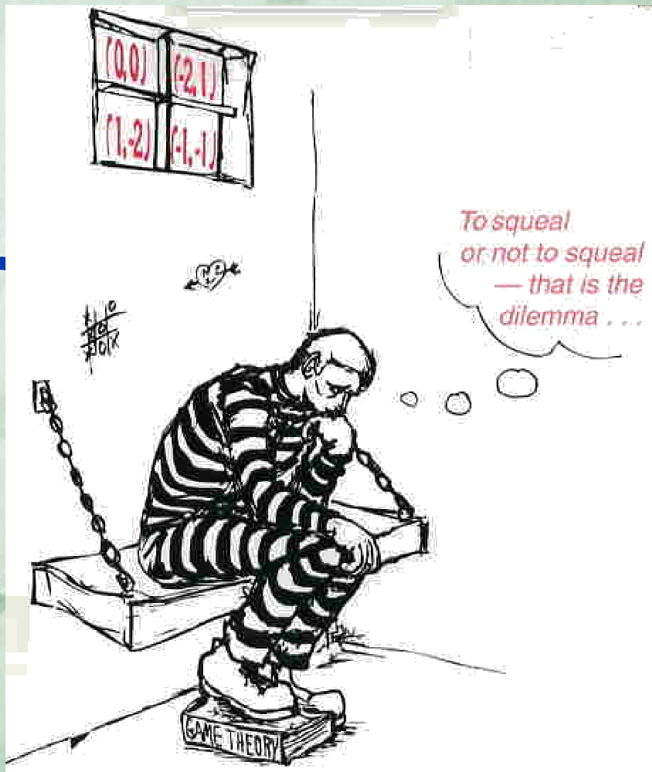
Game Theor


(0,0) (-2,1)
(1,-2) (-1,-1)
To squeal or not to squeal — that is the dilemma . . .
GAME THEORY

Outline

- Game Theory
- Decision Theory
- Markov Game
- Markov Decision Processes
- Conclusion

Game Theory (1/3)

- Game theory is a branch of economics.
  - von Neumann, J. and Morgenstern, O. “*Theory of Games and Economic Behavior,*” Princeton University Press, 1944.
- Game theory (for modeling cooperation and competition in multi-agent system).

Game Theory (2/3)

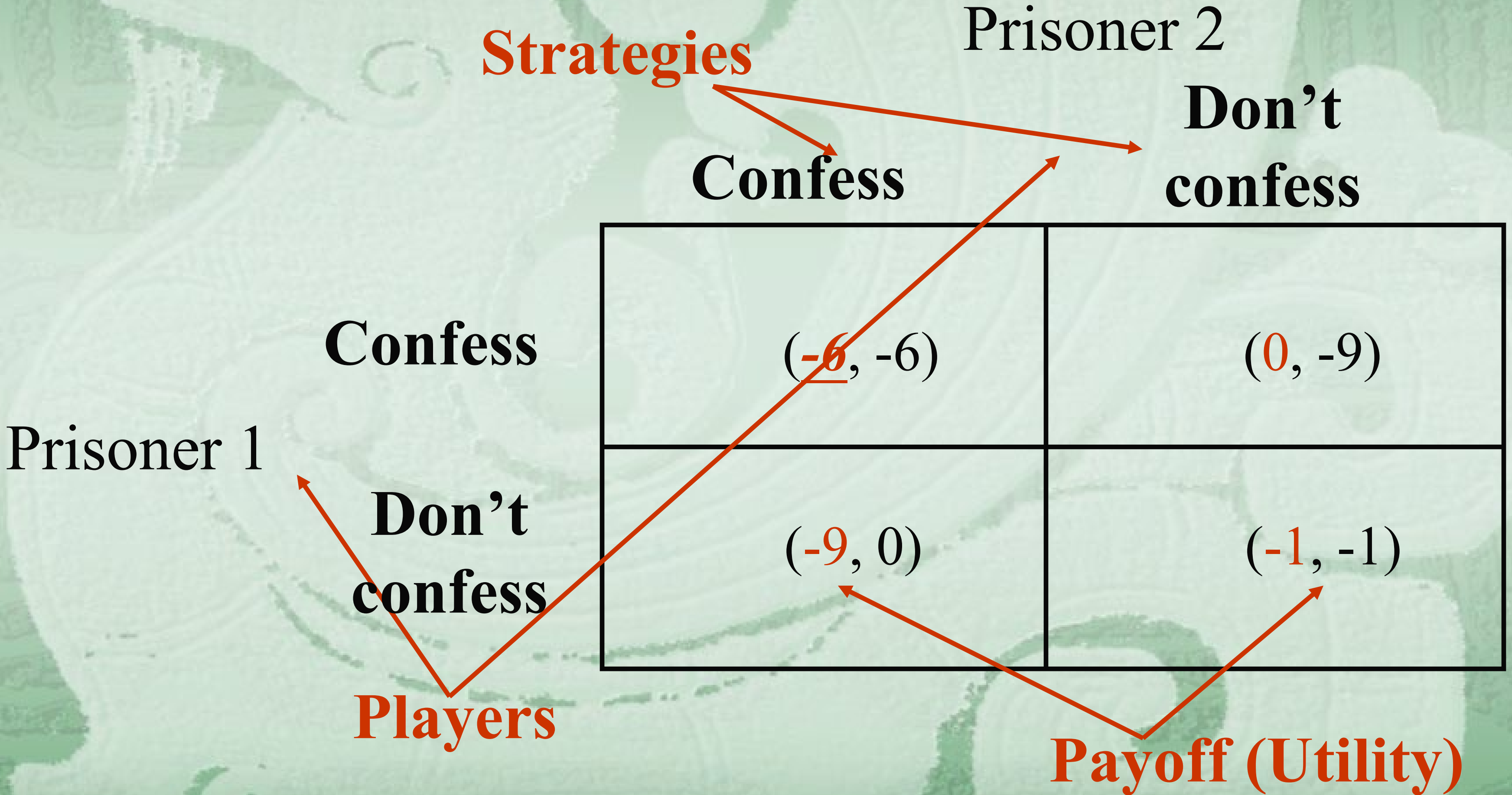
- Key assumption
  - Players are rational
  - Players choose their strategies solely to promote their own welfare (no compassion for the opponent)
- Goal: To find an equilibrium:
  - Equilibrium: *local optimum* in the space of policies

Game Theory (3/3)

- The elements of such a game include:
  - *Players (Agents)*：decision makers in the game
  - *Strategies* ： predetermined rules that tell a player which action to take at each stage of the game
  - *Payoffs (table)*： utilities (dollars) that each player realizes for a particular outcome

- *Equilibrium* ： stable results. Here stable results mean that each player behaves in the desired manner and will *not change its decision*.

Prisoners Dilemma



Strategies

		Prisoner 2	
		Confess	**Don’t confess**
Prisoner 1	**Confess**	(-6, -6)	(0, -9)
	Don’t confess	(-9, 0)	(-1, -1)

Players

Payoff (Utility)

Example

Value of the game, Saddle-point, Nash Equilibrium

Player B

Player A

	1	2	3	4	**Row: Minimum**
1	8	2	9	5	2
2	6	***5***	7	18	***5***
3	7	3	-4	10	-4
Column: Maximum	8	***5***	9	18	

Maximin

Minimax

Classification of Game Theory

- Two-person, zero-sum games
  - One player wins = The other one loses
- Two-person, constant-sum games
- N-person game
- Nonzero-sum game

Outline

- Game Theory
- Decision Theory
- Markov Game
- Markov Decision Processes
- Conclusion

Decision Theory (1/2)

Probability Theory	Describes what an agent should believe based on evidence.
+	
Utility Theory	Describes what an agent wants.
=	
Decision Theory	Describes what an agent should do.

Decision Theory (2/2)

- The *decision maker* needs to choose one of the possible *actions*
- Each combination of an action and a state of nature would result in a *payoff (table)*
- This payoff table should be used to find an *optimal action* for the decision maker according to an appropriate criterion

Outline

- Game Theory
- Decision Theory
- Markov Game
- Markov Decision Processes
- Conclusion

Markov Game

- *Markov games* is an extension of game theory to *MDP like environments*
  - Markov game assumption such that the decisions of users are only based on the current state

Outline

- Game Theory
- Decision Theory
- Markov Game
- Markov Decision Processes
- Conclusion

Markov Decision Processes (1/2)

- Markov decision processes (MDPs) theory has developed substantially in the last three decades and become an established topic within many operational research.
- Modeling of (infinite) sequence of recurring decision problems (general behavioral strategies)
- MDPs defined
  - Objective functions
    - Utility function
      - Revenue
      - Cost
  - Policies
    - Set of decision
      - Dynamic (MDPs)

Markov Decision Processes (2/2)

- Three components: Initial state, transition model, reward function
- Policy: Specifies what an agent should do in every state
- Optimal policy: The policy that yields the highest expected utility

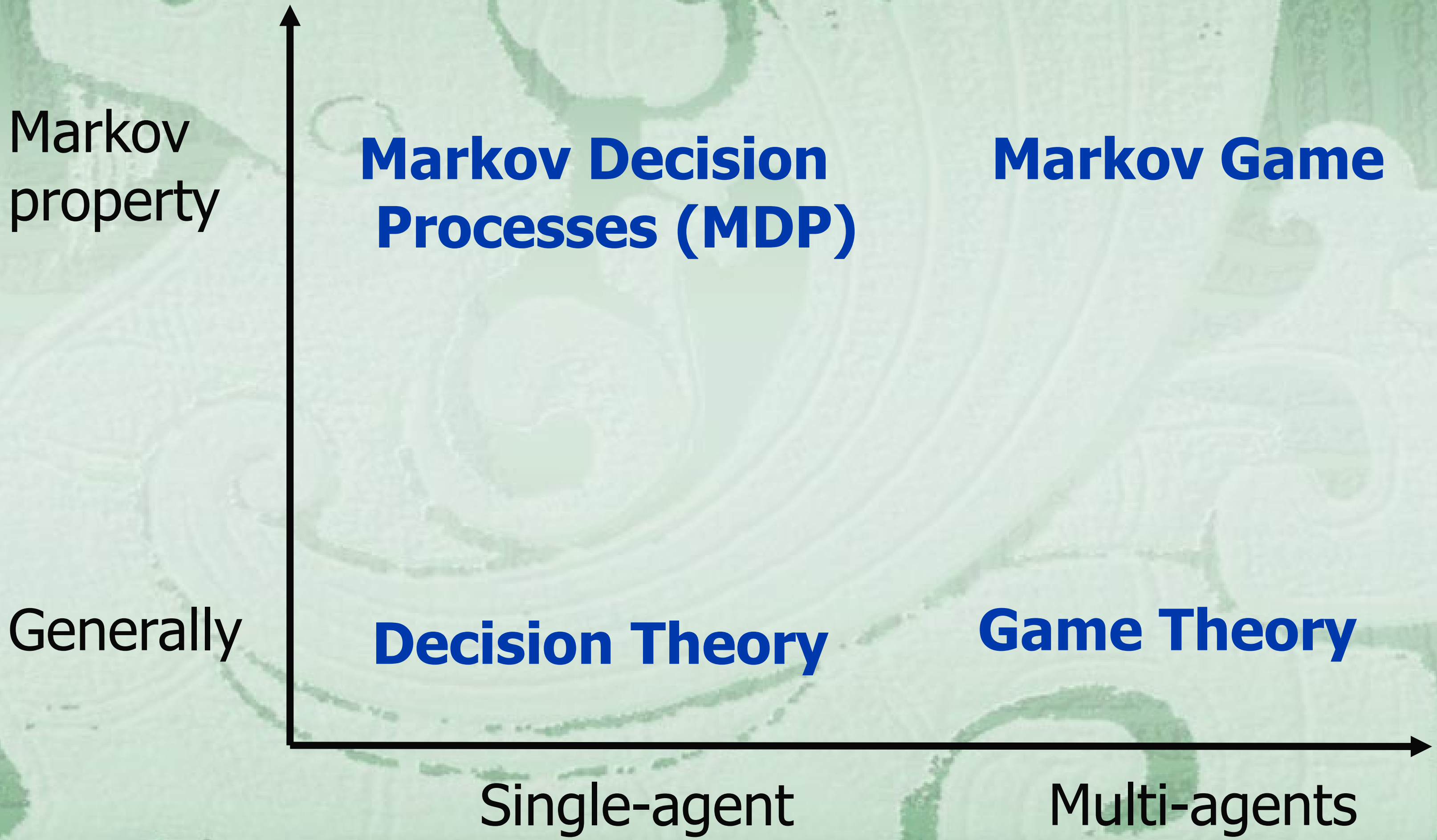
MDP vs. MG

- Single agent: Markov Decision Process
  - MDP is capable of describing only single-agent environments
- Multi-agent: Markov Game
  - n-player Markov Game
  - optimal policy: Nash equilibrium

Outline

- Game Theory
- Decision Theory
- Markov Game
- Markov Decision Processes
- Conclusion

Conclusion



	Single-agent	Multi-agents
Markov property	**Markov Decision Processes (MDP)**	**Markov Game**
Generally	**Decision Theory**	**Game Theory**

References (1/3)

1. **Hamdy A. Taha, “Operations Research: an Introduction,” third edition, 1982.**
2. **Hillier and Lieberman,”Introduction to Operations Research,” fourth edition, Holden-Day, Inc, 1986.**
3. R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, “Network Flows,” Prentice-Hall, 1993.
4. Leslie Pack Kaelbling, “Techniques in Artificial Intelligence: Markov Decision Processes,” MIT OpenCourseWare, Fall 2002.
5. **Ronald A. Howard, “Dynamic Programming and Markov Processes,” Wiley, New York, 1970.**
6. D. J. White, “Markov Decision Processes,” Wiley, 1993.
7. Dean L. Isaacson and Richard W. Madsen, “Markov Chains Theory and Applications,” Wiley, 1976
8. M. H. A. Davis “Markov Models and Optimization,” Chapman & Hall, 1993.
9. Martin L. Puterman, “Markov Decision Processes: Discrete Stochastic Dynamic Programming,” Wiley, New York, 1994.
10. Hsu-Kuan Hung, Adviser：Yeong-Sung Lin ,“Optimization of GPRS Time Slot Allocation”, June, 2001.
11. Hui-Ting Chuang, Adviser：Yeong-Sung Lin ,“Optimization of GPRS Time Slot Allocation Considering Call Blocking Probability Constraints”, June, 2002.

References (2/3)

12. 高孔廉，「作業研究--管理決策之數量方法」，三民總經銷，民國74年四版。
13. 李朝賢，「作業研究概論」，弘業文化實業股份有限公司出版，民國66年8月。
14. 楊超然，「作業研究」，三民書局出版，民國66年9月初版。
15. 葉若春，「作業研究」，中興管理顧問公司出版，民國86年8月五版。
16. 薄喬萍，「作業研究決策分析」，復文書局發行，民國78年6月初版。
17. 葉若春，「線性規劃理論與應用」，民國73年9月增定十版。
18. Leonard Kleinrock, “Queueing Systems Volume I: Threory,” Wiley, New York, 1975.
19. Chiu, Hsien-Ming, “Lagrangian Relaxation,” Tamkang University, Fall 2003.
20. L. Cheng, E. Subrahmanian, A. W. Westerberg, “Design and planning under uncertainty: issues on problem formulation and solution”, Computers and Chemical Engineering, 27, 2003, pp.781-801.
21. Regis Sabbadin, “Possibilistic Markov Decision Processes”, Engineering Application of Artificial Intelligence, 14, 2001, pp.287-300.
22. K. Karen Yin, Hu Liu, Neil E. Johnson, “Markovian Inventory Policy with Application to the Paper Industry”, Computers and Chemical Engineering, 26, 2002, pp.1399-1413.

References (3/3)

23. Wal, J. van der. “*Stochastic Dynamic Programming*,” Mathematical Centre Tracts No. 139, Mathematisch Centrum, Amsterdam, 1981.
24. Von Neumann, J. and Morgenstern, O. “*Theory of Games and Economic Behavior,”* Princeton University Press, 1947.
25. Grahman Romp, “Game Theory :Introduction and Applications ,” Oxford university Press, 1997.
26. Straffin, Philip D. “Game Theory and Strategy,” Washington :Mathematical Association of America, 1993.
27. Watson, Joel. “Strategy :An Introduction to Game Theory ,” New York :W.W. Norton, 2002.