# Operations Management Unit-IV Inventory Management 



## Outline

$\square$ Global Company Profile: Amazon.com
$\square$ Functions of Inventory
V Types of Inventory
V Inventory Management

- ABC Analysis
$\square$ Record Accuracy
■ Cycle Counting
ஏ Control of Service Inventories


## Outline - Continued

V Inventory Models
অ Independent vs. Dependent Demand
๒ Holding, Ordering, and Setup Costs

## Outline - Continued

『 Inventory Models for Independent Demand
$\square$ The Basic Economic Order Quantity (EOQ) Model
$\square$ Minimizing Costs
$\square$ Reorder Points
『 Production Order Quantity Model
■ Quantity Discount Models

## Outline - Continued

『 Probabilistic Models and Safety Stock
$\square$ Other Probabilistic Models
マ Fixed-Period (P) Systems

## Learning Objectives

## When you complete this chapter you should be able to:

1. Conduct an ABC analysis
2. Explain and use cycle counting
3. Explain and use the EOQ model for independent inventory demand
4. Compute a reorder point and safety stock

## Learning Objectives

## When you complete this chapter you should be able to:

5. Apply the production order quantity model
6. Explain and use the quantity discount model
7. Understand service levels and probabilistic inventory models

## Amazon.com

Amazon.com started as a "virtual" retailer - no inventory, no warehouses, no overhead; just computers taking orders to be filled by others
$\square$ Growth has forced Amazon.com to become a world leader in warehousing and inventory management

## Amazon.com

1. Each order is assigned by computer to the closest distribution center that has the product(s)
2. A"flow meister" at each distribution center assigns work crews
3. Lights indicate products that are to be picked and the light is reset
4. Items are placed in crates on a conveyor. Bar code scanners scan each item 15 times to virtually eliminate errors.

## Amazon.com

5. Crates arrive at central point where items are boxed and labeled with new bar code
6. Gift wrapping is done by hand at 30 packages per hour
7. Completed boxes are packed, taped, weighed and labeled before leaving warehouse in a truck
8. Order arrives at customer within a week

## Inventory

$\checkmark$ One of the most expensive assets of many companies representing as much as $50 \%$ of total invested capital
$\square$ Operations managers must balance inventory investment and customer service

## Functions of Inventory

1. To decouple or separate various parts of the production process
2. To decouple the firm from fluctuations in demand and provide a stock of goods that will provide a selection for customers
3. To take advantage of quantity discounts
4. To hedge against inflation

## Types of Inventory

$\square$ Raw material
V Purchased but not processed
చ Work-in-process
■ Undergone some change but not completed
V function of cycle time for a product
■ Maintenance/repair/operating (MRO)
$\boxtimes$ Necessary to keep machinery and processes productive
$\square$ Finished goods
$\boxtimes$ Completed product awaiting shipment

## The Material Flow Cycle


$\xrightarrow{\text { Input }} \begin{gathered}\text { Wait for } \\ \text { inspection }\end{gathered} \begin{gathered}\text { Wait to } \\ \text { be moved }\end{gathered} \begin{gathered}\text { Move Wait in queue Setup } \\ \text { time }\end{gathered} \begin{gathered}\text { Run } \\ \text { foperator time }\end{gathered} \quad \xrightarrow{\text { time }} \xrightarrow{\text { Output }}$


Figure 12.1

## Inventory Management

$\nabla$ How inventory items can be classified

- How accurate inventory records can be maintained


## ABC Analysis

## $\nabla$ Divides inventory into three classes

 based on annual dollar volumeV Class A - high annual dollar volume
$\square$ Class B-medium annual dollar volume
V Class C - Iow annual dollar volume
$\boxtimes$ Used to establish policies that focus on the few critical parts and not the many trivial ones

## ABC Analysis

| Item Stock Number | Percent of Number of Items Stocked | Annual Volume (units) | $x$ | Unit Cost | = | Annual Dollar Volume | Percent of Annual Dollar Volume |  | Class |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \#10286 | 20\% | 1,000 |  | \$ 90.00 |  | \$ 90,000 | 38.8\% |  | A |
| \#11526 |  | 500 |  | 154.00 |  | 77,000 | 33.2\% |  | A |
| \#12760 |  | 1,550 |  | 17.00 |  | 26,350 | 11.3\% |  | B |
| \#10867 | 30\% | 350 |  | 42.86 |  | 15,001 | 6.4\% | 23\% | $B$ |
| \#10500 |  | 1,000 |  | 12.50 |  | 12,500 | 5.4\% |  | $B$ |

## ABC Analysis

| Item <br> Stock <br> Number | Percent of Number of Items Stocked | Annual Volume (units) | $\boldsymbol{x}$ | Unit Cost | = | Annual Dollar Volume | Percent of Annual Dollar Volume |  | Class |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \#12572 |  | 600 |  | \$ 14.17 |  | \$ 8,502 | 3.7\% |  | C |
| \#14075 |  | 2,000 |  | . 60 |  | 1,200 | . $5 \%$ |  | C |
| \#01036 | 50\% | 100 |  | 8.50 |  | 850 | .4\% $\}$ |  | C |
| \#01307 |  | 1,200 |  | . 42 |  | 504 | .2\% |  | C |
| \#10572 |  | 250 |  | . 60 |  | 150 | .1\% |  | C |
|  |  | 8,550 |  |  |  | \$232,057 | 100.0\% |  |  |

## ABC Analysis

Percent of annual dollar usage


Percent of inventory items
Figure 12.2

## ABC Analysis

『 Other criteria than annual dollar volume may be used
$\square$ Anticipated engineering changes

- Delivery problems
$\square$ Quality problems
$\boxtimes$ High unit cost


## ABC Analysis

$\square$ Policies employed may include
$\boxtimes$ More emphasis on supplier development for A items
च Tighter physical inventory control for A items
$\boxtimes$ More care in forecasting $A$ items

## Record Accuracy

$\square$ Accurate records are a critical ingredient in production and inventory systems
$\square$ Allows organization to focus on what is needed

অ Necessary to make precise decisions about ordering, scheduling, and shipping

V Incoming and outgoing record keeping must be accurate

V Stockrooms should be secure

## Cycle Counting

■ Items are counted and records updated on a periodic basis
$\checkmark$ Often used with ABC analysi to determine cycle
$\square$ Has several advantages


V Eliminates shutdowns and interruptions
V Eliminates annual inventory adjustment
Trained personnel audit inventory accuracy
$\square$ Allows causes of errors to be identified and corrected

V Maintains accurate inventory records

## Cycle Counting Example

5,000 items in inventory, 500 A items, 1,750 B items, 2,750 C items

Policy is to count A items every month (20 working days), B items every quarter (60 days), and C items every six months (120 days)

| Item <br> Class | Quantity | Cycle Counting Policy | Number of Items <br> Counted per Day |
| :---: | :---: | :--- | ---: |
| A | 500 | Each month | $500 / 20=25 /$ day |
| $B$ | 1,750 | Each quarter | $1,750 / 60=29 /$ day |
| C | 2,750 | Every 6 months | $2,750 / 120=23 /$ day |
|  |  |  | 77/day |

## Control of Service Inventories

$\nabla$ Can be a critical component of profitability
$\downarrow$ Losses may come from shrinkage or pilferage

$\square$ Applicable techniques include

1. Good personnel selection, training, and discipline
2. Tight control on incoming shipments
3. Effective control on all goods leaving facility

## Independent Versus Dependent Demand

V Independent demand - the demand for item is independent of the demand for any other item in inventory
V Dependent demand - the demand for item is dependent upon the demand for some other item in the inventory

## Holding, Ordering, and Setup Costs

$\square$ Holding costs - the costs of holding or "carrying" inventory over time
$\nabla$ Ordering costs - the costs of placing an order and receiving goods
$\square$ Setup costs - cost to prepare a machine or process for manufacturing an order

## Holding Costs

| Category | Cost (and range) <br> as a Percent of <br> Inventory Value |
| :--- | ---: |
| Housing costs (building rent or <br> depreciation, operating costs, taxes, <br> insurance) | $6 \%$ (3-10\%) |
| Material handling costs (equipment lease or <br> depreciation, power, operating cost) | $3 \%$ (1-3.5\%) |
| Labor cost | $3 \%(3-5 \%)$ |
| Investment costs (borrowing costs, taxes, <br> and insurance on inventory) <br> Pilferage, space, and obsolescence | $11 \%$ (6-24\%) |
| Overall carrying cost | $3 \%(2-5 \%)$ |

Table 12.1

## Holding Costs

Holding costs vary considerably depending on the business, location, and interest rates. Generally greater than $15 \%$, some high tech items have holding costs greater than $50 \%$.

## Inventory Models for Independent Demand

## Need to determine when and how much to order

$\square$ Basic economic order quantity
V Production order quantity
マ Quantity discount model

## Basic EOQ Model

## Important assumptions

1. Demand is known, constant, and independent
2. Lead time is known and constant
3. Receipt of inventory is instantaneous and complete
4. Quantity discounts are not possible
5. Only variable costs are setup and holding
6. Stockouts can be completely avoided

## Inventory Usage Over Time



Figure 12.3

## Minimizing Costs

## Objective is to minimize total costs



## The EOQ Model

Q = Number of pieces per order
Q* = Optimal number of pieces per order (EOQ)
D = Annual demand in units for the inventory item
S = Setup or ordering cost for each order
H = Holding or carrying cost per unit per year

Annual setup cost $=$ (Number of orders placed per year) x (Setup or order cost per order)

$$
\begin{aligned}
& =\left(\frac{\text { Annual demand }}{\text { Number of units in each order }}\right)\binom{\text { Setup or order }}{\text { cost per order }} \\
& =\left[\frac{D}{Q}\right](S) \\
& \text { Annual setup cost }=\frac{D}{Q} s
\end{aligned}
$$

## The EOQ Model

Q = Number of pieces per order
Q* = Optimal number of pieces per order (EOQ)
D = Annual demand in units for the inventory item
S = Setup or ordering cost for each order
H = Holding or carrying cost per unit per year

Annual holding cost $=$ (Average inventory level) x (Holding cost per unit per year)

$$
\begin{aligned}
& =\left(\frac{\text { Order quantity }}{2}\right) \text { (Holding cost per unit per year) } \\
& =\left[\frac{Q}{2}\right](H) \quad \text { Annual setup } \cos t=\frac{D}{Q} s \\
& \text { Annual holding } \operatorname{cost}=\frac{Q}{2} H
\end{aligned}
$$

## The EOQ Model

Q = Number of pieces per order
$Q^{*}=$ Optimal number of pieces per order (EOQ)
D = Annual demand in units for the inventory item
S = Setup or ordering cost for each order
H = Holding or carrying cost per unit per year
Optimal order quantity is found when annual setup cost equals annual holding cost

$$
\frac{D}{Q} S=\frac{Q}{2} H
$$

Solving for $Q^{*}$

$$
\begin{gathered}
2 D S=Q^{2} H \\
Q^{2}=2 D S / H \\
Q^{*}=\sqrt{2 D S / H}
\end{gathered}
$$

$$
\begin{gathered}
\text { Annual setup cost }=\frac{D}{Q} s \\
\text { Annual holding cost }=\frac{Q}{2} H
\end{gathered}
$$

## An EOQ Example

Determine optimal number of needles to order
$D=1,000$ units
$S=\$ 10$ per order
H = \$. 50 per unit per year

$$
Q^{*}=\sqrt{\frac{2 D S}{H}}
$$

$Q^{*}=\sqrt{\frac{2(1,000)(10)}{0.50}}=\sqrt{40,000}=200$ units

## An EOQ Example

Determine optimal number of needles to order $D=1,000$ units Q* $=200$ units
$S=\$ 10$ per order
$H=\$ .50$ per unit per year
$\begin{aligned} & \text { Expected } \\ & \begin{array}{c}\text { number of } \\ \text { orders }\end{array}\end{aligned}=N=\frac{\text { Demand }}{\text { Order quantity }}=\frac{D}{Q^{*}}$

$$
N=\frac{1,000}{200}=5 \text { orders per year }
$$

## An EOQ Example

Determine optimal number of needles to order
$D=1,000$ units
$S=\$ 10$ per order
$H=\$ .50$ per unit per year

Q* $=200$ units
N = 5 orders per year

Number of working
Expected days per year
time between $=T=$ orders

$$
T=\frac{250}{5}=50 \text { days between orders }
$$

## An EOQ Example

Determine optimal number of needles to order
$D=1,000$ units
$S=\$ 10$ per order
$H=\$ .50$ per unit per year $\quad T=50$ days

Total annual cost $=$ Setup cost + Holding cost

$$
\begin{aligned}
& T C=\frac{D}{Q} S+\frac{Q}{2} H \\
& T C=\frac{1,000}{200}(\$ 10)+\frac{200}{2}(\$ .50)
\end{aligned}
$$

$$
T C=(5)(\$ 10)+(100)(\$ .50)=\$ 50+\$ 50=\$ 100
$$

## Robust Model

■ The EOQ model is robust
$\square$ It works even if all parameters and assumptions are not met
$\checkmark$ The total cost curve is relatively flat in the area of the EOQ

## An EOQ Example

Management underestimated demand by 50\% $D=7,000$ unitc 1,500 units $Q^{*}=200$ units
$S=\$ 10$ per order $\quad N=5$ orders per year
$H=\$ .50$ per unit per year $\quad T=50$ days

$$
\begin{aligned}
& T C=\frac{D}{Q} S+\frac{Q}{2} H \\
& T C=\frac{1,500}{200}(\$ 10)+\frac{200}{2}(\$ .50)=\$ 75+\$ 50=\$ 125
\end{aligned}
$$

Total annual cost increases by only 25\%

## An EOQ Example

Actual EOQ for new demand is 244.9 units $D=7,0000$ unitc 1,500 units $Q^{*}=244.9$ units
$S=\$ 10$ per order $\quad N=5$ orders per year $H=\$ .50$ per unit per year $\quad T=50$ days

$$
\begin{aligned}
& T C=\frac{D}{Q} S+\frac{Q}{2} H \\
& T C=\frac{1,500}{244.9}(\$ 10)+\frac{244.9}{2}(\$ .50)
\end{aligned}
$$

$T C=\$ 61.24$ + $\$ 61.24=\$ 122.48$

Only 2\% less than the total cost of \$125 when the order quantity was 200

## Reorder Points

■ EOQ answers the "how much" question
V The reorder point (ROP) tells when to order

$$
\begin{aligned}
R O P & =\binom{\text { Demand }}{\text { per day }}\binom{\text { Lead time for a }}{\text { new order in days }} \\
& =d \times L
\end{aligned}
$$

$$
d=\frac{D}{\text { Number of working days in a year }}
$$

## Reorder Point Curve

Figure 12.5


## Reorder Point Example

Demand = 8,000 iPods per year 250 working day year
Lead time for orders is 3 working days

$$
\begin{aligned}
d & =\frac{D}{\text { Number of working days in a year }} \\
& =8,000 / 250=32 \text { units }
\end{aligned}
$$

$$
R O P=d x L
$$

$=32$ units per day $\times 3$ days $=96$ units

## Production Order Quantity Model

$\nabla$ Used when inventory builds up over a period of time after an order is placed
U Used when units are produced and sold simultaneously

## Production Order Quantity Model



Figure 12.6

## Production Order Quantity Model

$Q=$ Number of pieces per order $\quad p=$ Daily production rate
$H=$ Holding cost per unit per year d= Daily demand/usage rate
$t=$ Length of the production run in days
$\binom{$ Annual inventory }{ holding cost }$=\left(\right.$ Average inventory level) $\times\left[\begin{array}{c}\text { Holding cost } \\ \text { per unit per year }\end{array}\right)$
$\binom{$ Annual inventory }{ level }$=($ Maximum inventory level $) / 2$
$\binom{$ Maximum }{ inventory level }$=\binom{$ Total produced during }{ the production run }$-\binom{$ Total used during }{ the production run }

$$
=p t-d t
$$

## Production Order Quantity Model

$$
\begin{aligned}
Q & =\text { Number of pieces per order } \quad p=\text { Daily production rate } \\
H & =\text { Holding cost per unit per year } \quad d=\text { Daily demand/usage rate } \\
t & =\text { Length of the production run in days }
\end{aligned}
$$

$$
\binom{\text { Maximum }}{\text { inventory level }}=\binom{\text { Total produced during }}{\text { the production run }}-\binom{\text { Total used during }}{\text { the production run }}
$$

$$
=p t-d t
$$

However, $Q=$ total produced $=p t ;$ thus $t=Q / p$
$\binom{$ Maximum }{ inventory level }$=p\left(\frac{Q}{p}\right)-d\left(\frac{Q}{p}\right)=Q\left(1-\frac{d}{p}\right)$
Holding cost $=\frac{\text { Maximum inventory level }}{2}(H)=\frac{Q}{2}\left[1-\left(\frac{d}{p}\right)\right] H$

## Production Order Quantity Model

Q = Number of pieces per order<br>$H=$ Holding cost per unit per year<br>\[ \begin{aligned} \& p=Daily production rate<br>\& d=Daily demand/usage rate \end{aligned} \]

Setup cost = (D/Q)S Holding cost $=\frac{1}{2} H Q[1-(d / p)]$

$$
(D / Q) S=\frac{1}{2} H Q[1-(d / p)]
$$

$$
Q^{2}=\frac{2 D S}{H[1-(d / p)]}
$$

$$
Q_{p}^{*}=\sqrt{\frac{2 D S}{H[1-(d / p)]}}
$$

## Production Order Quantity Example

$D=1,000$ units
$S=\$ 10$
$H=\$ 0.50$ per unit per year

$$
\begin{aligned}
Q^{*} & =\sqrt{\frac{2 D S}{H[1-(d / p)]}} \\
Q^{*} & =\sqrt{\frac{2(1,000)(10)}{0.50[1-(4 / 8)]}}=\sqrt{80,000} \\
& =282.8 \text { or } 283 \text { hubcaps }
\end{aligned}
$$

## Production Order Quantity Model

Note:

$$
d=4=\frac{D}{\text { Number of days the plant is in operation }}=\frac{1,000}{250}
$$

When annual data are used the equation becomes

$$
Q^{*}=\sqrt{\frac{2 D S}{H\left(1-\frac{\text { annual demand rate }}{\text { annual production rate }}\right)}}
$$

## Quantity Discount Models

$\checkmark$ Reduced prices are often available when larger quantities are purchased
$\square$ Trade-off is between reduced product cost and increased holding cost

Total cost $=$ Setup cost + Holding cost + Product cost

$$
T C=\frac{D}{Q} S+\frac{Q}{2} H+P D
$$

## Quantity Discount Models

A typical quantity discount schedule

| Discount <br> Number | Discount Quantity | Discount (\%) | Discount <br> Price ( $P$ ) |
| :---: | :---: | :---: | :---: |
| 1 | 0 to 999 | no discount | $\$ 5.00$ |
| 2 | 1,000 to 1,999 | 4 | $\$ 4.80$ |
| 3 | 2,000 and over | 5 | $\$ 4.75$ |

Table 12.2

## Quantity Discount Models

## Steps in analyzing a quantity discount

1. For each discount, calculate $Q^{*}$
2. If $Q^{*}$ for a discount doesn't qualify, choose the smallest possible order size to get the discount
3. Compute the total cost for each $Q^{*}$ or adjusted value from Step 2
4. Select the $Q^{*}$ that gives the lowest total cost

## Quantity Discount Models



## Quantity Discount Example

Calculate $Q^{*}$ for every discount

$$
Q^{*}=\sqrt{\frac{2 D S}{I P}}
$$

$$
\begin{aligned}
& Q_{1}{ }^{*}=\sqrt{\frac{2(5,000)(49)}{(.2)(5.00)}}=700 \text { cars/order } \\
& Q_{2}{ }^{*}=\sqrt{\frac{2(5,000)(49)}{(.2)(4.80)}}=714 \text { cars/order } \\
& Q_{3}{ }^{*}=\sqrt{\frac{2(5,000)(49)}{(.2)(4.75)}}=718 \text { cars/order }
\end{aligned}
$$

## Quantity Discount Example

Calculate $Q^{*}$ for every discount


$$
\begin{aligned}
& Q_{1}{ }^{*}=\sqrt{\frac{2(5,000)(49)}{(.2)(5.00)}}=700 \text { cars/order } \\
& Q_{2}{ }^{*}=\sqrt{\frac{2(5,000)(49)}{(.2)(4.80)}}=\begin{array}{l}
1,1,1 \mathrm{cars} / \text { order } \\
1,000-\text { adjusted }
\end{array}
\end{aligned}
$$

$$
Q_{3}{ }^{*}=\sqrt{\frac{2(5,000)(49)}{(.2)(4.75)}}=\begin{aligned}
& \quad, 18 \text { cars/order } \\
& 2,000-\text { adjusted }
\end{aligned}
$$

# Quantity Discount Example 

| Discount <br> Number | Unit <br> Price | Order <br> Quantity | Annual <br> Product <br> Cost | Annual <br> Ordering <br> Cost | Annual <br> Holding <br> Cost | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\$ 5.00$ | 700 | $\$ 25,000$ | $\$ 350$ | $\$ 350$ | $\$ 25,700$ |
| 2 | $\$ 4.80$ | 1,000 | $\$ 24,000$ | $\$ 245$ | $\$ 480$ | $\$ 24,725$ |
| 3 | $\$ 4.75$ | 2,000 | $\$ 23.750$ | $\$ 122.50$ | $\$ 950$ | $\$ 24,822.50$ |

Table 12.3
Choose the price and quantity that gives the lowest total cost Buy 1,000 units at $\$ 4.80$ per unit

## Probabilistic Models and Safety Stock

$\square$ Used when demand is not constant or certain

ஏ Use safety stock to achieve a desired service level and avoid stockouts

$$
R O P=d x L+s s
$$

Annual stockout costs $=$ the sum of the units short $x$ the probability $x$ the stockout cost/unit $x$ the number of orders per year

## Safety Stock Example

ROP = 50 units Orders per year = 6

Stockout cost $=\$ 40$ per frame
Carrying cost = \$5 per frame per year

| Number of Units | Probability |  |
| :---: | :---: | :---: |
|  | 30 | .2 |
|  | 40 | .2 |
| $R O P \rightarrow$ | 50 | .3 |
|  | 60 | .2 |
|  | 70 | .1 |
|  |  | 1.0 |

## Safety Stock Example

ROP = 50 units
Orders per year = 6

Stockout cost $=\$ 40$ per frame
Carrying cost = \$5 per frame per year

| Safety <br> Stock | Additional <br> Holding Cost | Stockout Cost |  | Total <br> Cost |
| :---: | :---: | :---: | :---: | :---: |
| 20 | $(20)(\$ 5)=\$ 100$ | $\$ 0$ | $\$ 100$ |  |
| 10 | $(10)(\$ 5)=\$ 50$ | $(10)(.1)(\$ 40)(6)$ | $=\$ 240$ | $\$ 290$ |
| 0 | $\$ 0$ | $(10)(.2)(\$ 40)(6)+(20)(.1)(\$ 40)(6)=\$ 960$ | $\$ 960$ |  |

A safety stock of 20 frames gives the lowest total cost $R O P=50+20=70$ frames

## Probabilistic Demand



## Probabilistic Demand



## Probabilistic Demand

Use prescribed service levels to set safety stock when the cost of stockouts cannot be determined
$R O P=$ demand during lead time $+\mathrm{Z} \sigma_{d L T}$
where

$$
\begin{aligned}
\mathrm{Z} & =\text { number of standard deviations } \\
\sigma_{d L T}= & \text { standard deviation of demand } \\
& \text { during lead time }
\end{aligned}
$$

## Probabilistic Example

Average demand $=\mu=350$ kits
Standard deviation of demand during lead time $=\sigma_{d L T}=10$ kits 5\% stockout policy (service level = 95\%)

## Using Appendix I, for an area under the curve of $95 \%$, the $Z=1.65$

$$
\text { Safety stock }=Z \sigma_{d L T}=1.65(10)=16.5 \text { kits }
$$

Reorder point $=$ expected demand during lead time + safety stock
$=350$ kits + 16.5 kits of safety stock
$=366.5$ or 367 kits

## Other Probabilistic Models

When data on demand during lead time is not available, there are other models available

1. When demand is variable and lead time is constant
2. When lead time is variable and demand is constant
3. When both demand and lead time are variable

# Other Probabilistic Models 

## Demand is variable and lead time is constant

$$
\begin{aligned}
R O P= & (\text { average daily demand } \\
& x \text { lead time in days })+\mathrm{Z}_{\text {dLT }}
\end{aligned}
$$

where

$$
\begin{aligned}
\sigma_{d} & =\text { standard deviation of demand per day } \\
\sigma_{d L T} & =\sigma_{d} \sqrt{\text { lead time }}
\end{aligned}
$$

## Probabilistic Example

Average daily demand (normally distributed) $=15$ Standard deviation = 5 Lead time is constant at 2 days $\quad Z$ for $90 \%=1.28$ 90\% service level desired

From Appendix I

$$
\begin{aligned}
R O P & =(15 \text { units } x 2 \text { days })+Z \sigma_{\text {dlt }} \\
& =30+1.28(5)(\sqrt{2}) \\
& =30+9.02=39.02 \approx 39
\end{aligned}
$$

Safety stock is about 9 iPods

## Other Probabilistic Models

Lead time is variable and demand is constant

$$
\begin{aligned}
R O P= & (\text { daily demand } x \text { average lead } \\
& \text { time in days }) \\
= & Z x(\text { daily demand }) x \sigma_{L T}
\end{aligned}
$$

where

$$
\sigma_{L T}=\text { standard deviation of lead time in days }
$$

## Probabilistic Example

Daily demand $($ constant $)=10$
Z for 98\% = 2.055
From Appendix I
Average lead time $=6$ days
Standard deviation of lead time $=\sigma_{L T}=3$
98\% service level desired

$$
\begin{aligned}
R O P & =(10 \text { units } \times 6 \text { days })+2.055(10 \text { units })(3) \\
& =60+61.65=121.65
\end{aligned}
$$

Reorder point is about 122 cameras

## Other Probabilistic Models

## Both demand and lead time are variable

$$
\begin{aligned}
R O P= & \text { (average daily demand } \\
& x \text { average lead time) }+Z \sigma_{d L T}
\end{aligned}
$$

where

$$
\begin{aligned}
\sigma_{d} & =\text { standard deviation of demand per day } \\
\sigma_{L T} & =\text { standard deviation of lead time in days } \\
\sigma_{d L T} & =\sqrt{\begin{array}{l}
\left(\text { average lead time } x \sigma_{d}^{2}\right) \\
+(\text { average daily demand })^{2} \times \sigma_{L T}^{2}
\end{array}}
\end{aligned}
$$

## Probabilistic Example

Average daily demand (normally distributed) $=150$ Standard deviation $=\sigma_{d}=16$ Average lead time 5 days (normally distributed) Standard deviation $=\sigma_{L T}=1$ day 95\% service level desired
$Z$ for $95 \%=1.65$
From Appendix I
$R O P=(150$ packs $x 5$ days $)+1.65 \sigma_{d L T}$

$$
\begin{aligned}
& =(150 \times 5)+1.65 \sqrt{\left(5 \text { days } \times 16^{2}\right)+\left(150^{2} \times 1^{2}\right)} \\
& =750+1.65(154)=1,004 \text { packs }
\end{aligned}
$$

## Fixed-Period (P) Systems

$\boxtimes$ Orders placed at the end of a fixed period
च Inventory counted only at end of period
$\square$ Order brings inventory up to target level
$\square$ Only relevant costs are ordering and holding
■ Lead times are known and constant
$\square$ Items are independent from one another

## Fixed-Period (P) Systems



Time
Figure 12.9

## Fixed-Period (P) Example

3 jackets are back ordered It is time to place an order

No jackets are in stock Target value $=50$

Order amount $(Q)=\operatorname{Target}(T)$ - Onhand inventory - Earlier orders not yet received + Back orders

$$
Q=50-0-0+3=53 \text { jackets }
$$

## Fixed-Period Systems

$\boxtimes$ Inventory is only counted at each review period
$\downarrow$ May be scheduled at convenient times
$\checkmark$ Appropriate in routine situations
$\square$ May result in stockouts between periods
$\downarrow$ May require increased safety stock

