Unsteady Heat Transfer

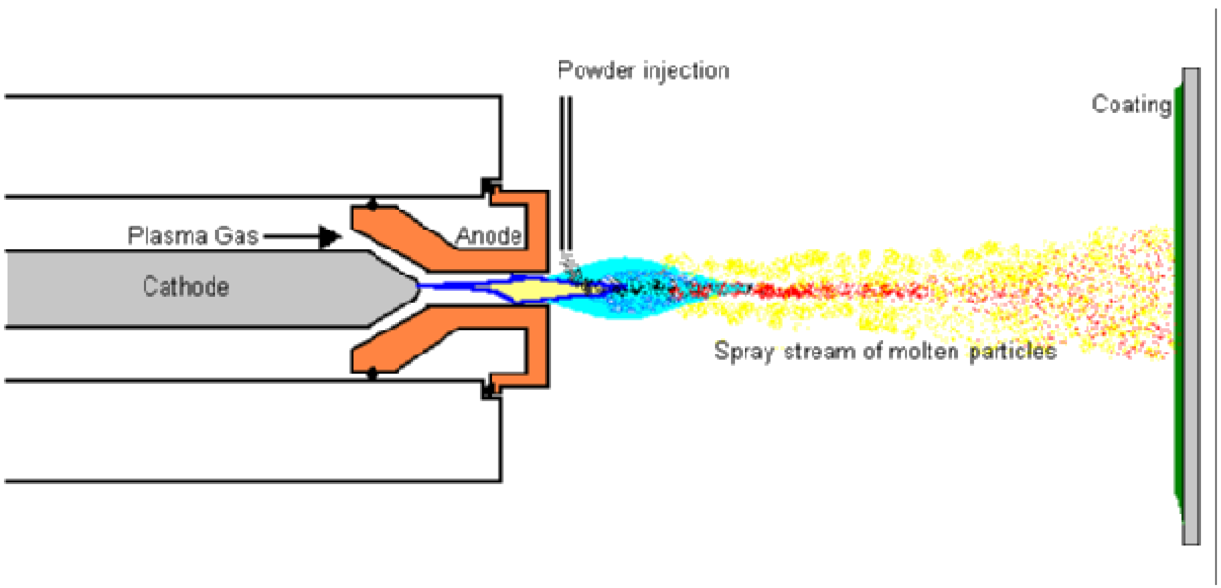
Many heat transfer problems require the understanding of the complete time history of the temperature variation. For example, in metallurgy, the heat treating process can be controlled to directly affect the characteristics of the processed materials. Annealing (slow cool) can soften metals and improve ductility. On the other hand, quenching (rapid cool) can harden the strain boundary and increase strength. In order to characterize this transient behavior, the full unsteady equation is needed:

$$\rho c \frac{\partial T}{\partial t} = k\nabla^2 T, \text{ or } \frac{1}{\alpha}\frac{\partial T}{\partial t} = \nabla^2 T$$

where $\alpha = \frac{\mathrm{k}}{\rho \mathrm{c}}$ is the thermal diffusivity

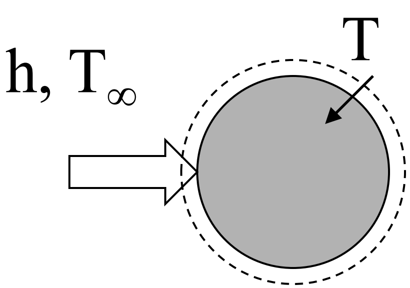
Lumped Capacitance Method (LCM)

The simplest situation in an unsteady heat transfer process is to neglect the temperature distribution inside the solid and only deals with the heat transfer between the solid and the ambient fluids. In other words, we are going to assume the temperature inside the solid is constant and equals to the surface temperature. Let us look at a practical example about a plasma spray process involving the injection of tiny particles into a plasma jet at a very high temperature (see web page for more information). These particles will eventually melt and impinging on the processing surface and solidify to form a layer of protecting coating.



Example: (Plasma Spray)

Assume spherical alumina particles are used in the plasma jet. (diameter D=50 mm, density ρ=3970 kg/m3, thermal conductivity k=10.5 W/m.K and specific heat c_p=1560 J/kg, and an initial temperature of 300 K) The plasma is maintained at a temperature of 10,000 K and has a convection coefficient of h=30,000 W/m^2.K. The melting temperature of the particle is 2318 K and the latent heat of fusion is 3577 kJ/kg. (a) Determine the time required to heat a particle to its melting point, (b) determine the time for the particle to melt completely after it reaches the melting temperature. (Special notes: why the particles follow the plasma jet? Does the particles travel at the same velocity as the local jet velocity? Does the jet has a uniform velocity?



Energy balance: energy in = energy storage in solid

$$hA_S(T_\infty - T) = \frac{dE}{dt} = \frac{d(mc_PT)}{dt}$$

$$hA_S(T_\infty - T) = \rho c_P V \frac{dT}{dt}$$

wher A_S is the surface area of the sphere

Example (cont.)

Define a new variable θ=T-T_∞

$$\rho V c_P \frac{d\theta}{dt} = -hA_S\theta, \ \frac{d\theta}{\theta} = -\frac{hA_S}{\rho V c_P} dt$$

Integrate with respect to time and apply the initial condition

T(t=0)=T_i=300(K) or θ(t=0)=θ_i=T_i-T_∞=300-10000=-9700(K)

$$\ln\theta = \frac{-hA_S}{\rho V c_P} t + C_1, \ \ \theta(t) = C_2 \exp\left[-\left(\frac{hA_S}{\rho V c_P}\right)t\right]$$

Substitute the initial condition: $\theta(0)=\theta_i = C_2$

The general solution for lumped capacitance method

$$\frac{\theta(\mathrm{t})}{\theta_\mathrm{i}} = \frac{T(t)-T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_S}{\rho V c_P}\right)t\right]$$

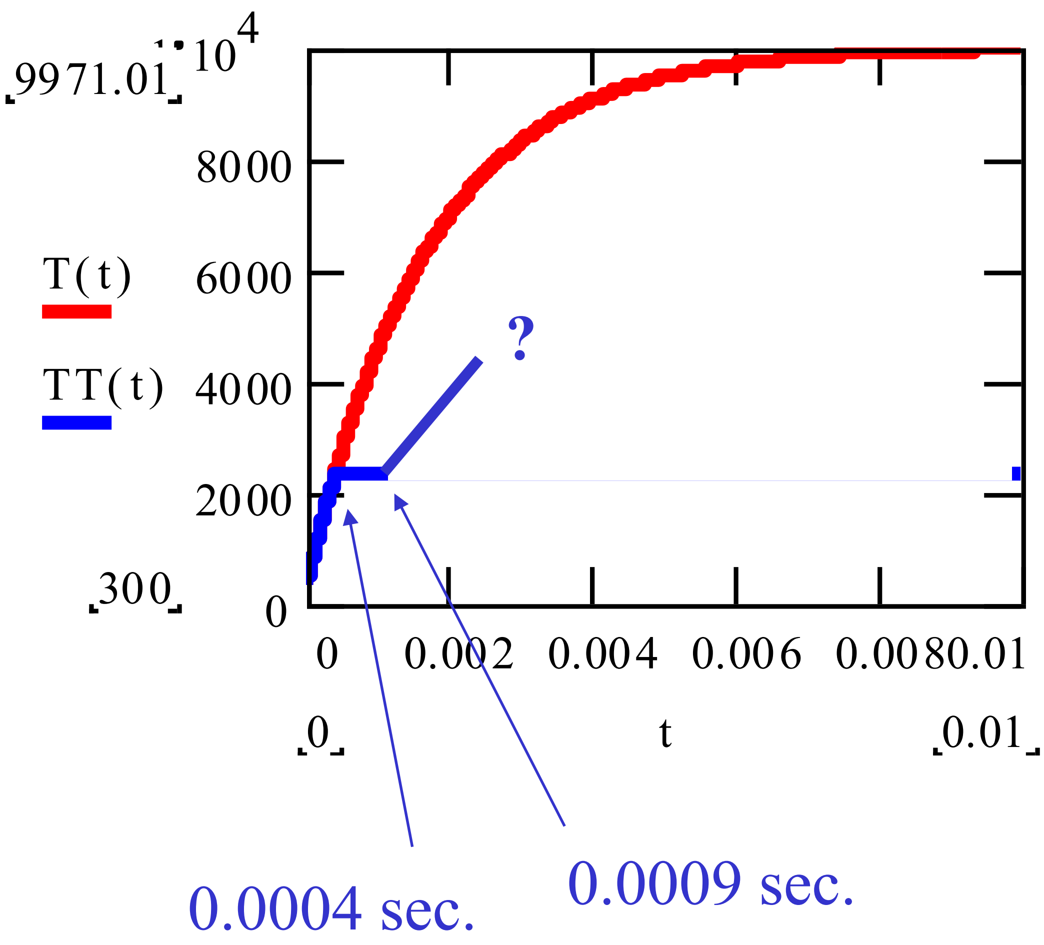
$$\frac{T(t)-10000}{-9700} = \exp\left[-\left(\frac{30000 * \pi D^2}{3970 * (1/6)\pi D^3 * 1560}\right)t\right] = \exp(-581.3t)$$

$$T(t) = 10000 - 9700\exp(-581.3t)$$

Example (cont.)

Temperature distribution as a function of time:

$$T(t) = 10000 - 9700\exp(-581.3t)$$



- Temperature of the particle increases exponentially from 300 K to 10000K in a very short time (<0.01 sec.)
- It only takes 0.0004 sec. To reach the melting temperature
- Therefore, the true temperature variation is described by the blue curve. (why?)

Example (cont.)

After the particle reaches its melting temperature, the heat input will not increase the temperature of the particle anymore. Rather, the heat will be absorbed by the solid particle as latent heat of melting in order for it to melt.

$$\int_{t_1}^{t_2} q_{conv} dt == \Delta E_{sf} = m h_{sf}$$

$$h A_S (T_\infty - T_{melt})(t_2 - t_1) = \rho V h_{sf}$$

$$t_2 - t_1 = \frac{\rho V h_{sf}}{h A_S (T_\infty - T_{melt})} = \frac{3970(50 \times 10^{-6})(3577000)}{6(30000)(10000 - 2318)} = 5 \times 10^{-4} (s)$$

It will take an additional 5×10^{-4} s. to melt the particle

The total time to completely melt the particle will be 9×10^{-4} s.