Compound amount factor(SPCAF).
- ▣ Reverse of SPCAF is SPPWF(SP Present worth factor).
- ▣ The amount A is annual payment needed to return initial borrowed sum P plus interest on the investment at a rate I, over n years can be calculated as.

$$A = P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] = P(CRF)$$

$CRF = \text{capital recovery factor}$

Sinking Fund Factor

- ▣ SFF is related to sum of money put aside at the end of each year A , so that after n years the accumulated fund, with interest compound will be worth F .
- ▣ The value of A is calculated according to

$$A = F \left[\frac{i}{(1+i)^n - 1} \right] = F(SFF)$$

- ▣ Do example 7.1 and 7.2 for above in the book

Comparing alternatives on Cost Basis

- ▣ Present worth method: Convert all cash flow to a single sum equivalent at time zero using $i = \text{MARR}$
- ▣ Following example is illustrating the PW method

Annual Worth Method

- ▣ It converts all cash flow to an equivalent uniform series of cash flow over the planning horizon using $i = \text{MARR}$
- ▣ $AW = PW * CRF$
- ▣ Following is the example of above.

Payback Period Method

- ▣ It involves the calculation of length of time required to recover the initial capital investment based on zero interest rate.
- ▣ For example a plant of initial cost Rs.100000 and o/p worth is Rs. 25500 per year.
- ▣ Then what is the PBP=?

Capitalized worth method

- ▣ A special case of present worth method.
- ▣ Best suited for long term projects.
- ▣ It is used in economic decision making because it provide a method that is independent of time period of various alternatives.
- ▣ If P is deposited into a fund at I rate, so that a payment of amount A may be withdrawn at each period indefinitely, then
- ▣ $A=C*I$
- ▣ This means that a deposit of Rs. 10000 at 8% annually will be required to fund a payment of Rs. 8000 each year indefinitely.

Depreciation and taxes

- ▣ $TI = GI - E - G$
- ▣ TI = annual taxable income
- ▣ GI = gross or total annual income
- ▣ E = allowable annual expenses
- ▣ G = annual depreciation charge

As taxes are cash flow, they should be included in economical analysis as wages etc. are included.

Type of taxes

- ▣ Property taxes

- ▣ Sales taxes

- ▣ Excise taxes

- ▣ Income taxes

$$D = \frac{V_0 - V_s}{N}$$

- ▣ V represent original and salvage values of assets

- ▣ N is estimate life of asset.

- ▣ Booked value v_b is original value less than accumulated depreciation at a given time.

$$V_b = V_0 - nD = V_0 - \frac{n}{N}(V_0 - V_s)$$

Declined balance method

- ▣ Another model for calculating depreciation.
- ▣ A fixed % R is multiplied times the book value of asset at the beginning of year to determine depreciation charge for that year.

$$D_n = R(R - 1)^{n-1} \cdot V_0$$

$$\text{book.value} = B_n = (1 - R)^n \cdot V_0$$

Benefit-cost analysis

- ▣ B/C ratio if unity, represent the minimum justification for adopting a new project.
- ▣ Conventional B/C = $\frac{B_n}{C_c + C_m}$
- ▣ Modified B/C = $\frac{B_n - C_m}{C_c}$
- ▣ B_n is net benefit.
- ▣ C represent owners net Capital cost and net operating and maintainance cost.

Value analysis

- ▣ Use value and esteem value are two type of values.
- ▣ Procedure for value analysis.
 - Identify primary or secondary function.
 - Evaluate worth of functions by comparisons.
 - Develop value alternatives.

Chapter 8

Optimization using Calculus

The profit function

Suppose that a business firm has estimated its profit (π) function (based on marketing and production studies) as follows:

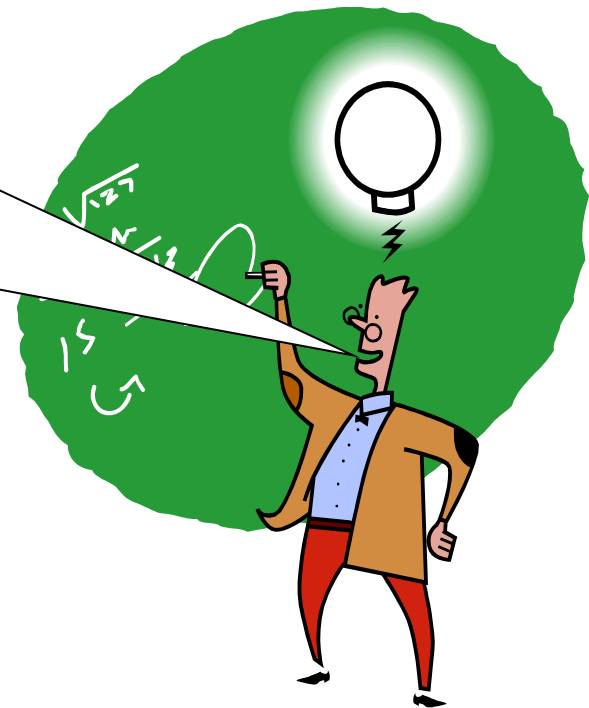
$$\pi = 2Q - 0.1Q^2 - 3.6$$

Where π is profit (in thousands of dollars) and Q is quantity (in thousands of units).

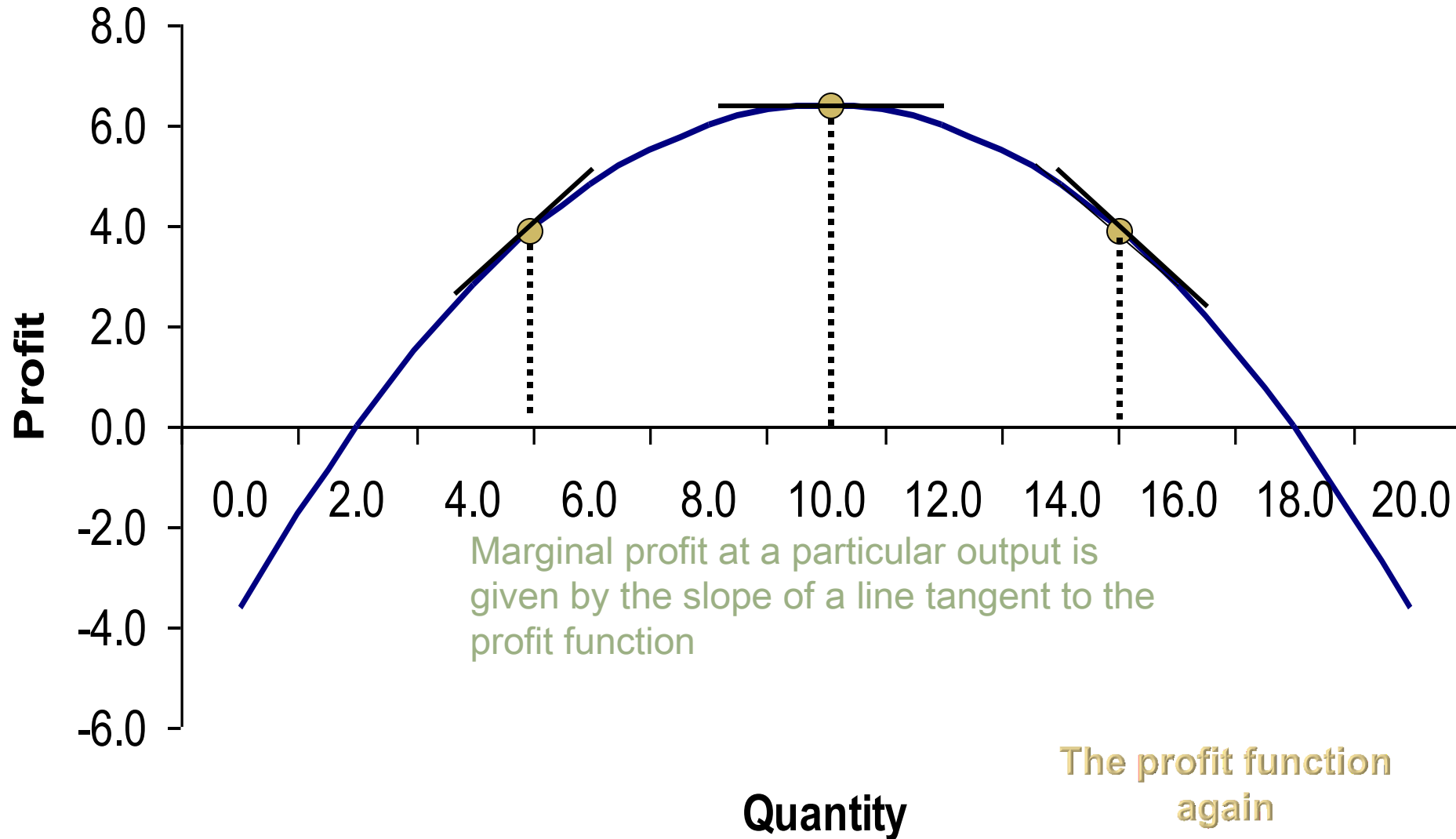
Thus the problem for management is to set its quantity(Q) at the level that maximizes profits (π).

What is an objective function?

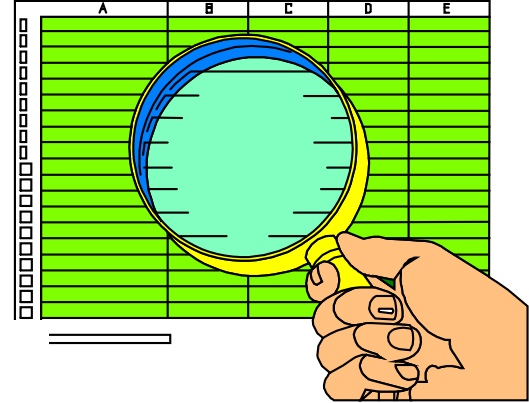
The profit function shows the relationship between the manager's decision variable (Q) and her objective (π). That is why we call it the *objective function*.



The Profit Function



Computing profit at various output levels using a spreadsheet



- Recall our profit function is given by:

$$\pi = 2Q - .1Q^2 - 3.6$$

- Fill in the “quantity” column with 0, 2, 4, . . .
- Assume that you typed zero in column cell a3 of your spreadsheet
- Place your cursor in the the cell b3 (it now contains the bolded number -3.6) – just to the right of cell a3.
- Type the following in the formula bar:

$$=(2*a3)-(.1*a3^2)-3.6$$

and click on the check mark to the left of the formula bar.

- Now move your cursor to the southeast corner of cell b3 until you see a small cross (+). Now move your cursor down through cells b4, b5, b6 . . . to compute profit at various levels of output.

Quantity (000s)	Profit (000s)
0.0	-3.6
2.0	0.0
4.0	2.8
6.0	4.8
8.0	6.0
10.0	6.4
12.0	6.0
14.0	4.8
16.0	2.8
18.0	0.0
20.0	-3.6

Rules of calculus



Rule 1: *The derivative of a constant is zero.*

Example:

Let $Y = 7$

Thus:

$$dy/dx = 0$$



Rule 2: *The derivative of a constant times a variable is simply the constant.*

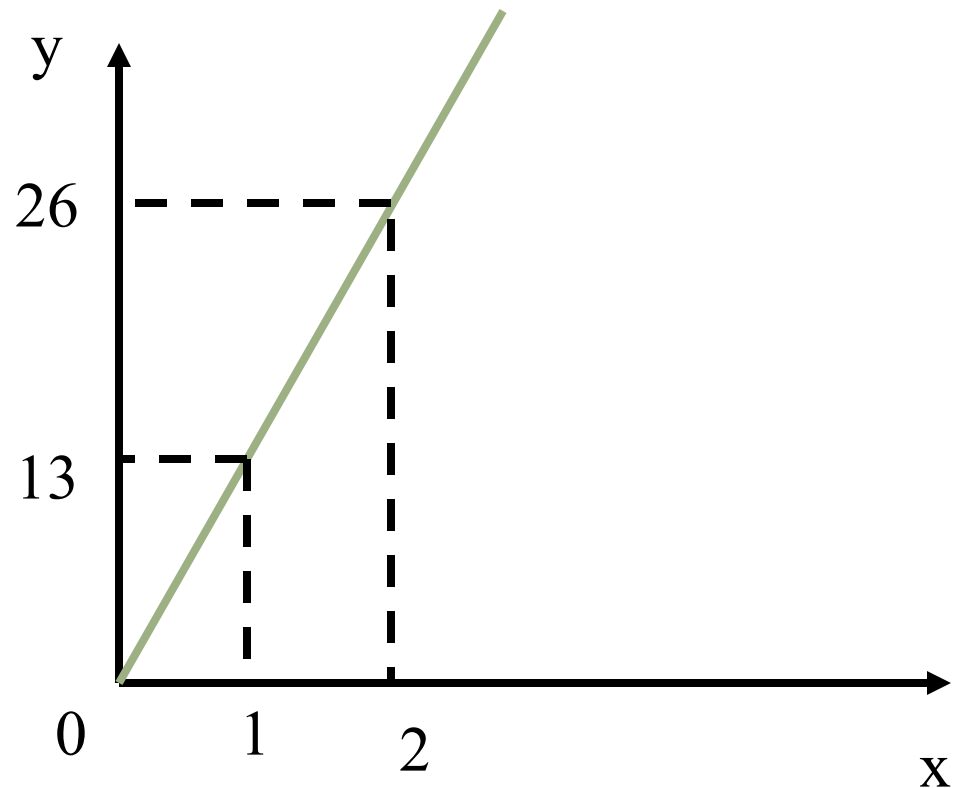
Example:

Let $y = 13x$

Thus:

$dy/dx = 13$

Rule 2



Rule 3: *A power function has the form $y = ax^n$, where a and n are constants. The derivative of a power function is:*

$$\frac{dy}{dx} = n \cdot ax^{n-1}$$

Example:

$$\text{Let } y = 4x^3$$

Thus:

$$dy / dx = 12x^2$$

Rule 3

Special cases of the power function

Note the following:

$y = 1/x^2$ is equivalently written as $y = x^{-2}$
and

$y = \sqrt{x}$ can be written $y = x^{1/2}$

Hence by rule 3 (or the *power rule*), the respective derivatives are given by:

$$dy/dx = -2x^{-3}$$

And

$$dy/dx = .5x^{-1/2} = .5 / \sqrt{x}$$

Power
functions

Rule 4: *Suppose the product of two functions :
 $y = f(x)g(x)$. Then we have:*

$$\frac{dy}{dx} = \left(\frac{df}{dx} \right)(g) + \left(\frac{dg}{dx} \right)(f)$$

Example:

$$\text{Let } y = (4x)(3x^2)$$

Thus:

$$dy/dx = (4)(3x^2) + (6x)(4x) = 36x^2$$



Rule 5

Rule 5: *The derivative of the sum of functions is equal to the sum of the derivatives.*

If $y = f(x) + g(x)$, then:

$$dy/dx = df/dx + dg/dx$$

Example:

$$\text{Let: } y = .1x^2 - 2x^3$$

Thus:

$$dy/dx = .2x - 6x^2$$

Rule 6: *Suppose y is a quotient: $y = f(x)/g(x)$.
Then we have:*

$$\frac{dy}{dx} = \frac{(df/dx)(g) - (dg/dx)(f)}{g^2}$$

Example

Suppose we have: $y = x/(8 + x)$

Thus:

$$dy/dx = [1 \cdot (8 + x) - 1 \cdot (x)] / (8 + x)^2 = 8 / (8 + x)^2$$

The marginal profit ($M\pi$) function

Let the profit function be given by:

$$\pi = 2Q - .1Q^2 - 3.6$$

To obtain the marginal profit function, we take the first derivative of profit π with respect to output (Q):

$$M\pi = d\pi/dQ = 2 - .2Q$$

To solve for the output level that maximizes profits, set $M\pi = 0$.

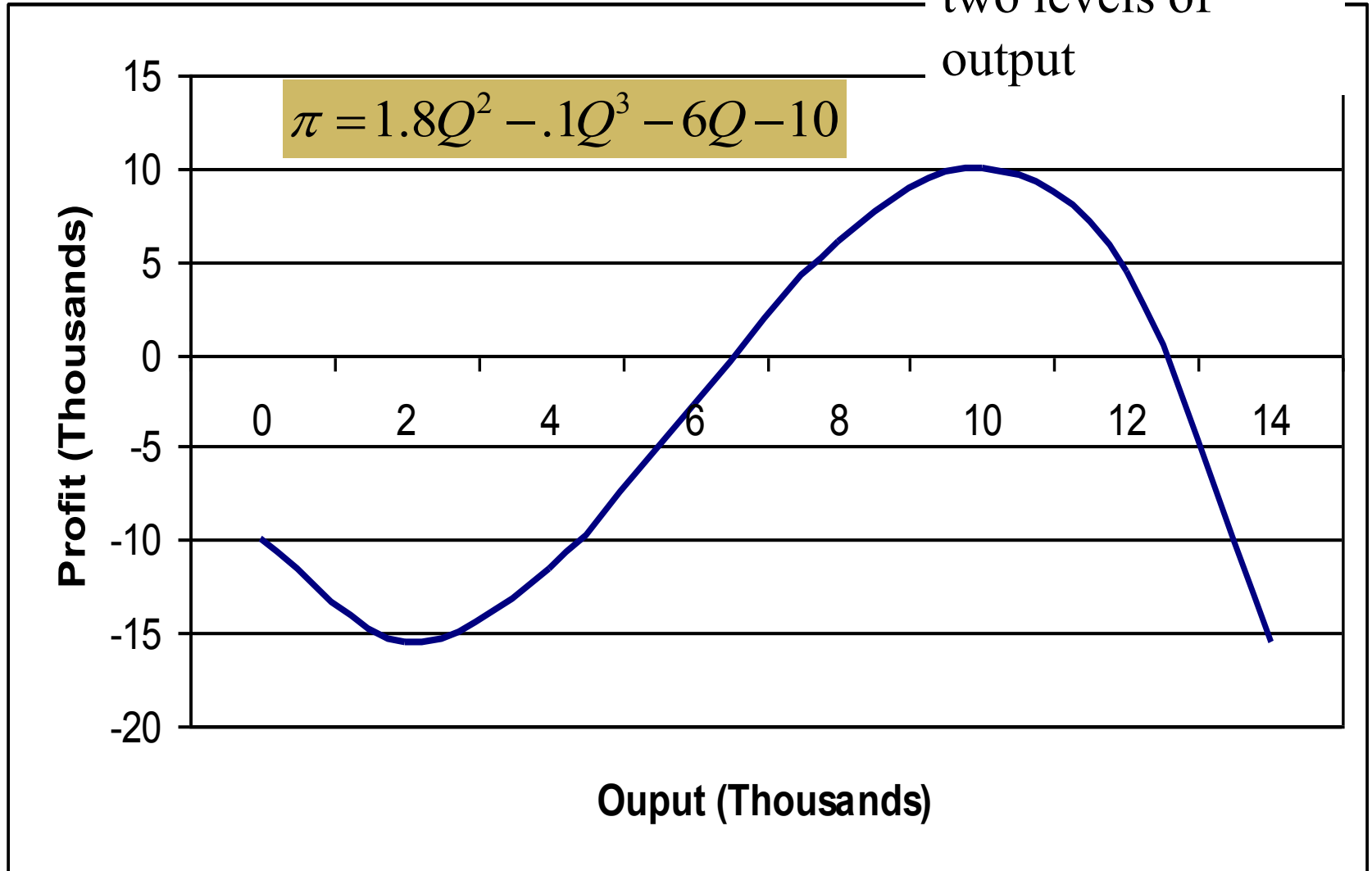
$$2 - .2Q = 0 \text{ Thus: } Q = 10$$

The second derivative

We know that the slope of the profit function is zero at its maximum point. So the first derivative of the profit function with respect to Q will be zero at that output. Problem is, how do we know we have a maximum instead of a minimum?

A more complicated profit function

Notice this function has a slope of zero at two levels of output



Taking the second derivative



Our profit function (π) is given by:

$$\pi = 1.8Q^2 - .1Q^3 - 6Q - 10$$

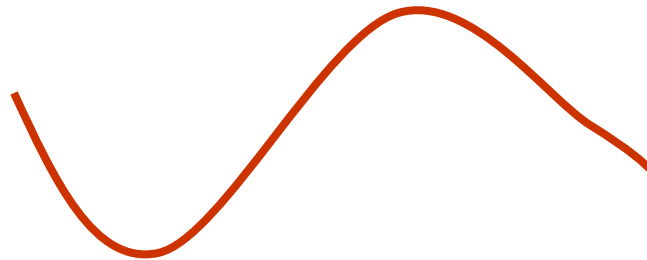
Now let's derive the marginal profit ($M\pi$) function

$$d\pi / dQ = 3.6Q - .3Q^2 - 6$$

We can verify that $M\pi = 0$ when $Q = 2$ and $Q = 10$.

Maximum or minimum?

Notice at the minimum point of the function, the slope is turning from zero to positive. Notice also at the maximum point, the slope is changing from zero to *negative*



To insure a maximum, check to see that the second derivative is negative

To take the second derivative of the profit function:

$$\frac{d^2\pi}{dQ^2} = \frac{d(d\pi / dQ)}{dQ} = \frac{dM\pi}{dQ}$$

Thus we have:

$$\frac{d^2\pi}{dQ^2} = \frac{d(3.6Q - .3Q^2)}{dQ} = 3.6 - .6Q$$

Hence, when $Q = 2$, we find that $d^2\pi / dQ^2 = 3.6 - .6(2) = 2.4$

When $Q = 10$, we find that $d^2\pi / dQ^2 = 3.6 - .6(10) = -2.4$

Marginal Revenue and Marginal Cost

$$\mathbf{MR = MC}$$

Marginal profit ($M\pi$) is zero when marginal revenue (MR) is equal to marginal cost (MC), or alternatively, when $MR - MC = 0$.

Hence to find the profit maximizing output, set the first derivative of the revenue function equal to the first derivative of the cost functions

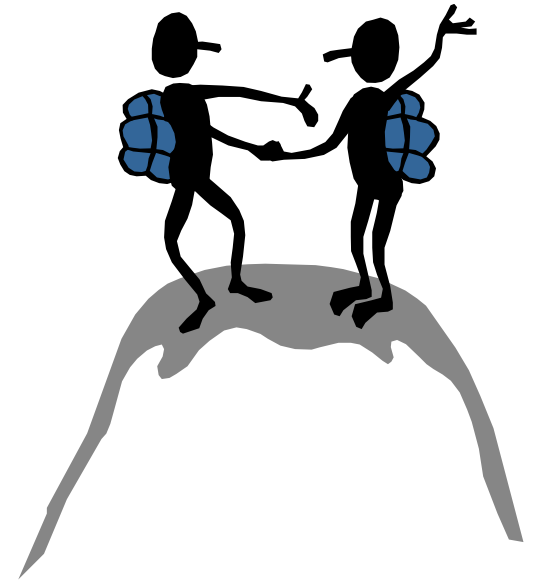
Solving for the profit maximizing output

Let $\pi(Q) = R(Q) - C(Q)$,

where R is sales revenue and C is cost

Thus we have

$$\frac{d\pi}{dQ} = \frac{dR}{dQ} - \frac{dC}{dQ} = MR - MC = 0$$



Multivariable functions

$$y = f(x, z, w)$$

Suppose we have a *multivariate* function such as the following: $\pi = f(P, A)$, where P is market price and A is the advertising budget. Our function has been estimated as follows:

$$\pi = 20 + 2P - 2P^2 + 4A - A^2 + 2PA$$

We would like to know:

- How sensitive are profits to a change in price, other things being equal (or *ceteris paribus*)?
- How sensitive are profits to a change in the advertising budget, *ceteris paribus*?

Partial derivatives

We get the answer to question 1 by taking the first *partial* derivative of π with respect to P .


$$\longrightarrow \frac{\partial \pi}{\partial P} = 2 - 4p + 2A$$

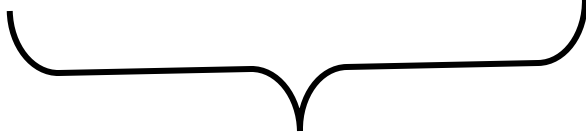
We can find the answer to question 2 by taking the first partial derivative of π with respect to A :

$$\frac{\partial \pi}{\partial A} = 4 - 2A + 2P$$

Solving for the P and A that maximize π

We know that profits will be maximized when the first partial derivatives are equal to zero. Hence, we set them equal to zero and obtain a linear equation system with 2 unknowns (P and A)


$$2 - 4P + 2A = 0$$

$$4 - 2A + 2P = 0$$


The solution is:
 $P = 3$ and $A = 5$

Constrained optimization

So far we have looked at problems in which the decision maker maximizes some variable (π) but faces no constraints. We call this “unconstrained optimization.” Often, however, we seek to maximize (or minimize) some variable subject to one or more constraints.

Examples:

- Maximize profits subject to the constraint that output is equal to or greater than some minimum level.
- Maximize output subject to the constraint that cost must be equal to or less than some maximum value.

Example 1

Suppose we are seeking to maximize the following profit function subject to the constraint that $Q \geq 7$.

$$\pi = 40Q - 4Q^2$$

What happens if we take the first derivative, set equal to zero, and solve for Q ?

$$\frac{d\pi}{dQ} = 40 - 8Q$$

Solving to maximize π , we get $Q = 5$.

But that violates our constraint.

Another example



A firm has a limited amount of output and must decide what quantities (Q_1 and Q_2) to sell in two different market segments. Suppose its profit (π) function is given by:

$$\pi = (20Q_1 - .5Q_1^2) + (40Q_2 - Q_2^2)$$

The firm's output cannot exceed 25—that is, it seeks to maximize π subject to $Q \leq 25$.

If we set the marginal profit functions equal to zero and solved for Q_1 and Q_2 , we would get: $Q_1 = 20$ and $Q_2 = 20$, so that $Q_1 + Q_2 = 40$. Again, *this violates the constraint that total output cannot exceed 25.*

Method of Lagrange Multipliers



This technique entails creating a new variable (the *Lagrange multiplier*) for each constraint. We then determine optimal values for each decision variable and the Lagrange multiplier.

Lagrange technique: Example 1

Recall [example 1](#) . Our constraint was given by $Q = 7$. We can restate this constraint as:

$$7 - Q = 0$$

Our new variable will be denoted by z . Our Lagrange (L) function can be written:

$$L = \pi + z(7 - Q) = 40Q - 4Q^2 + z(7 - Q)$$

Taking the partials of L with respect to Q and z

Now we just take the first partial derivative of L with respect to Q and z , set them equal to zero, and solve.

$$\frac{\partial L}{\partial Q} = 40 - 8Q - z = 0$$

$$\frac{\partial L}{\partial z} = 7 - Q = 0$$

Solving for Q and z simultaneously, we obtain:

$$Q = 7 \text{ and } z = -16$$

Interpretation of the Lagrange multiplier (z)

You may interpret the result that $z = -16$ as follows: *marginal profit ($M\pi$) at the constrained optimum output is -16* — that is, the last unit produced subtracted \$16 for our profit

Lagrange technique: Example 2

Recall [example 2](#). Our constraint was given by:

$$Q_1 + Q_2 = 25$$

Our Lagrange (**L**) function can be written as :

$$L = (20Q_1 - .5Q_1^2) + (40Q_2 - Q_2^2) + z(25 - Q_1 - Q_2)$$

Taking the partials of L with respect to Q_1 , Q_2 and z

This time we take the first partial derivative of L with respect to Q_1 , Q_2 , and z , set them equal to zero, and solve.

$$\frac{\partial L}{\partial Q_1} = 20 - Q_1 - z = 0$$

$$\frac{\partial L}{\partial Q_2} = 40 - 2Q_2 - z = 0$$

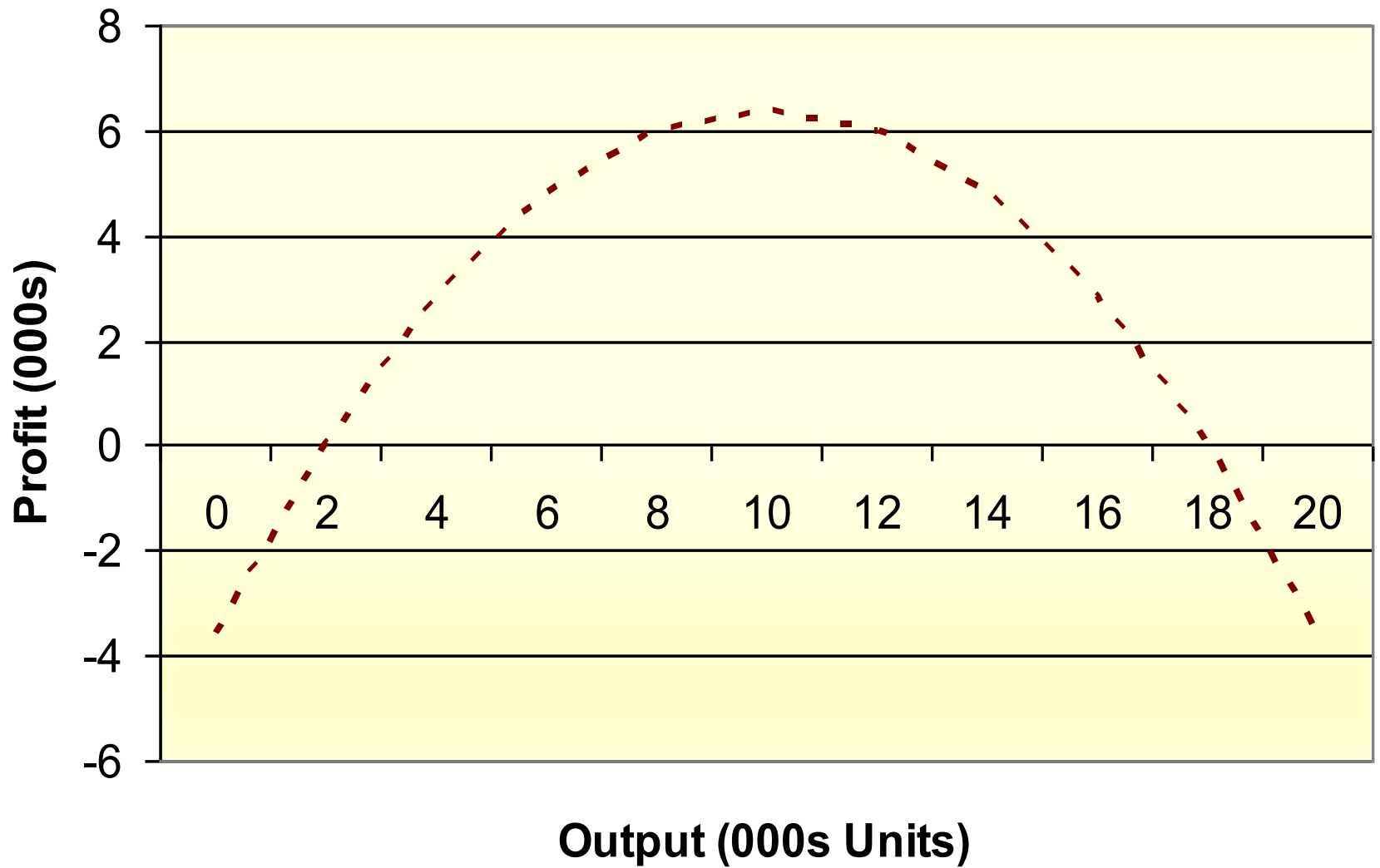
$$\frac{\partial L}{\partial z} = 25 - Q_1 - Q_2 = 0$$

The solutions are:

$$Q_1 = 10$$

$$Q_2 = 15$$

$$z = 10$$



SYSTEM SIMULATION

Chapter 10

Introduction

- ▣ Simulation is a very powerful and widely used management science technique for the analysis and study of complex systems.
- ▣ Simulation may be defined as a technique that imitates the operation of a real-world system as it evolves over time. This is normally done by developing a simulation model. A simulation model usually takes the form of a set of assumptions about the operation of the system, expressed as mathematical or logical relations between the objects of interest in the system.

Definition of simulation

- ▣ **Simulation** is the imitation of the operation of a real-world process or system over time. The act of simulating something first requires that a model be developed; this model represents the key characteristics or behaviors of the selected physical or abstract system or process. The model represents the system itself, whereas the simulation represents the operation of the system over time.

Simulation models

- ▣ **Simulation modeling** is the process of creating and analyzing a digital prototype of a physical model to predict its performance in the real world. Simulation modeling is used to help designers and engineers understand whether, under what conditions, and in which ways a part could fail and what loads it can withstand. Simulation modeling can also help predict fluid flow and heat transfer patterns.

Iconic Models

- ▣ Look like real things.
- ▣ Four type of iconic models:
 - Proof of concept model
 - Scale model
 - Experimental model
 - Prototype model

Analog model

- ▣ **Analogical models** are a method of representing a phenomenon of the world, often called the "target system" by another, more understandable or analyzable system. They are also called **dynamical analogies**.
- ▣ They are based on symbols manipulation and algebraic techniques.

Simulation process

- ▣ It contains:
 - Model development
 - Model execution
 - Model output
 - ▣ A model is created to represent a manufacturing system. Creation of model will need:
 - Define nature of purpose
 - Specification of constraints.

Different type of simulation

- ▣ Statistical simulation
- ▣ Continuous simulation
- ▣ Discrete event simulation
- ▣ Combined simulation

Simulation Methodology

- ▣ Developing a dedicated model
- ▣ Using simulation software

Terms used in simulation

- ▣ Time
- ▣ Entities and resources
- ▣ Events
- ▣ States and queues
- ▣ Activities
- ▣ Executives

Computer simulation

- ▣ A **computer simulation** run on a single computer, or a network of computers, to reproduce behavior of a system.
- ▣ The simulation uses an abstract model (a **computer model**, or a **computational model**) to simulate the system. Computer simulations have become a useful part of mathematical modeling of many natural systems in physics (computational physics), astrophysics, climatology, chemistry and biology, human systems in economics, psychology, social science, and engineering.
- ▣ Simulation of a system is represented as the running of the system's model. It can be used to explore and gain new insights into new technology and to estimate the performance of systems too complex for analytical solutions.

Monte Carlo Methods

- A Monte Carlo simulation creates samples from a known distribution
- For example, if you know that a coin is weighted so that heads will occur 90% of the time, then you might assign the following values:

X	0	1
$f_X(x)$	0.10	0.90

Monte Carlo Methods

- Define problem
- Construct appropriate model
- Specify value to variables
- Collect info required
- Define starting
- Define a coding system that correlate factors indentified in problem defining
- Select random no. generator and no.
- Correlate the generated random no. with factor indentified in step 1 to 6.
- Summarize result
- Evaluate
- Formulate proposal to management

Waiting line simulation

- ▣ This module simulates client/server relationships. The random events for customer arrivals are generated according to Poisson distribution whose parameter λ can be chosen from the pull-down menu "Arrival Rate"; the random events for server services are generated according to negative exponential distribution whose parameter m can be chosen from the pull-down menu "Service Rate".

Structure of waiting line simulation

- ▣ SCSP
- ▣ MCSP
- ▣ SCMP
- ▣ MCMP

Advantages of waiting line theory

- ▣ Good service to customer
- ▣ Optimization
- ▣ Better understanding of queues
- ▣ Better models

Limitations

- ▣ Complex
- ▣ Discipline may be an issue
- ▣ Distributions may not fit
- ▣ Difficult analyzing