Operations Management

Unit-IV Inventory Management



Outline

- ☑ Global Company Profile: Amazon.com
- ✓ Functions of Inventory

☑ Types of Inventory

☑ Inventory Management

☑ ABC Analysis

☑ Record Accuracy

☑ Cycle Counting

✓ Control of Service Inventories

Outline – Continued

☑ Inventory Models

☑ Independent vs. Dependent Demand☑ Holding, Ordering, and Setup Costs

Outline – Continued

☑ Inventory Models for Independent Demand

- ☑ The Basic Economic Order Quantity (EOQ) Model
- ✓ Minimizing Costs
- *I* Reorder Points
- **☑** Production Order Quantity Model
- **☑** Quantity Discount Models

Outline – Continued

☑ Probabilistic Models and Safety Stock

☑ Other Probabilistic Models

✓ Fixed-Period (P) Systems

Learning Objectives

When you complete this chapter you should be able to:

- 1. Conduct an ABC analysis
- 2. Explain and use cycle counting
- 3. Explain and use the EOQ model for independent inventory demand
- 4. Compute a reorder point and safety stock

Learning Objectives

When you complete this chapter you should be able to:

- 5. Apply the production order quantity model
- 6. Explain and use the quantity discount model
- 7. Understand service levels and probabilistic inventory models

Amazon.com

Amazon.com started as a "virtual" retailer - no inventory, no warehouses, no overhead; just computers taking orders to be filled by others Growth has forced Amazon.com to become a world leader in warehousing and inventory management

Amazon.com

1. Each order is assigned by computer to the closest distribution center that has the product(s)

- 2. A "flow meister" at each distribution center assigns work crews
- 3. Lights indicate products that are to be picked and the light is reset

4. Items are placed in crates on a conveyor. Bar code scanners scan each item 15 times to virtually eliminate errors.

Amazon.com

- Crates arrive at central point where items are boxed and labeled with new bar code
 Gift wrapping is done by hand at 30
 - packages per hour
- 7. Completed boxes are packed, taped, weighed and labeled before leaving warehouse in a truck
- 8. Order arrives at customer within a week

Inventory

- One of the most expensive assets of many companies representing as much as 50% of total invested capital
- ☑ Operations managers must balance inventory investment and customer service

Functions of Inventory

- 1. To decouple or separate various parts of the production process
- 2. To decouple the firm from fluctuations in demand and provide a stock of goods that will provide a selection for customers
- 3. To take advantage of quantity discounts
- 4. To hedge against inflation

Types of Inventory

✓ Raw material

☑ Purchased but not processed

☑ Work-in-process

☑ Undergone some change but not completed

☑ A function of cycle time for a product

- ✓ Maintenance/repair/operating (MRO)
 - ☑ Necessary to keep machinery and processes productive
- ✓ Finished goods
 - ☑ Completed product awaiting shipment

The Material Flow Cycle



Figure 12.1

Inventory Management

✓ How inventory items can be classified

✓ How accurate inventory records can be maintained

- ☑ Divides inventory into three classes based on annual dollar volume
 - ✓ Class A high annual dollar volume
 - ☑ Class B medium annual dollar volume
 - ✓ Class C low annual dollar volume
- ✓ Used to establish policies that focus on the few critical parts and not the many trivial ones

ltem Stock Number	Percent of Number of Items Stocked	Annual Volume (units)	X	Unit Cost	=	Annual Dollar Volume	Percent of Annual Dollar Volume	Class
#10286	20%	1,000		\$ 90.00		\$ 90,000	38.8%	A
#11526		500		154.00		77,000	33.2% ∫ ⁷²	A
#12760		1,550		17.00		26,350	11.3%	В
#10867	30%	350		42.86		15,001	6.4% 23	% B
#10500		1,000		12.50		12,500	5.4%	В

ltem Stock Number	Percent of Number of Items Stocked	Annual Volume (units)	x	Unit Cost	=	Annual Dollar Volume	Percent of Annual Dollar Volume		Class
#12572		600		\$ 14.17		\$ 8,502	3.7%		С
#14075		2,000		.60		1,200	.5%		С
#01036	50%	100		8.50		850	.4%	> 5%	С
#01307		1,200		.42		504	.2%		С
#10572		250		.60		150	.1%		С
		8,550				\$232,057	100.0%		



- ☑ Other criteria than annual dollar volume may be used
 - ☑ Anticipated engineering changes
 - ☑ Delivery problems
 - ☑ Quality problems
 - ✓ High unit cost

✓ Policies employed may include

- More emphasis on supplier development for A items
- ☑ Tighter physical inventory control for A items
- ☑ More care in forecasting A items

Record Accuracy

- Accurate records are a critical ingredient in production and inventory systems
- ☑ Allows organization to focus on what is needed
- ☑ Necessary to make precise decisions about ordering, scheduling, and shipping
- ☑ Incoming and outgoing record keeping must be accurate
- ✓ Stockrooms should be secure

Cycle Counting

- ☑ Items are counted and records updated on a periodic basis
- Often used with ABC analysi to determine cycle



- ✓ Has several advantages
 - ☑ Eliminates shutdowns and interruptions
 - ☑ Eliminates annual inventory adjustment
 - ☑ Trained personnel audit inventory accuracy
 - ☑ Allows causes of errors to be identified and corrected
 - **☑** Maintains accurate inventory records

Cycle Counting Example

5,000 items in inventory, 500 A items, 1,750 B items, 2,750 C items

Policy is to count A items every month (20 working days), B items every quarter (60 days), and C items every six months (120 days)

ltem Class	Quantity	Cycle Counting Policy	Number of Items Counted per Day
Α	500	Each month	500/20 = 25/ <i>day</i>
В	1,750	Each quarter	1,750/60 = 29/ <i>day</i>
С	2,750	Every 6 months	2,750/120 = 23/ <i>day</i>
			77/day

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Control of Service Inventories

- ✓ Can be a critical component of profitability
- ✓ Losses may come from shrinkage or pilferage



- ☑ Applicable techniques include
 - 1. Good personnel selection, training, and discipline
 - 2. Tight control on incoming shipments
 - 3. Effective control on all goods leaving facility

Independent Versus Dependent Demand

- Independent demand the demand for item is independent of the demand for any other item in inventory
- Dependent demand the demand for item is dependent upon the demand for some other item in the inventory

Holding, Ordering, and Setup Costs

- ☑ Holding costs the costs of holding or "carrying" inventory over time
- Ordering costs the costs of placing an order and receiving goods
- Setup costs cost to prepare a machine or process for manufacturing an order

Holding Costs

	Cost (and range) as a Percent of
Category	Inventory Value
Housing costs (building rent or depreciation, operating costs, taxes, insurance)	6% (3 - 10%)
Material handling costs (equipment lease or depreciation, power, operating cost)	3% (1 - 3.5%)
Labor cost	3% (3 - 5%)
Investment costs (borrowing costs, taxes, and insurance on inventory)	11% (6 - 24%)
Pilferage, space, and obsolescence	3% (2 - 5%)
Overall carrying cost	26%

Table 12.1

Holding Costs

Holding costs vary considerably depending on the business, location, and interest rates. Generally greater than 15%, some high tech items have holding costs greater than 50%. Inventory Models for Independent Demand

Need to determine when and how much to order

☑ Basic economic order quantity

☑ *Production order quantity*

☑ Quantity discount model

Basic EOQ Model

Important assumptions

- 1. Demand is known, constant, and independent
- 2. Lead time is known and constant
- 3. Receipt of inventory is instantaneous and complete
- 4. Quantity discounts are not possible
- 5. Only variable costs are setup and holding
- 6. Stockouts can be completely avoided

Inventory Usage Over Time



Figure 12.3

Minimizing Costs

Objective is to minimize total costs



The EOQ Model

- **Q** = Number of pieces per order
- **Q*** = Optimal number of pieces per order (EOQ)
 - **D** = Annual demand in units for the inventory item
 - **S** = Setup or ordering cost for each order
 - *H* = Holding or carrying cost per unit per year

Annual setup cost = (Number of orders placed per year) x (Setup or order cost per order)

$$= \left(\frac{Annual \ demand}{Number \ of \ units \ in \ each \ order}\right) \left(\begin{array}{c} \text{Setup or order} \\ \text{cost per order} \end{array}\right)$$
$$= \left(\frac{D}{Q}\right)(S) \qquad \qquad \text{Annual setup } \cos t = \frac{D}{Q}S$$

The EOQ Model

- **Q** = Number of pieces per order
- **Q*** = Optimal number of pieces per order (EOQ)
 - **D** = Annual demand in units for the inventory item
 - **S** = Setup or ordering cost for each order
 - *H* = Holding or carrying cost per unit per year

Annual holding cost = (Average inventory level) x (Holding cost per unit per year)

$$= \left(\frac{Order \ quantity}{2}\right) (Holding \ cost \ per \ unit \ per \ year)$$
$$= \left(\frac{Q}{2}\right) (H)$$
Annual setup \ cost = $\frac{D}{Q}$ S
Annual holding \ cost = $\frac{Q}{2}$

The EOQ Model

- **Q** = Number of pieces per order
- **Q**^{*} = Optimal number of pieces per order (EOQ)
 - **D** = Annual demand in units for the inventory item
 - **S** = Setup or ordering cost for each order
 - *H* = Holding or carrying cost per unit per year

Optimal order quantity is found when annual setup cost equals annual holding cost

Solving for Q*

$$\frac{D}{Q}S = \frac{Q}{2}H$$
$$2DS = Q^{2}H$$
$$Q^{2} = 2DS/H$$
$$Q^{*} = \sqrt{2DS/H}$$

Annual setup cost =
$$\frac{D}{Q}S$$

Annual holding cost = $\frac{Q}{2}H$
Determine optimal number of needles to order D = 1,000 units S = \$10 per order H = \$.50 per unit per year

$$Q^* = \sqrt{\frac{2DS}{H}}$$
$$Q^* = \sqrt{\frac{2(1,000)(10)}{0.50}} = \sqrt{40,000} = 200 \text{ units}$$

Determine optimal number of needles to orderD = 1,000 unitsQ* = 200 unitsS = \$10 per orderH = \$.50 per unit per year

Expected
number of = N =
$$\frac{Demand}{Order \ quantity} = \frac{D}{Q^*}$$

orders
 $N = \frac{1,000}{200} = 5 \ orders \ per \ year$

Determine optimal number of needles to orderD = 1,000 units $Q^* = 200$ unitsS = \$10 per orderN = 5 orders per yearH = \$.50 per unit per year

Expected time between = $T = \frac{Number of working}{days per year}$ orders $T = \frac{250}{5} = 50$ days between orders

Determine optimal number of needles to orderD = 1,000 units $Q^* = 200$ unitsS = \$10 per orderN = 5 orders per yearH = \$.50 per unit per yearT = 50 days

Total annual cost = Setup cost + Holding cost

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$
$$TC = \frac{1,000}{200}(\$10) + \frac{200}{2}(\$.50)$$

TC = (5)(\$10) + (100)(\$.50) = \$50 + \$50 = \$100

Robust Model

✓ The EOQ model is robust

- ✓ It works even if all parameters and assumptions are not met
- ☑ The total cost curve is relatively flat in the area of the EOQ

Management underestimated demand by 50%D = 1,000 units1,500 units $Q^* = 200$ unitsS = \$10 per orderN = 5 orders per yearH = \$.50 per unit per yearT = 50 days

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$
$$TC = \frac{1,500}{200}(\$10) + \frac{200}{2}(\$.50) = \$75 + \$50 = \$125$$

Total annual cost increases by only 25%

Actual EOQ for new demand is 244.9 unitsD = 1,000 units1,500 units $Q^* = 244.9$ unitsS = \$10 per orderN = 5 orders per yearH = \$.50 per unit per yearT = 50 days

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$
$$TC = \frac{1,500}{244.9}(\$10) + \frac{244.9}{2}(\$.50)$$
$$TC = \$61.24 + \$61.24 = \$122.48$$

Only 2% less than the total cost of \$125 when the order quantity was 200

Reorder Points

- ✓ EOQ answers the "how much" question
- ✓ The reorder point (ROP) tells when to order

$$ROP = \begin{pmatrix} Demand \\ per day \end{pmatrix} \begin{pmatrix} Lead time for a \\ new order in days \end{pmatrix}$$
$$= d x L$$
$$d = \frac{D}{Number of working days in a year}$$

Reorder Point Curve



Reorder Point Example

Demand = 8,000 iPods per year 250 working day year Lead time for orders is 3 working days

 $d = \frac{D}{Number of working days in a year}$

= 8,000/250 = 32 *units*

 $ROP = d \times L$

= 32 *units per day x* 3 *days* = 96 *units*

☑ Used when inventory builds up over a period of time after an order is placed

✓ Used when units are produced and sold simultaneously



Figure 12.6

Q = Number of pieces per orderp = Daily production rateH = Holding cost per unit per yeard = Daily demand/usage ratet = Length of the production run in days

(Annual inventory) = (Average inventory level) x (Holding cost holding cost

$$\begin{pmatrix} Maximum \\ inventory \ level \end{pmatrix} = \begin{pmatrix} Total \ produced \ during \\ the \ production \ run \end{pmatrix} - \begin{pmatrix} Total \ used \ during \\ the \ production \ run \end{pmatrix}$$
$$= pt - dt$$

Q = Number of pieces per orderp = Daily production rateH = Holding cost per unit per yeard = Daily demand/usage ratet = Length of the production run in days

 $\begin{pmatrix} Maximum \\ inventory \ level \end{pmatrix} = \begin{pmatrix} Total \ produced \ during \\ the \ production \ run \end{pmatrix} - \begin{pmatrix} Total \ used \ during \\ the \ production \ run \end{pmatrix}$ = pt - dt

However, **Q** = *total produced* = *pt ; thus t* = **Q**/*p*

$$\begin{pmatrix} Maximum\\ inventory \ level \end{pmatrix} = p \left(\frac{Q}{p} \right) - d \left(\frac{Q}{p} \right) = Q \left(1 - \frac{d}{p} \right)$$

$$Holding \ cost = \frac{Maximum \ inventory \ level}{2} \ (H) = \frac{Q}{2} \left[1 - \left(\frac{d}{p} \right) \right] H$$

Q = Number of pieces per order p = Daily production rate *H* = *Holding cost per unit per year* D = Annual demand

d = Daily demand/usage rate

Setup cost = (D/Q)SHolding cost = $\frac{1}{2}HQ[1 - (d/p)]$ $(D/Q)S = \frac{1}{2}HQ[1 - (d/p)]$ $Q^2 = \frac{2DS}{H[1 - (d/p)]}$ $Q_{p}^{*} = \sqrt{\frac{2DS}{H[1 - (d/p)]}}$

Production Order Quantity Example

- *D* = 1,000 *units S* = \$10 *H* = \$0.50 *per unit per year*
- *p* = 8 *units per day d* = 4 *units per day*

$$Q^* = \sqrt{\frac{2DS}{H[1 - (d/p)]}}$$

$$Q^* = \sqrt{\frac{2(1,000)(10)}{0.50[1-(4/8)]}} = \sqrt{80,000}$$

= 282.8 or 283 hubcaps

Note:



When annual data are used the equation becomes

$$Q^{*} = \sqrt{\frac{2DS}{H\left(1 - \frac{annual\ demand\ rate}{annual\ production\ rate}\right)}}$$

- Reduced prices are often available when larger quantities are purchased
- It add Trade-off is between reduced product cost and increased holding cost

Total cost = Setup cost + Holding cost + Product cost

$$TC = \frac{D}{Q}S + \frac{Q}{2}H + PD$$

A typical quantity discount schedule

Discount Number	Discount Quantity	Discount (%)	Discount Price (P)
1	0 <i>to</i> 999	no discount	\$5.00
2	1,000 <i>to</i> 1,999	4	\$4.80
3	2,000 and over	5	\$4.75

Table 12.2

Steps in analyzing a quantity discount

- 1. For each discount, calculate Q*
- 2. If Q* for a discount doesn't qualify, choose the smallest possible order size to get the discount
- 3. Compute the total cost for each Q* or adjusted value from Step 2
- 4. Select the Q* that gives the lowest total cost



Quantity Discount Example

Calculate Q* for every discount

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_{1}^{*} = \sqrt{\frac{2(5,000)(49)}{(.2)(5.00)}} = 700 \ cars/order$$
$$Q_{2}^{*} = \sqrt{\frac{2(5,000)(49)}{(.2)(4.80)}} = 714 \ cars/order$$
$$Q_{3}^{*} = \sqrt{\frac{2(5,000)(49)}{(.2)(4.75)}} = 718 \ cars/order$$

Quantity Discount Example

Calculate Q* for every discount

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_{1}^{*} = \sqrt{\frac{2(5,000)(49)}{(.2)(5.00)}} = 700 \ cars/order$$
$$Q_{2}^{*} = \sqrt{\frac{2(5,000)(49)}{(.2)(4.80)}} = 744 \ cars/order$$
$$1,000 - adjusted$$
$$Q_{3}^{*} = \sqrt{\frac{2(5,000)(49)}{(.2)(4.75)}} = 748 \ cars/order$$
$$2,000 - adjusted$$

Quantity Discount Example

Discount Number	Unit Price	Order Quantity	Annual Product Cost	Annual Ordering Cost	Annual Holding Cost	Total
1	\$5.00	700	\$25,000	\$350	\$350	\$25,700
2	\$4.80	1,000	\$24,000	\$245	\$480	\$24,725
3	\$4.75	2,000	\$23.750	\$122.50	\$950	\$24,822.50

Table 12.3

Choose the price and quantity that gives the lowest total cost

Buy 1,000 units at \$4.80 per unit

Probabilistic Models and Safety Stock

- ☑ Used when demand is not constant or certain
- ✓ Use safety stock to achieve a desired service level and avoid stockouts

ROP = d x L + ss

Annual stockout costs = the sum of the units short x the probability x the stockout cost/unit x the number of orders per year

Safety Stock Example

ROP = 50 *units Orders per year* = 6 Stockout cost = \$40 per frame Carrying cost = \$5 per frame per year

Number of Units		Probability	
	30	.2	
	40	.2	
$ROP \rightarrow$	50	.3	
	60	.2	
	70	.1	
	-	1.0	

Safety Stock Example

ROP = 50 *units Orders per year* = 6 Stockout cost = \$40 per frame Carrying cost = \$5 per frame per year

Safety Stock	Additional Holding Cost	Stockout Cost		Total Cost
20	(20)(\$5) = \$100		\$0	\$100
10	(10)(\$5) = \$ 50	(10)(.1)(\$40)(6)	= \$240	\$290
0	\$ 0	(10)(.2)(\$40)(6) + (20)(.1)(\$40)(6)	= \$960	\$960

A safety stock of 20 frames gives the lowest total cost ROP = 50 + 20 = 70 frames

Probabilistic Demand



Probabilistic Demand



Probabilistic Demand

Use prescribed service levels to set safety stock when the cost of stockouts cannot be determined

ROP = demand during lead time + Z\sigma_{dLT}

- where **Z** = number of standard deviations
 - σ_{dLT} = standard deviation of demand during lead time

Probabilistic Example

Average demand = μ = 350 kits Standard deviation of demand during lead time = σ_{dLT} = 10 kits 5% stockout policy (service level = 95%)

Using Appendix I, for an area under the curve of 95%, the Z = 1.65

Safety stock = $Z_{\sigma_{dLT}}$ = 1.65(10) = 16.5 kits

Reorder point = expected demand during lead time + safety stock

- = 350 kits + 16.5 kits of safety stock
- = 366.5 or 367 kits

Other Probabilistic Models

When data on demand during lead time is not available, there are other models available

- 1. When demand is variable and lead time is constant
- 2. When lead time is variable and demand is constant
- 3. When both demand and lead time are variable

Other Probabilistic Models

Demand is variable and lead time is constant

ROP = (average daily demand x lead time in days) + $Z\sigma_{dLT}$

where $\sigma_d = standard deviation of demand per day$ $<math>\sigma_{dLT} = \sigma_d \sqrt{lead time}$

Probabilistic Example

Average daily demand (normally distributed) = 15Standard deviation = 5Lead time is constant at 2 daysZ for 90% = 1.2890% service level desiredFrom Appendix I

ROP = (15 units x 2 days) + $Z\sigma_{dlt}$ = 30 + 1.28(5)($\sqrt{2}$) = 30 + 9.02 = 39.02 ≈ 39

Safety stock is about 9 iPods

Other Probabilistic Models

Lead time is variable and demand is constant

- ROP = (daily demand x average lead time in days)
 - = Z x (daily demand) $x \sigma_{LT}$
- where σ_{LT} = standard deviation of lead time in days

Probabilistic Example

Z for 98% = 2.055Daily demand (constant) = 10From Appendix IAverage lead time = 6 daysStandard deviation of lead time = σ_{LT} = 398% service level desired

ROP = (10 units x 6 days) + 2.055(10 units)(3)= 60 + 61.65 = 121.65

Reorder point is about 122 cameras
Other Probabilistic Models

Both demand and lead time are variable

ROP = (average daily demand x average lead time) + $Z\sigma_{dLT}$

where
$$\sigma_d = standard deviation of demand per day$$

 $\sigma_{LT} = standard deviation of lead time in days$
 $\sigma_{dLT} = \sqrt{(average lead time x \sigma_d^2)}$
 $+ (average daily demand)^2 x \sigma_{LT}^2$

Probabilistic Example

Average daily demand (normally distributed) = 150Standard deviation = σ_d = 16Average lead time 5 days (normally distributed)Standard deviation = σ_{LT} = 1 day95% service level desiredZ for 95% = 1.65

From Appendix I

$$ROP = (150 \ packs \ x \ 5 \ days) + 1.65\sigma_{dLT}$$

= (150 x 5) + 1.65 $\sqrt{(5 \ days \ x \ 16^2) + (150^2 \ x \ 1^2)}$
= 750 + 1.65(154) = 1,004 packs

Fixed-Period (P) Systems

- ✓ Orders placed at the end of a fixed period
- ☑ Inventory counted only at end of period
- ✓ Order brings inventory up to target level
 - **⊘** Only relevant costs are ordering and holding
 - ☑ Lead times are known and constant
 - ☑ Items are independent from one another

Fixed-Period (P) Systems



Figure 12.9

Fixed-Period (P) Example

3 jackets are back ordered It is time to place an order *No jackets are in stock Target value* = 50

Order amount (Q) = Target (T) - Onhand inventory - Earlier orders not yet received + Back orders

Q = 50 - 0 - 0 + 3 = 53 *jackets*

Fixed-Period Systems

- ☑ Inventory is only counted at each review period
- ☑ May be scheduled at convenient times
- **☑** Appropriate in routine situations
- ☑ May result in stockouts between periods
- ☑ May require increased safety stock