CONVECTIVE HEAT TRANSFER FROM/ TO FLUID FLOWING IN A STRAIGHT CIRCULAR DUCT (Re < 2000)

- Quantity of interest:
  - Duct length required to raise mean fluid temperature from T_0 to $T_b < T_w$
  - U → average axial fluid velocity
  - $\rho_0(\pi d_w^2/4)U$ → total mass flow rate
  - $v_z(r)$ parabolic (constant property flow Newtonian fluid far downstream of inlet)
  - T(r) then satisfies linear, 2nd-order PDE:

$$2U.\left[1-\left(\frac{2r}{d_w}\right)^2\right]\frac{\partial T}{\partial z}=\frac{\alpha}{r}.\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

(axial conduction neglected compared with radial)

CONVECTIVE HEAT TRANSFER FROM/ TO FLUID FLOWING IN A STRAIGHT CIRCULAR DUCT (Re < 2000)

- Boundary conditions:

$$T(r,0) \equiv T_0 = \text{constant}$$

$$T(d_w/2, z) \equiv T_w = \text{constant}$$

$$\left(\frac{\partial T}{\partial x}\right)_{r=0} = 0,$$

- T(r,z) found by Fourier method of "Separation-of-variables"
- Local heat flux

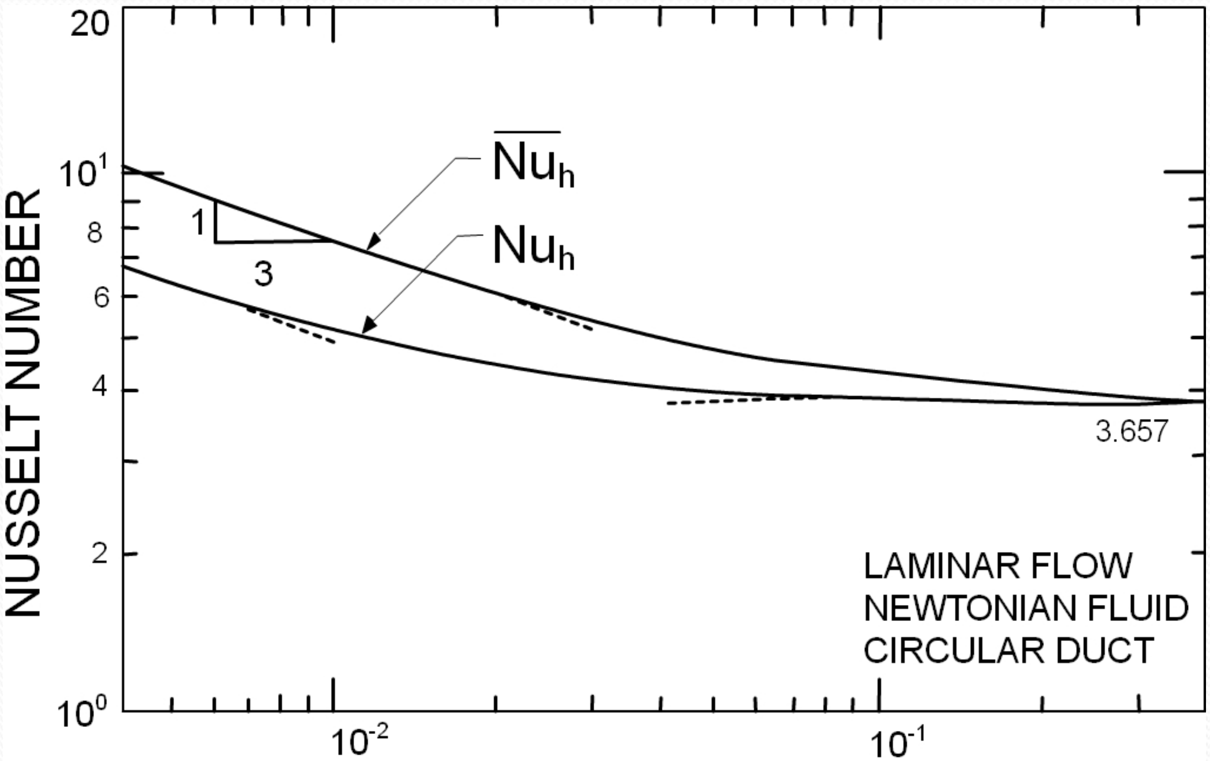
$$\dot{q}''_{ref} \equiv k\left[\frac{T_w - T_b(z)}{d_w}\right]$$

CONVECTIVE HEAT TRANSFER FROM/ TO FLUID FLOWING IN A STRAIGHT CIRCULAR DUCT (Re < 2000)

- $T_b(z)$ →mixing-cup average temperature at axial station z

  Fluid temperature that would result from severing the duct at z and adiabatically mixing the effluent

CONVECTIVE HEAT TRANSFER FROM/ TO FLUID FLOWING IN A STRAIGHT CIRCULAR DUCT (Re < 2000)



$$\xi_h \equiv \frac{1}{Re.Pr} \cdot \frac{Z}{d_w}$$

CONVECTIVE HEAT TRANSFER FROM/ TO FLUID FLOWING IN A STRAIGHT CIRCULAR DUCT (Re < 2000)

- Nu_h is a function of the dimensionless axial variable (Graetz, 1885)

$$\xi_h \equiv \frac{1}{Re\ Pr} \cdot \frac{z}{d_w}$$

and

$$Nu_h \approx const\ \xi_h^{-1/3} \qquad for\ \xi_h \to 0$$

$$Nu_h \to 3.657 \qquad for\ \xi_h \to \infty$$

CONVECTIVE HEAT TRANSFER FROM/ TO FLUID FLOWING IN A STRAIGHT CIRCULAR DUCT (Re < 2000)

- $Nu_h \rightarrow \infty$ at thermal inlet
- $Nu_h \rightarrow$ constant value far downstream (fully-developed or asymptotic Nusselt number)

$$\overline{Nu}_h(\xi_h) \equiv \frac{1}{\xi_h} \cdot \int_0^{\xi_h} Nu_h(\xi_h^{'})\, d\xi_h^{'}$$

Axial average heat-transfer coefficient

$$\overline{Nu}_h(\xi_h) = \overline{Nu}_h(\infty) \cdot \left[F\begin{pmatrix} thermal \\ entrance \end{pmatrix} \right]$$

where

$$F(thermal\ entrance) \approx \left[1 + (7.60\xi_h)^{-8/3} \right]^{1/8}$$

CONVECTIVE HEAT TRANSFER FROM/ TO FLUID FLOWING IN A STRAIGHT CIRCULAR DUCT (Re < 2000)

- Overall energy balance on fluid contained in Eulerian CV between 0 and z yields:

$$\frac{T_w - T_b(z)}{T_w - T_b(0)} = exp\left\{-4\xi_h \overline{Nu}_h(\infty).F(thermal\ entrance)\right\}$$

  - Enables calculation of length required to achieve any T_b

CONVECTIVE HEAT TRANSFER FROM/ TO FLUID FLOWING IN A STRAIGHT CIRCULAR DUCT (Re < 2000)

Alternatively:

$$\frac{\dot{q}/A_w}{\left\{\dfrac{k\left(\Delta T\right)_{mean}}{d_w}\right\}} = \overline{Nu_h}\left(\xi_{max}\right)$$

where

$$\dot{q} = \dot{m}c_p\left[T_b\left(L\right) - T_b\left(0\right)\right]$$

and

$$A_w = \pi d_w L$$

CONVECTIVE HEAT TRANSFER FROM/ TO FLUID FLOWING IN A STRAIGHT CIRCULAR DUCT (Re < 2000)

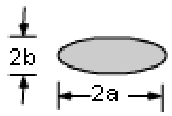
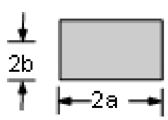
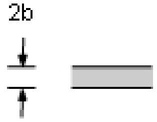
- Resulting in:

$$\Delta T_{mean} \equiv \frac{\left(T_w - T_b(0)\right) - \left(T_w - T_b(L)\right)}{ln\left[\dfrac{T_w - T_b(0)}{T_w - T_b(L)}\right]} \equiv LMTD$$

(log-mean temperature difference, LMTD)

- Other important internal-flow heat-transfer problems exhibit same general features:

CONVECTIVE HEAT TRANSFER FROM/ TO FLUID FLOWING IN A STRAIGHT CIRCULAR DUCT (Re < 2000)

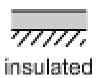
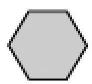
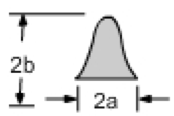
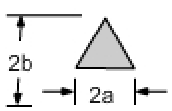
Geometry	b/a	Nu_T	Nu_{H1}	Nu_{H2}	$C_f R_e$	$\frac{j_{H1}}{C_f}^{+}$	$\frac{Nu_{H1}}{Nu_T}$
	1	3.657	4.364	4.364	16.000	0.307	1.19
	1/2	3.74	4.88		18.24	0.301	1.30
	1/8	3.72	5.09		19.15	0.299	1.37
	1	2.976	3.608	3.091	14.227	0.286	1.21
	1/2	3.391	4.123	3.017	15.548	0.299	1.22
	1/4	4.439	5.331	2.930	18.233	0.329	1.20
	1/8	5.597	6.490	2.904	20.585	0.355	1.16
	0	7.541	8.235	8.235	24.000	0.386	1.09

† This heading is the same as $Nu_{H1}Pr^{1/3}/C_f Re$ with Pr =0.7; Nu, C_f, Re based on $d_{eff} \equiv 4A/$ perimeter

a The subscripts T, H1 and H2 on the relevant Nusselt numbers mean, respectively:

T1: pertaining to constant wall temperature
H1: pertaining to actual constant heat flux
H2: pertaining to peripheral and axial constancy of heat flux

CONVECTIVE HEAT TRANSFER FROM/ TO FLUID FLOWING IN A STRAIGHT CIRCULAR DUCT (Re < 2000)

Geometry	b/a	Nu_T	Nu_{H1}	Nu_{H2}	$C_f R_e$	$\frac{j_{H1}^{+}}{C_f}$	$\frac{Nu_{H1}}{Nu_T}$
	0	4.861	5.385	5.385	24.000	0.253	1.11
		3.34*	4.002	3.862	15.054	0.299	1.20
	$\sqrt{3}/2$	2.39*	3.014	1.474	12.630	0.269	1.26
	$\sqrt{3}/2$	2.47	3.111	1.892	13.333	0.263	1.26

† This heading is the same as $Nu_{H1}Pr^{-1/3}/C_f Re$ with Pr =0.7; Nu,C_f, Re based on $d_{eff} \equiv 4A/$ perimeter

a The subscripts T, H1 and H2 on the relevant Nusselt numbers mean, respectively:

T1: pertaining to constant wall temperature
H1: pertaining to actual constant heat flux
H2: pertaining to peripheral and axial constancy of heat flux

HEAT EXCHANGE BETWEEN A FLUID AND A POROUS MEDIUM ("PACKING")

- Packed duct is equivalent of many tortuous interstitial ducts defined by spaces between bed particles
- Appropriate dimensionless energy-transport coefficient → $Nu_{h,bed}$
  - Depends on Re_{bed}, Pr

$$\mathrm{Re}_{bed} \equiv \frac{G_o d_{p,eff}}{\mu\left(1-\varepsilon\right)}$$

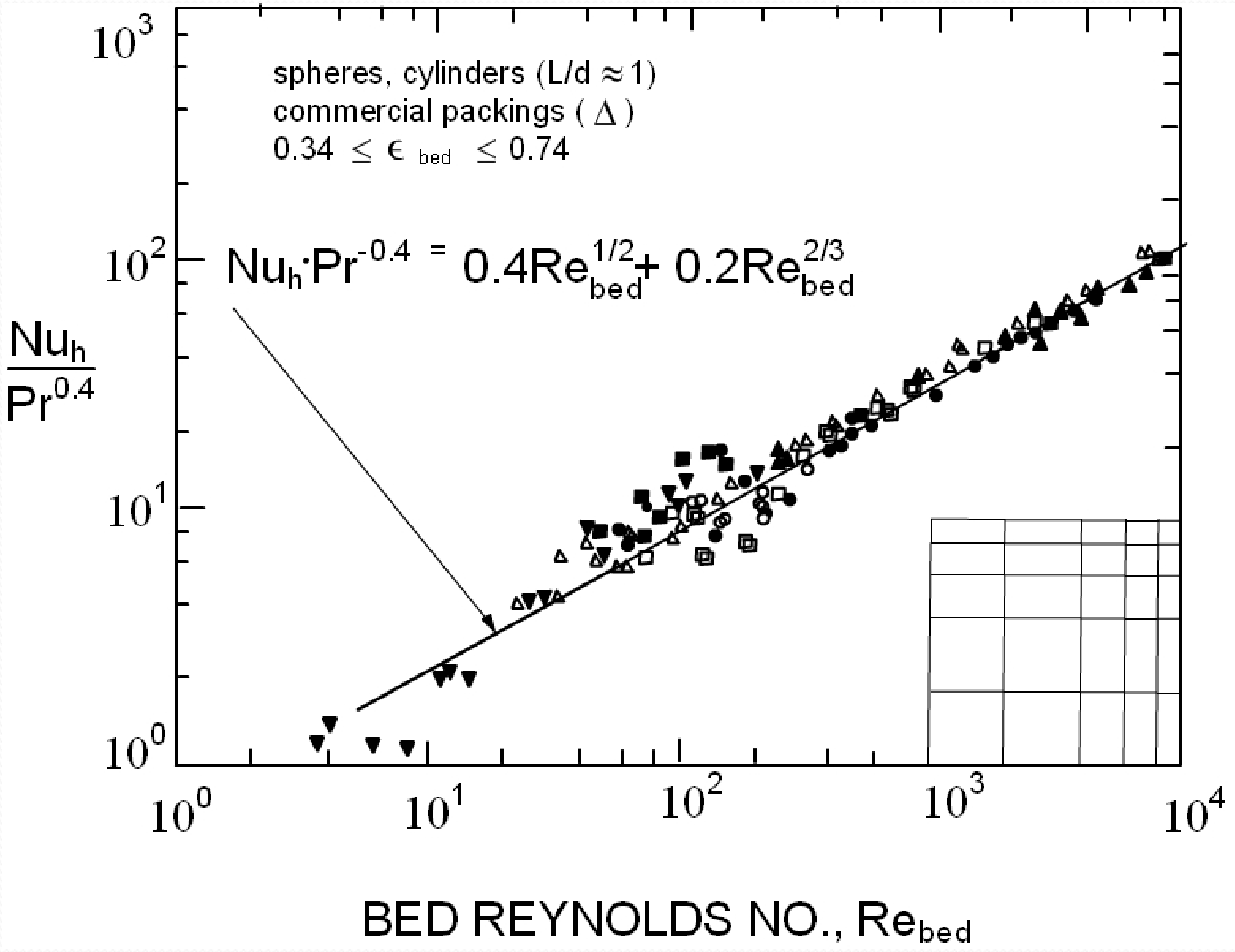
$G_0\left(\equiv \dot{m} / A_0\right)$ → "superficial" fluid mass velocity

HEAT EXCHANGE BETWEEN A FLUID AND A POROUS MEDIUM ("PACKING")

- dz → element of column height
- Total volume → $A_0 dz$
- Packing surface area → a''' $A_0 dz$
- Energy transferred from packing to fluid → $d\dot{q}$
- Average heat flux → $d\dot{q}$ /(a''' $A_0 dz$)
- Dimensionless heat transfer coefficient

$$Nu_{h,bed} = const.\frac{d\dot{q}/\left(a'''A_o dz\right)}{\left[k\left(T_w - T_b\right)\right]/d_{i,eff}}$$

HEAT EXCHANGE BETWEEN A FLUID AND A POROUS MEDIUM ("PACKING")



HEAT EXCHANGE BETWEEN A FLUID AND A POROUS MEDIUM ("PACKING")

- Analogous expressions apply for species mass exchange between fluid & fixed/ fluidized bed, tube bundle
  - e.g., fixed-bed catalytic reactor
  - Preliminary design based on "plug flow" in cylindrical vessel of constant cross-section
  - Refinements needed to take care of non-idealities
    - Radially non-uniform G_0, non-uniform packing temperature, variable fluid properties, etc.

DIFFUSION FROM A STEADY POINT SOURCE IN A UNIFORM MOVING STREAM

- Quantity of interest:
  - Thermal wake of point heat source of strength $\dot{q}$
    - In constant-property, uniform, laminar stream
- Relevant to combustion gases, distributed heat sources, pollutant transport (mass-transfer analog)
- Energy is convected downstream (+z direction), diffuses radially & axially

DIFFUSION FROM A STEADY POINT SOURCE IN A UNIFORM MOVING STREAM

- Local energy-balance equation:

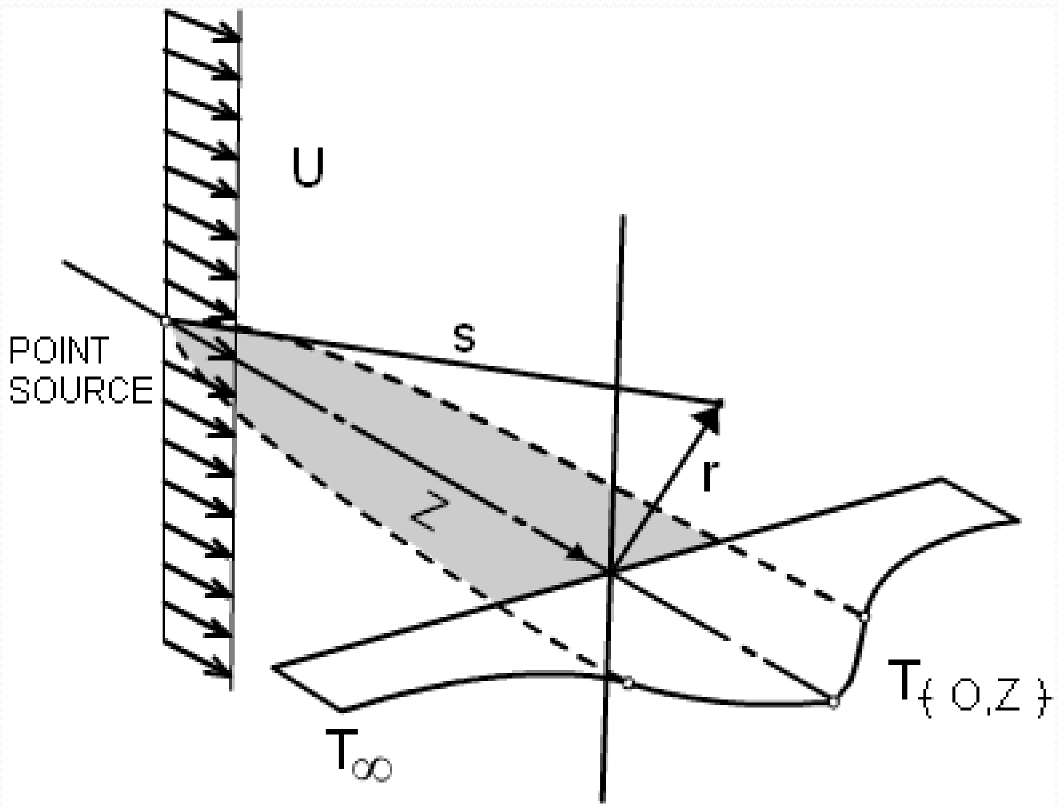
$$U\,\frac{\partial T}{\partial z} = \alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right] \quad (if\;\; \alpha = const).$$

Spherical radius

$$s \equiv \left(r^2 + z^2\right)^{1/2}$$

(measured from point source)

DIFFUSION FROM A STEADY POINT SOURCE IN A UNIFORM MOVING STREAM



Energy convection and diffusion from a continuous point source in a uniform stream

DIFFUSION FROM A STEADY POINT SOURCE IN A UNIFORM MOVING STREAM

- Boundary conditions:

$$\lim_{s \to \infty} T = T_{\infty} \qquad \text{(specified constant)}$$

$$\lim_{s \to 0} \left[-\left(4\pi s^2\right) k \left(\frac{\partial T}{\partial s} \right) \right] = \dot{q} \qquad \text{(specified source strength)}$$

$$\lim_{r \to 0} \left(\frac{\partial T}{\partial r} \right) = 0 \qquad for\ z > 0$$

Leading to:

$$T(r,z) - T_{\infty} = \frac{\dot{q}}{4\pi k s} . \exp\left[\frac{-U(s-z)}{2\alpha} \right]$$

DIFFUSION FROM A STEADY POINT SOURCE IN A UNIFORM MOVING STREAM

- Relation implies non-physical behavior: T → ∞ in immediate vicinity of point source
  - Meaningful at downstream distances >> source size
  - At any z >> r, radial temperature profiles are Gaussian in shape: $\dfrac{T(r,z)-T_\infty}{T(0,z)-T_\infty} \approx \exp\left[-\dfrac{Uz}{4\alpha}\left(\dfrac{r}{z}\right)^2\right]$
- Along the axis r = 0, $T(0,z)-T_\infty = \dfrac{\dot{q}}{4\pi kz}$
  - Can be used to determine k-values based on axial temperature decay data
  - No htc's since there are no boundary surfaces

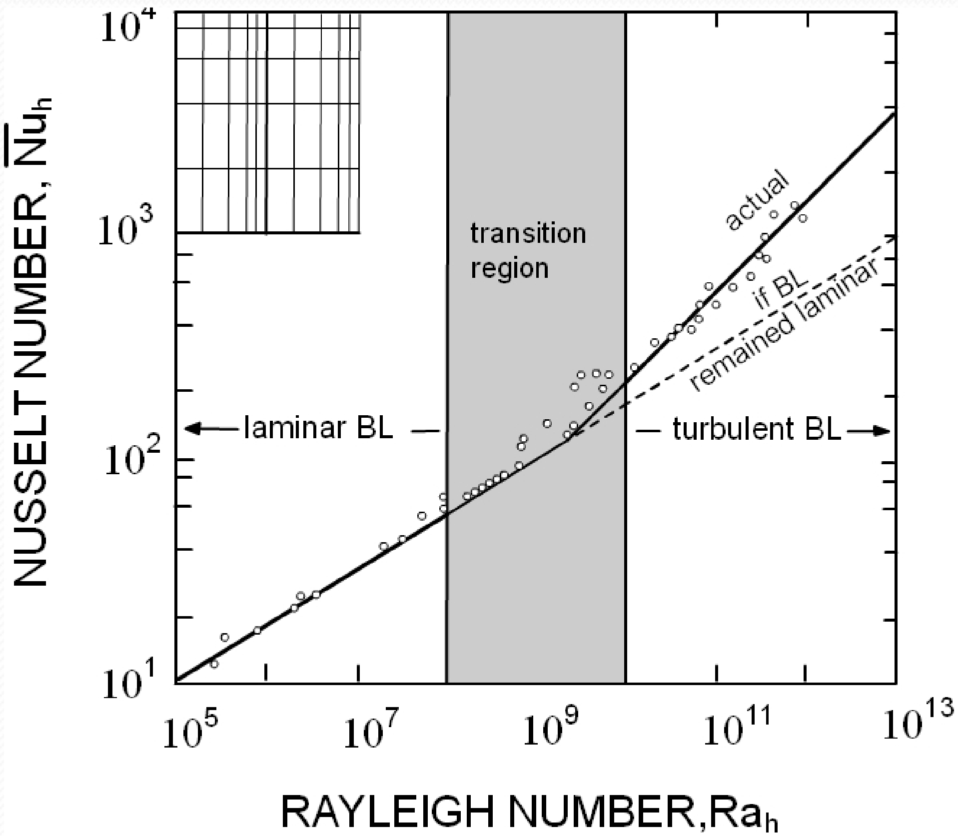
STEADY TURBULENT FLOWS

- Turbulence: localized non-steadiness associated with enhancement of time-averaged rates of momentum, energy & mass transport
  - For Re and Ra_h numbers above “transition” values
  - Extrapolation of laminar heat-transfer coefficient laws into turbulent region will result in serious underestimation of transport rates

STEADY TURBULENT FLOWS

- Turbulence: localized non-steadiness associated with enhancement of time-averaged rates of momentum, energy & mass transport
  - e.g., higher drag (undesirable), higher heat-exchange rates (desirable)– the two may be in conflict!
  - For Re and Ra_h numbers above “transition” values
  - Extrapolation of laminar heat-transfer coefficient laws into turbulent region will result in serious underestimation of transport rates

STEADY TURBULENT FLOWS



Vertical flat surfaces in quiescent Newtonian fluid

FORCED-CONVECTIVE TURBULENT HEAT TRANSFER FROM/ TO STRAIGHT, SMOOTH DUCTS

- Fully-developed Nusselt numbers not constant, but depend on Re, Pr

$$Nu_h(\infty) = \overline{Nu_h}(\infty) \cong 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}$$

or

$$St_h(\infty) = \overline{St_h}(\infty) \cong 0.023\,\mathrm{Re}^{-0.2}\,\mathrm{Pr}^{-2/3}$$

(for Re > 10^4, Pr ≥ 0.7)

- Apply to non-circular ducts as well (with d_{eff} = 4A/P)

FORCED-CONVECTIVE TURBULENT HEAT TRANSFER FROM/ TO STRAIGHT, SMOOTH DUCTS

- Thermal entrance effect is modified to:

$$F\left(thermal\ \ entrance\right) \approx 1 + \left(\frac{z}{d_w}\right)^{-2/3} \qquad \left(\mathrm{Re} > 10^4, \mathrm{Pr} > 0.6\right)$$

  - Above Re, Pr-dependencies are oversimplifications
  - Stanton number and pipe friction factor are closely related (extended Reynolds' analogy)

TURBULENT THERMAL BOUNDARY LAYERS ON A SMOOTH FLAT PLATE

- For $Re_L > 10^5$, viscous flow within a flat-plate forced-convection BL becomes turbulent at some x (transition)
  - Heat transfer across laminar BL for x < x(transition)
  - Heat transfer across turbulent BL for x(transition) ≤ x ≤ L
- If turbulence is triggered near leading-edge, turbulence dominant over entire plate, and for $10^5 \le Re_L \le 10^7$, skin friction distribution becomes:

$$\frac{\tau_w(x)}{\rho_\infty U^2} \equiv \tfrac{1}{2} c_f(x) = 0.0288\left(\mathrm{Re}_x\right)^{-1/5}$$

TURBULENT THERMAL BOUNDARY LAYERS ON A SMOOTH FLAT PLATE

- Stanton-number distribution:

$$St_h(x) \cong 0.0288(\mathrm{Re}_x)^{-1/5}\,\mathrm{Pr}^{-2/3}$$

or

$$\overline{Nu_h}(x) \cong 0.0288(\mathrm{Re}_x)^{4/5}\,\mathrm{Pr}^{1/3}$$

Averaging over entire plate on one side:

$$\overline{Nu_h} \cong 0.036(\mathrm{Re}_L)^{4/5}\,\mathrm{Pr}^{1/3}$$

TURBULENT THERMAL BOUNDARY LAYERS ON A SMOOTH FLAT PLATE

- Thermal turbulent BL slope thickness $\delta_h \cong x^{1/5}$
  - Laminar: $\delta_h \cong x^{1/2}$
- Time-averaged heat fluxes in turbulent BL fall off with distance as $x^{-1/5}$
- On non-isothermal curved surfaces, x → distance along surface
  - e.g., turbine blades, nozzle surfaces

TURBULENT THERMAL BOUNDARY LAYERS ON A SMOOTH FLAT PLATE

- Natural convection:
  - For Newtonian fluids with Prandtl numbers nearly equal to that of air, and $Ra_h > 10^9$:

$$Nu_h(x) \cong 0.10\left(Ra_h\right)^{1/3} \qquad \left(\Pr \approx 0.7\right)$$

  - Local heat fluxes nearly constant along plate surface, hence:

$$\overline{Nu}_h \cong 0.10\left(Ra_h\right)^{1/3} \qquad \left(for \quad \Pr \approx 0.7, \quad \mathrm{Ra}_{h,L} > 10^9\right)$$

- (reference length → L)

ISOLATED SPHERE AT HIGH Re

- For Re >> 3 X 10^5, turbulence within thermal BL amplifies total heat-transfer coefficient, Nu_h, for an isothermal sphere
  - Up to Re = 10^5 :

$$\overline{Nu_h} = 2 + \left[0.4(\mathrm{Re})^{1/2} + 0.06(\mathrm{Re})^{2/3}\right](\mathrm{Pr})^{0.4}$$

- For Re >> 3 X 10^5, reliable data & correlations not available
  - Separated flow introduces great complexity in theory

ISOLATED SPHERE AT HIGH Re

- Turbulence types:
  - Confined to BL adjacent to solid surfaces
  - Mainstream turbulence
- Empirical correction factors needed for:

  $I_{t,\infty}$ = mainstream turbulence "intensity" (ratio of root-mean-square (rms) velocity fluctuation to time-averaged approach velocity),

ISOLATED SPHERE AT HIGH Re

and

$L_{t,\infty} / L \equiv$ ratio of mainstream turbulence (macro-) scale to body dimension

- Mainstream turbulence has two effects:
  - Triggers earlier transition to turbulence within BL
  - Modifies time-averaged transport across laminar BL