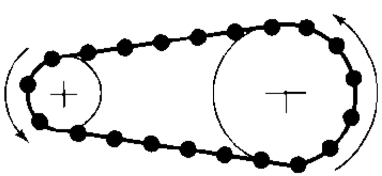
Kinematics of Machines (NME-502)

Unit – IV
Gears and Gear Train

Gear as a mechanical transmission devices

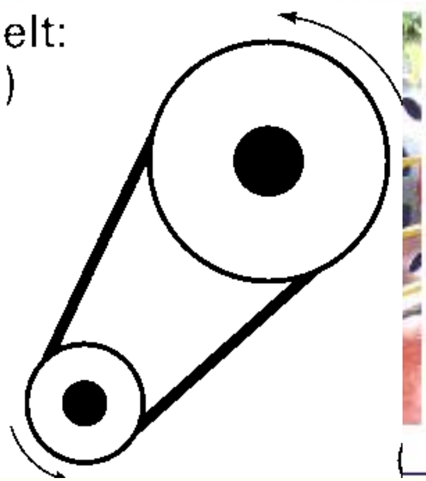
Chains

Sprocket and Chain:
(Parallel)



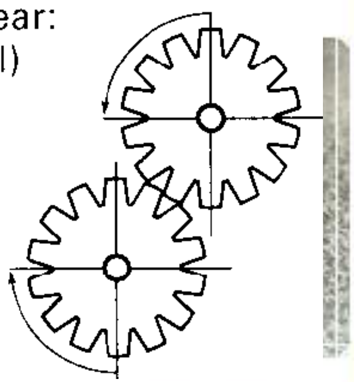
Belts

Pulley Belt:
(Parallel)



Gears

Spur Gear:
(Parallel)



Why Use Gears?

1. Reduce speed

   (Increase torque)

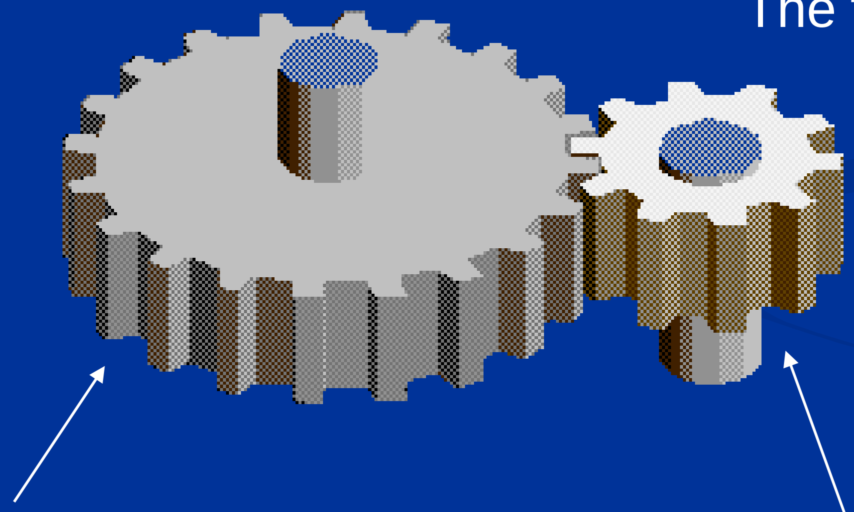
1. Move power from one point to another
2. Change direction of power
3. Split power

Generally this functionality is accomplished by many gears mounted in a gear box!

Gears: Definition

- Toothed member used to transmit motion by successfully engaging of teeth from one member to another member

The teeth prevent slippage.

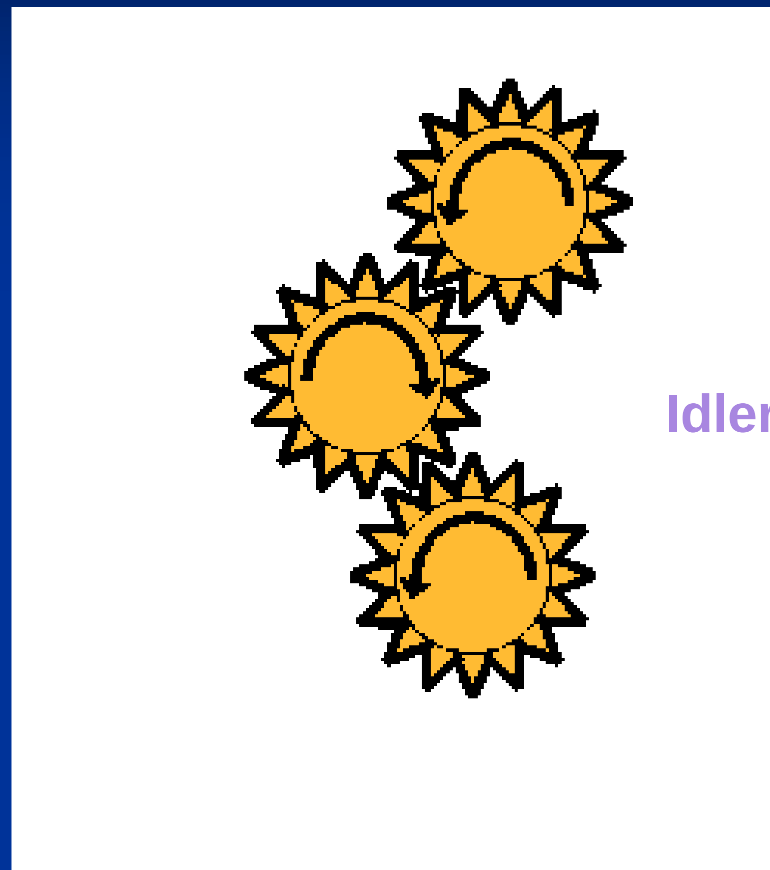


Driven gear

Driver gear

Gears

Idler gear



TYPES OF GEARS

1. According to the position of axes of the shafts.

a. Parallel

1.Spur Gear

2.Helical Gear

3. Double Helical (Herringbone)Gear

3.Rack and Pinion

b. Intersecting

Bevel Gear : Straight Bevel Gear

Spiral Bevel Gear

c. Non-intersecting and Non-parallel

worm and worm gears

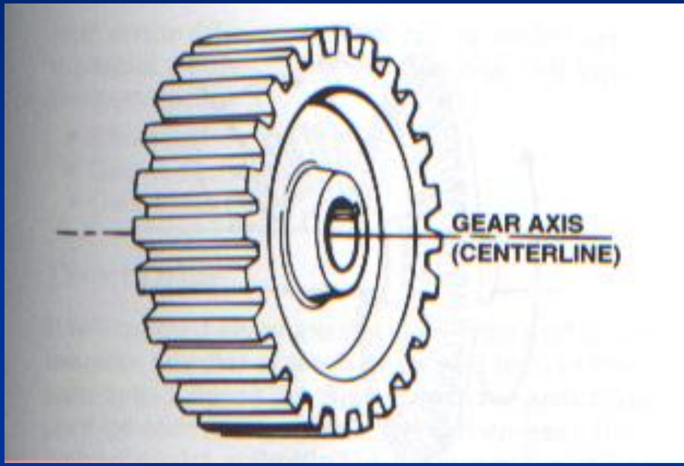
Crossed Helical gear

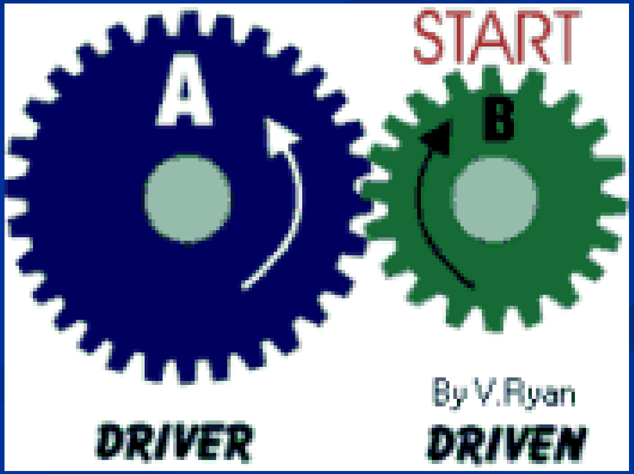
Hypoid Gears

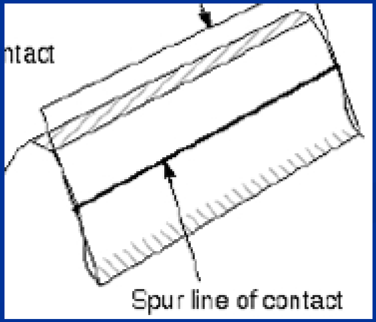
a) Parallel Shaft

1. SPUR GEAR

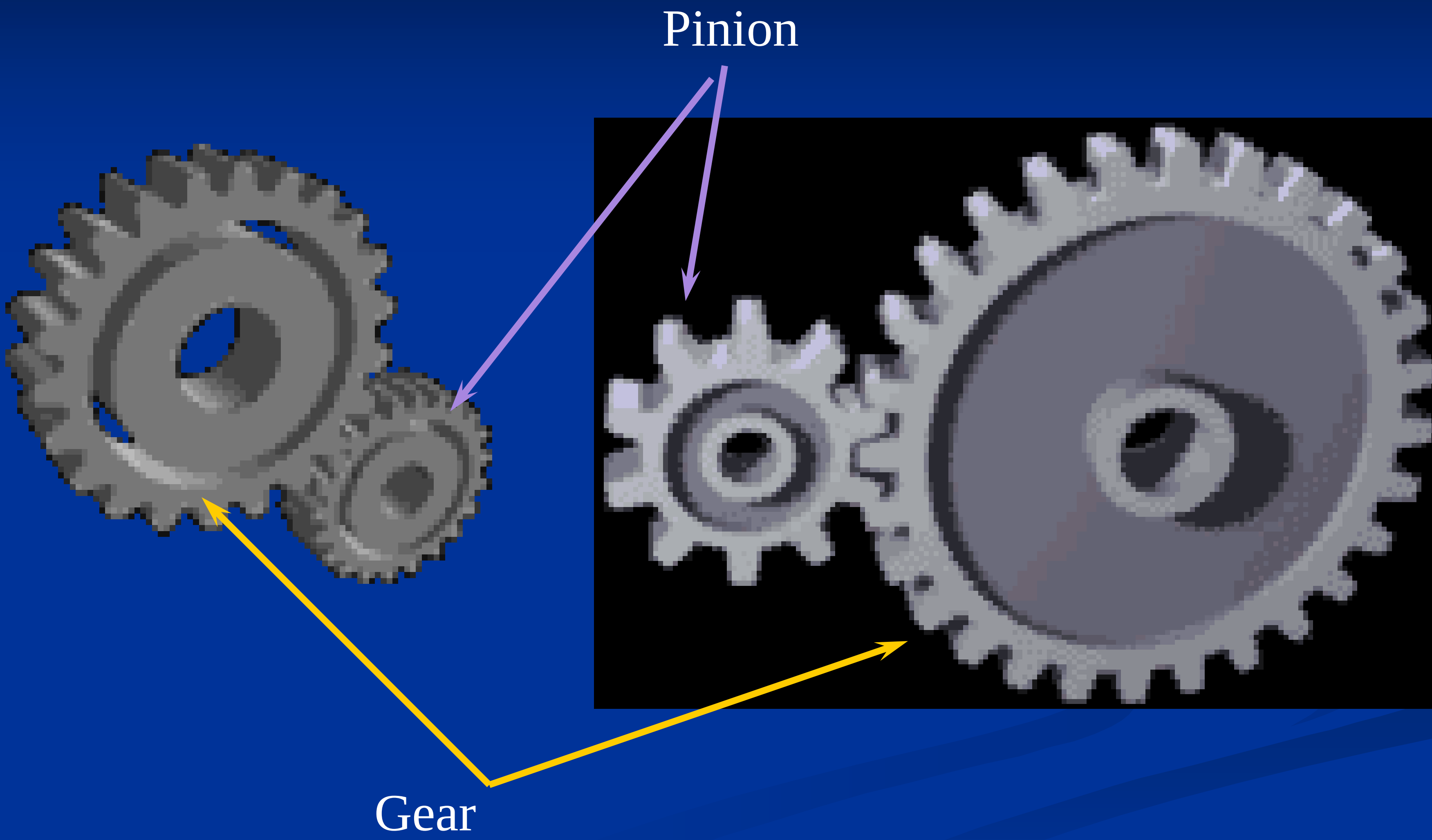
- Teeth is parallel to axis of rotation
- Transmit power from one shaft to another parallel shaft
- Spur gears only have one tooth in contact at a time
- Used in Electric screwdriver, Juice extracting machine, washing machine



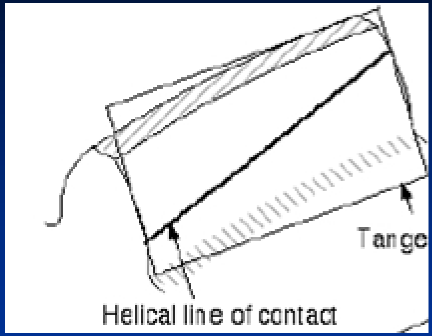




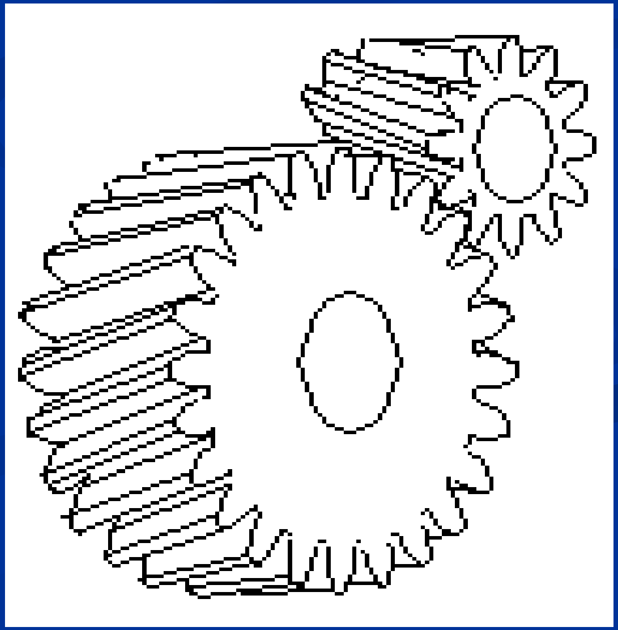
1.Spur Gears



2. Helical Gear

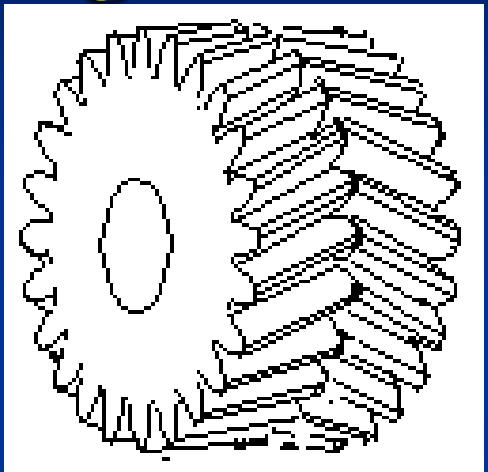


- Are quieter than spur gears
- Two teeth at a time contact
- Has a tendency to move shaft for and aft **(Due to axial Thrust)**
- Are left and right handed
- Opposites on parallel shafts



3. Double Helical Gear or Herringbone gears

- To avoid axial thrust, two helical gears of opposite hand can be mounted side by side, to cancel resulting thrust forces



- Herringbone gears are mostly used on heavy machinery.

4. Rack and pinion

- **Rack and pinion gears** are used to convert rotation (From the pinion) into linear motion (of the rack)

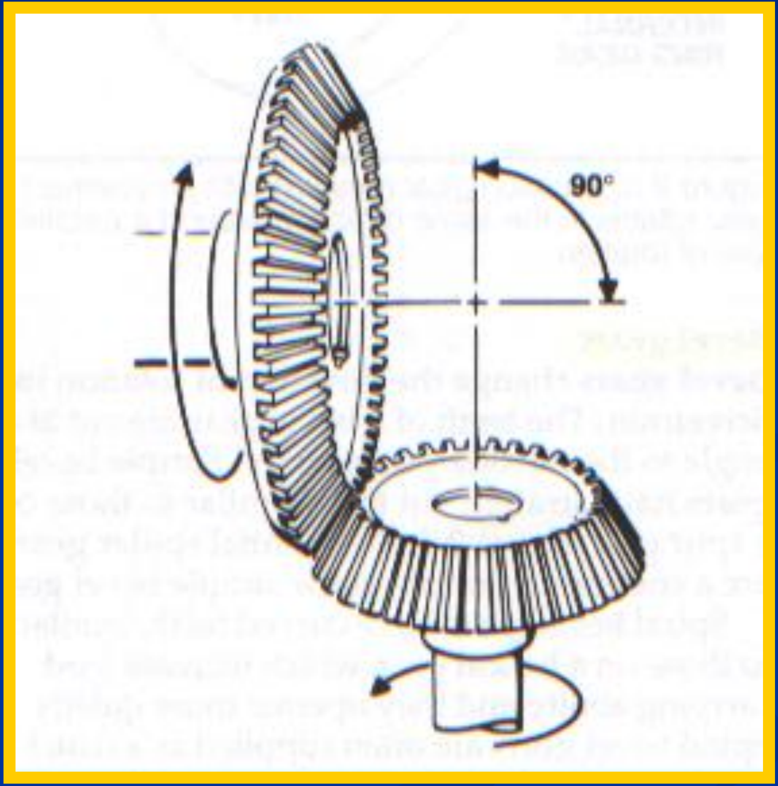
- A perfect example of this is the steering system on many cars

(b)Intersecting Shaft

Bevel Gear

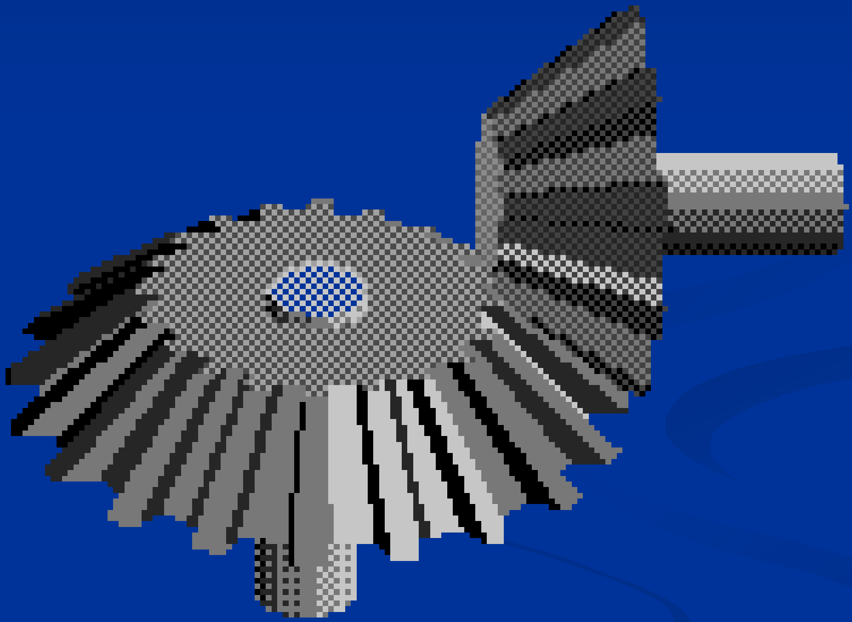
(b)Intersecting Shaft
Bevel gears

- **Used for change the direction of gear**
- They are usually mounted on shafts that are 90 degrees apart, but can be designed to work at other angles as well
- The teeth on bevel gears can be **straight**, **spiral**



- locomotives, automobiles, power plants etc.

Bevel Gears



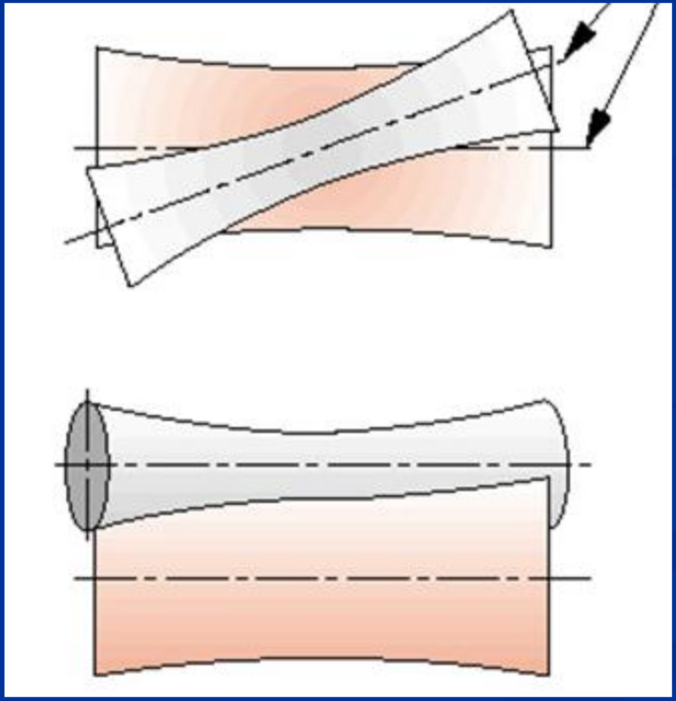
Bevel Gears

Straight and Spiral Bevel Gears

c) Skew Shaft

(Non parallel and Non intersecting Shaft)

1. WORM AND WORM GEAR
2. Crossed Helical Gear
3. Hypoid Gear

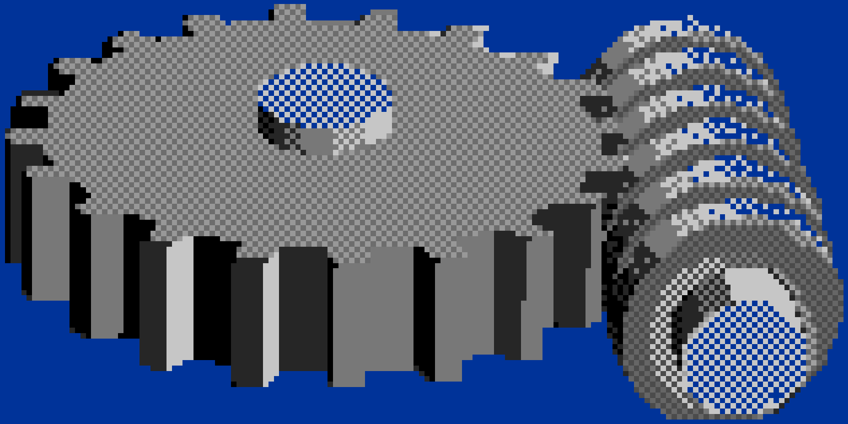


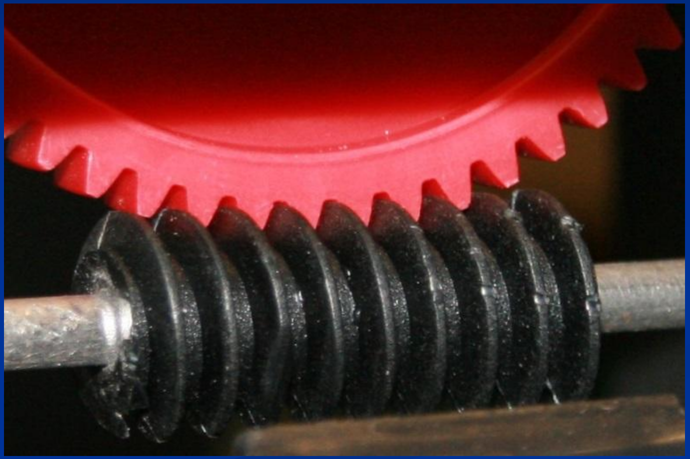
1.WORM AND WORM GEAR

- Large gear reductions
  20:1, and even up to 300:1
- An interesting property :
  The worm can easily turn the gear, but the gear cannot turn the worm
- Used in Speedometer cable drive mechanisms , Material Handling machinery

Worm gear and wheel

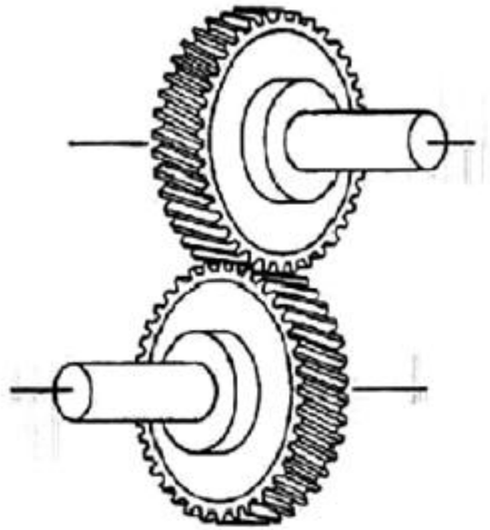
- The worm gear is always the drive gear



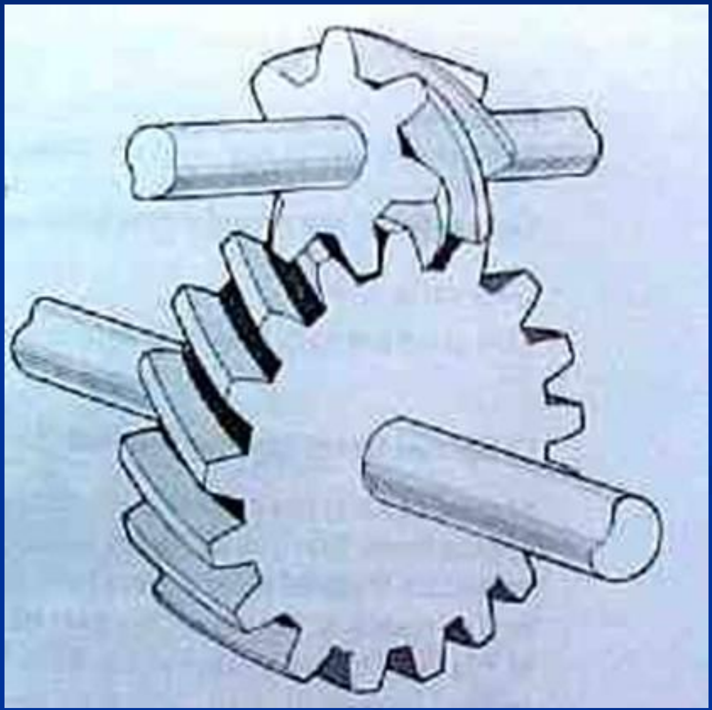


Worm and wheel

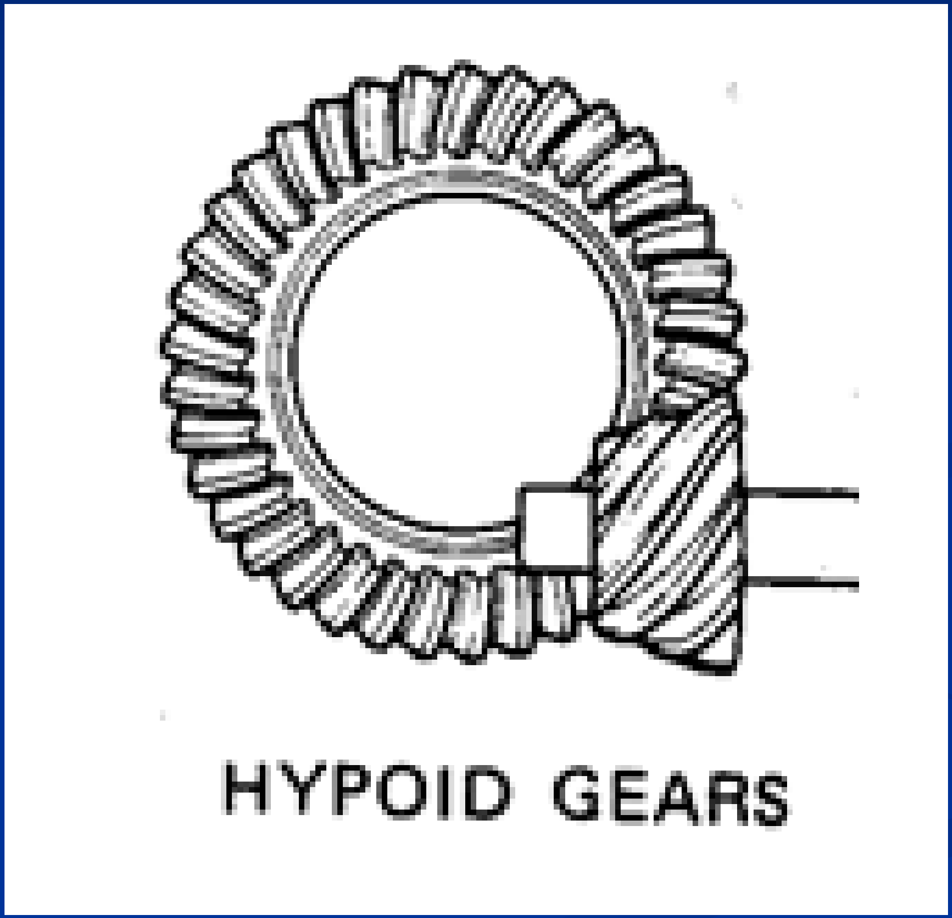
2.Crossed Helical Gear



Helical (Crossed Shaft)



3. Hypoid Gear

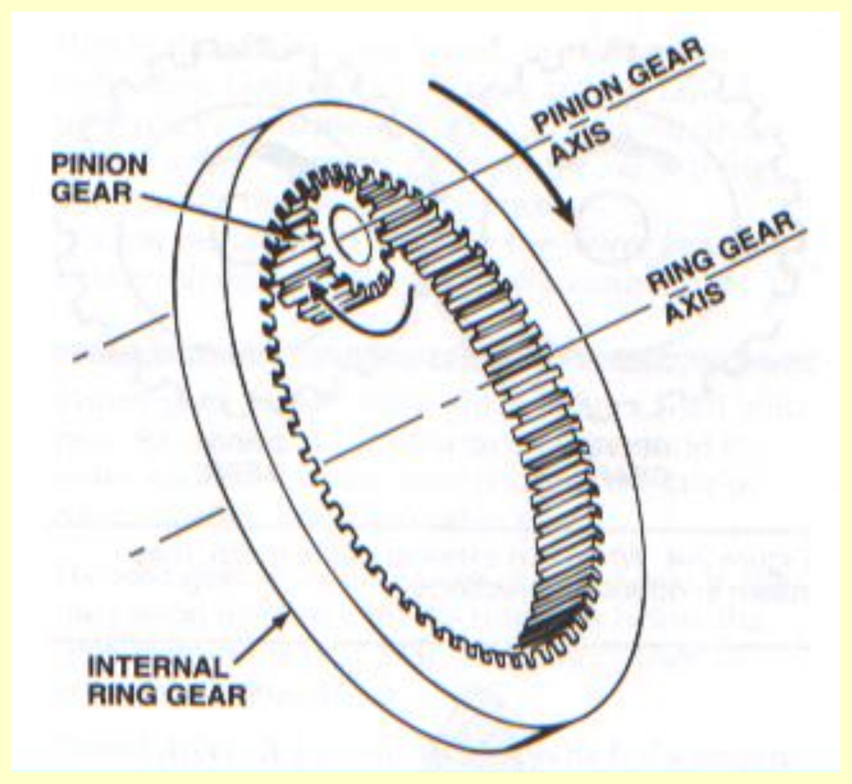


Classification bases on Types of Gears

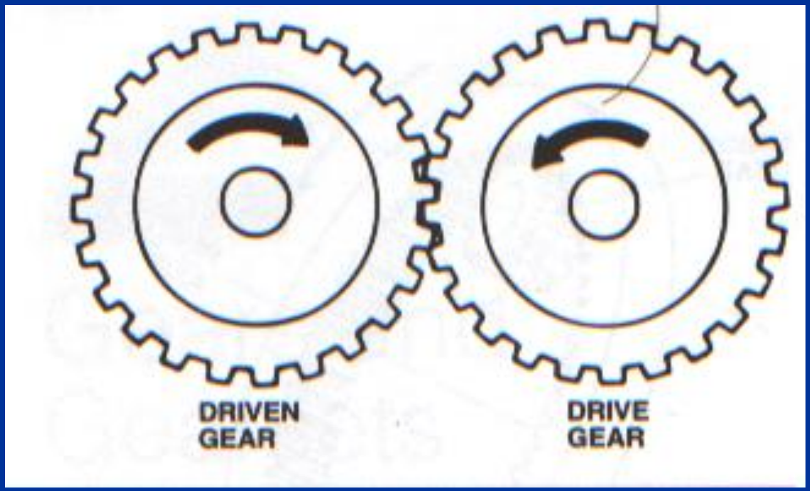
1. Internal Gear
2. External Gear
3. Rack and Pinion

1.Internal Gear

- Gear rotates in same direction



2.External Gears



- When two external gears mesh, they rotate in opposite directions

3.Rack and pinion

- **Rack and pinion gears** are used to convert rotation (From the pinion) into linear motion (of the rack)

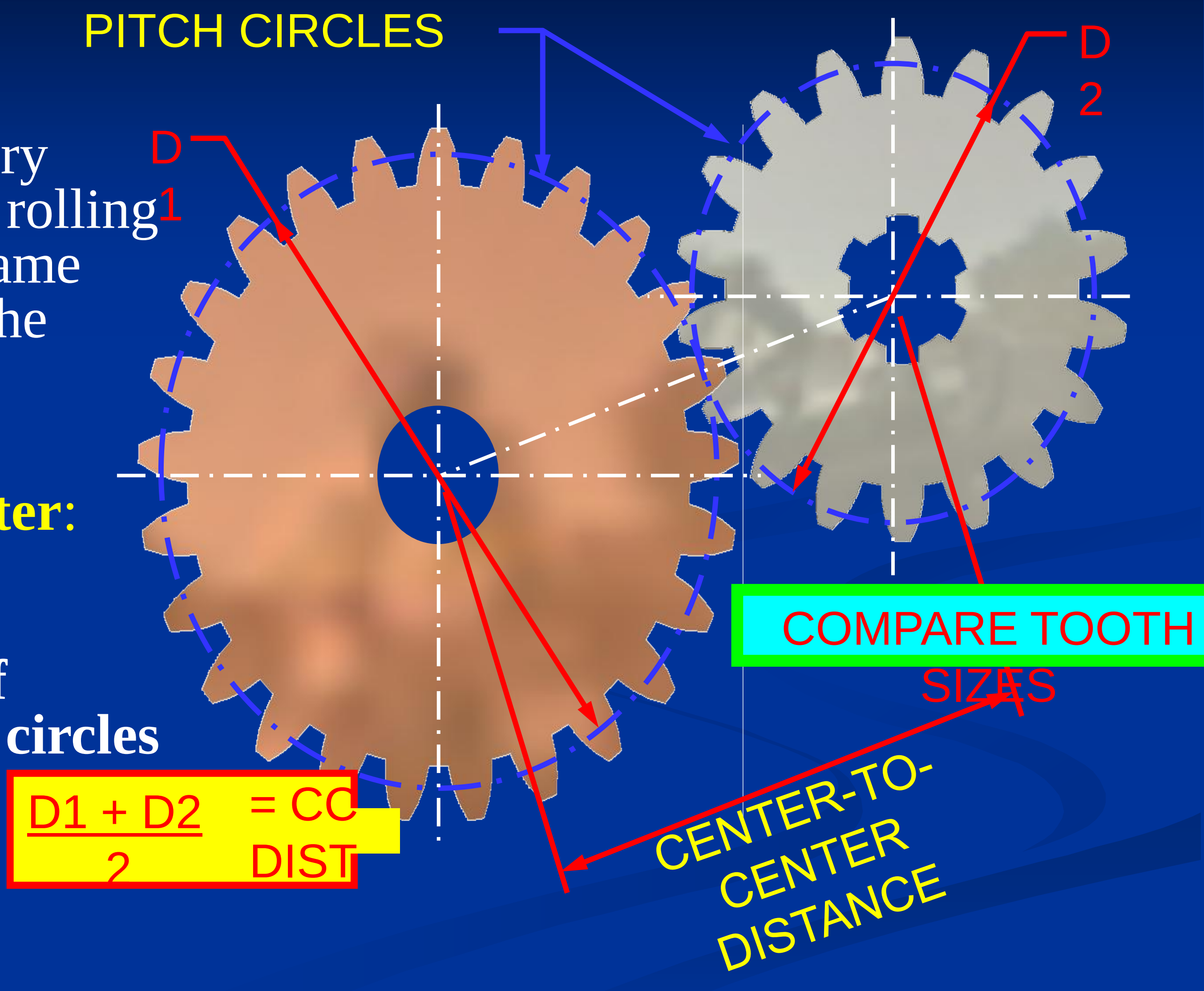
- Used in steering system on many cars

Gear Terminology

Pitch circle: Imaginary circle which by pure rolling Action would give same the same motion as the actual gear

Pitch circle Diameter: Dia of pitch circle

Pitch Point: Point of contact of two pitch circles of mating gear

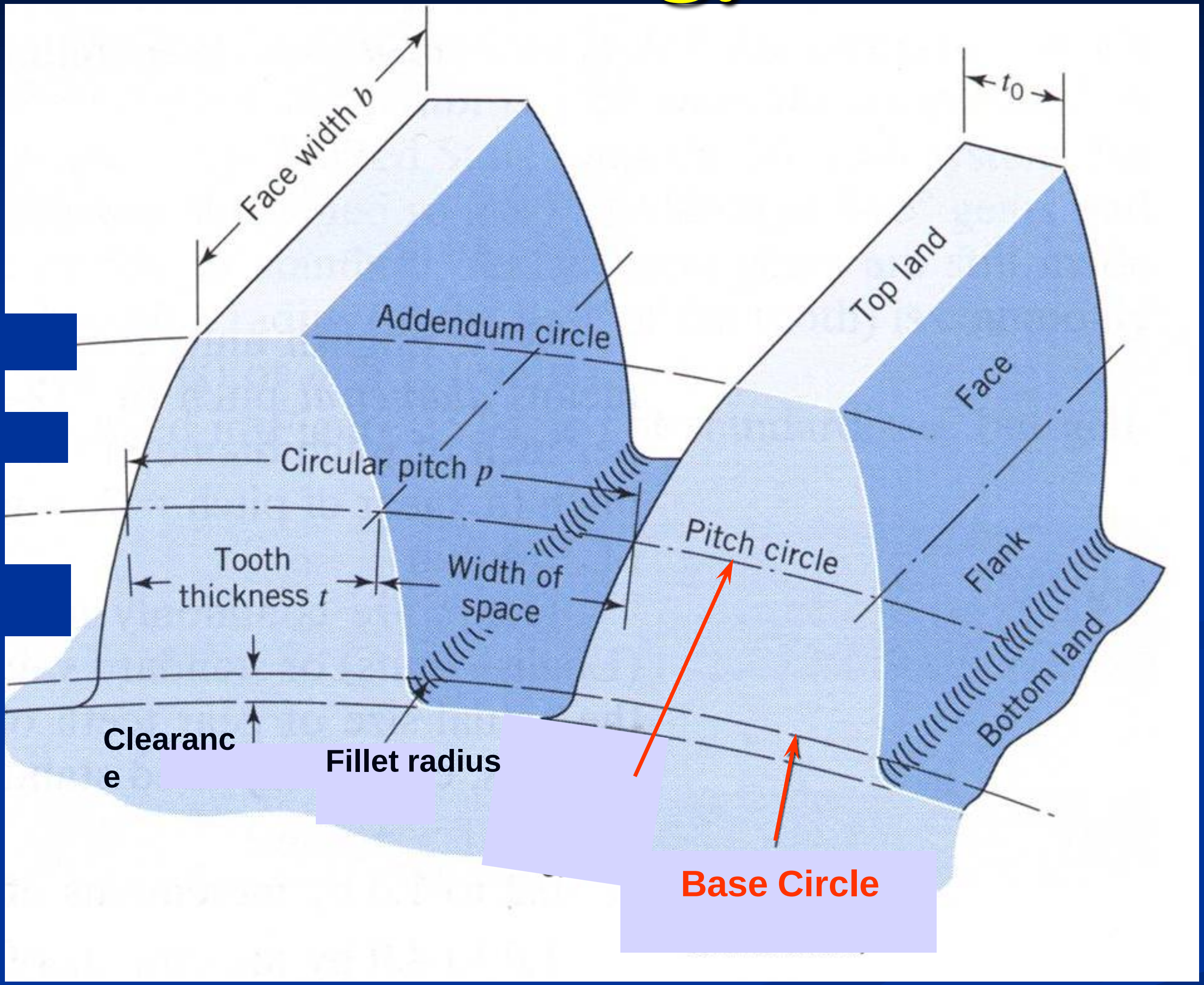


Gear Terminology

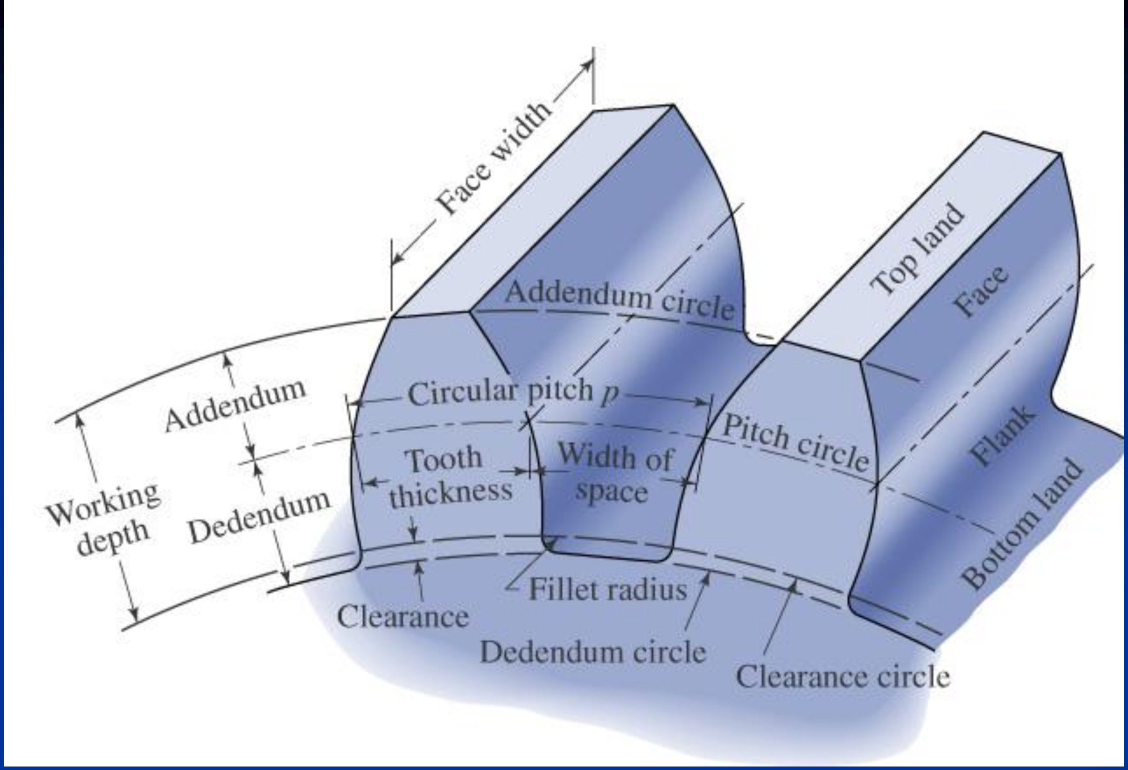
Base Circle: From which profile of teeth starts

Circular pitch: Distance from one teeth to the next, along the pitch circle.

$p_c = \pi d / T$

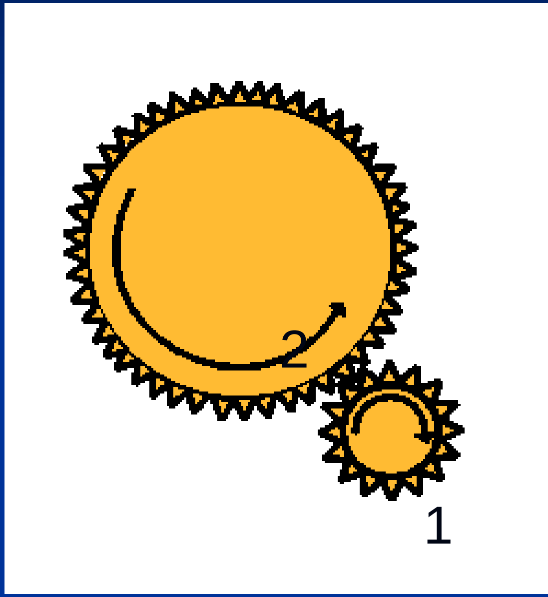


Diametral Pitch: NUMBER OF TEETH PER UNIT OF PITCH DIAMETER $p_d = T / D$; $Pc \times Pd = \pi$



Module = D/T (pitch circle diameter /number of teeth)

module is the inverse of diametral pitch.

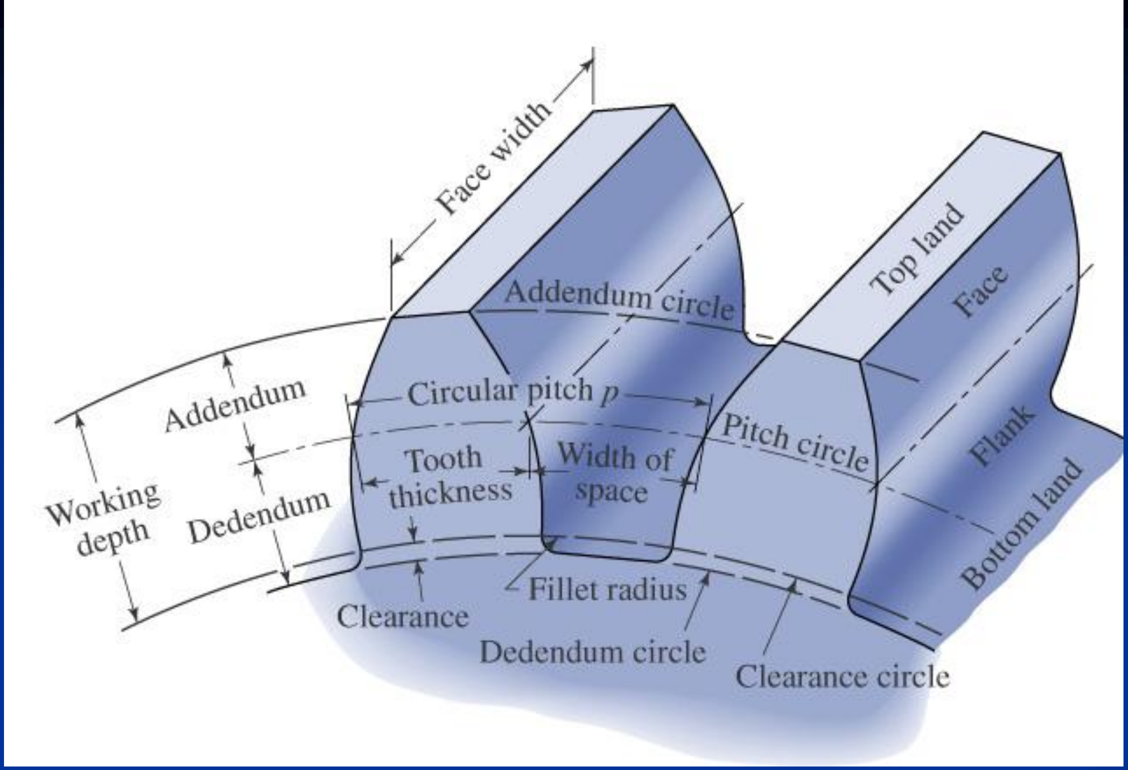


Gear ratio = Number of teeth on driven gear / Number of teeth on driver(pinion) gear

= T2/T1

Velocity ratio = Angular velocity of driver (Pinion)gear / Angular Velocity of driven gear

$= \omega 1 / \omega 2 = N1/N2 = T2/T1$ = Gear Ratio

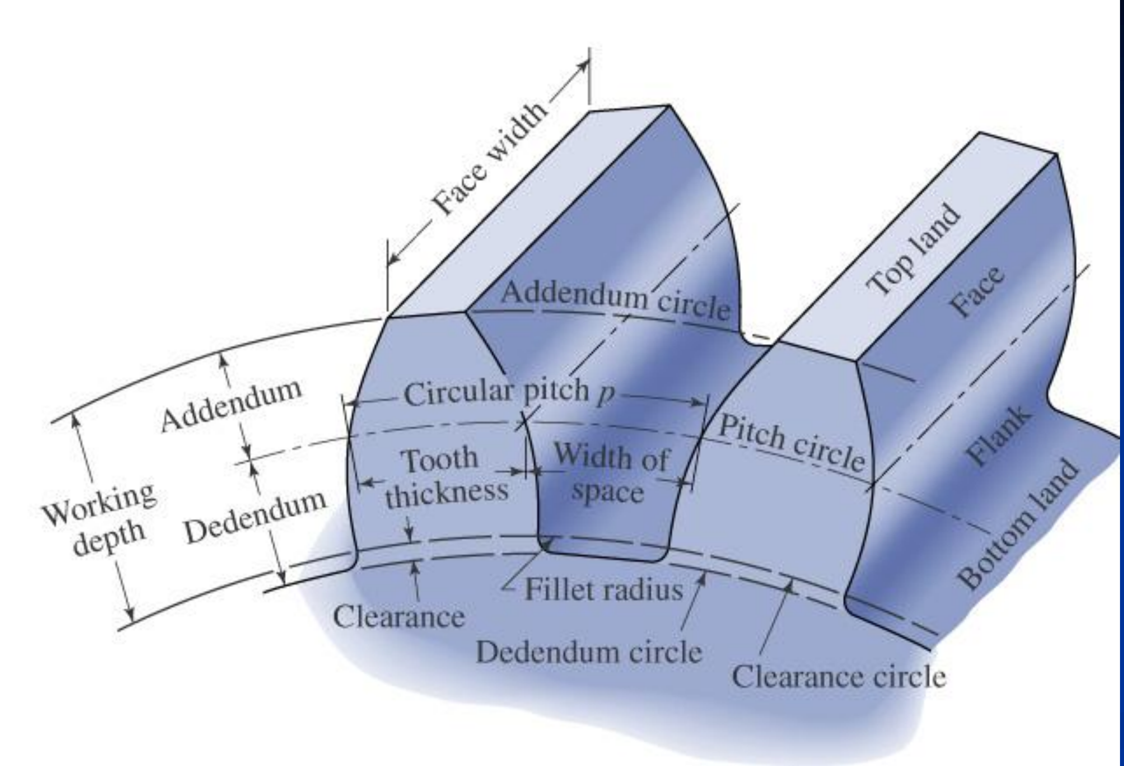


- **Addendum**: The radial distance between the pitch circle and the addendum circle.

Generally = 1 module

- **Dedendum**: The radial distance between the pitch circle and the root circle.

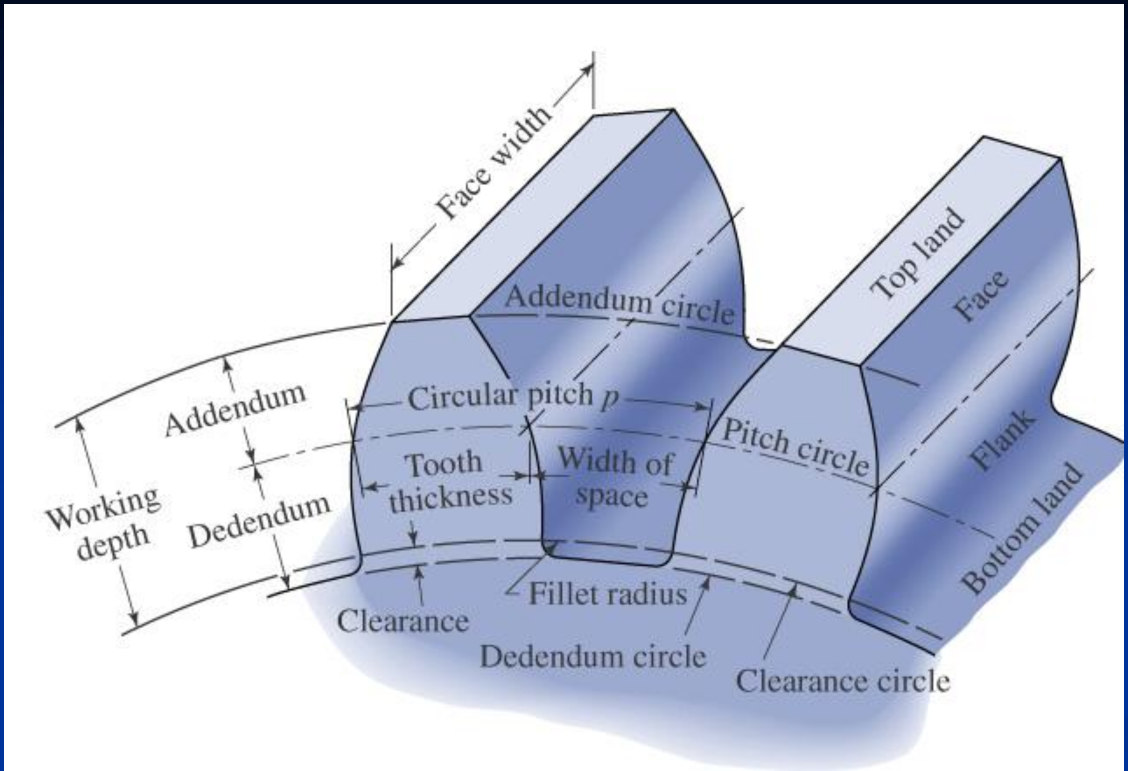
= 1.157 module



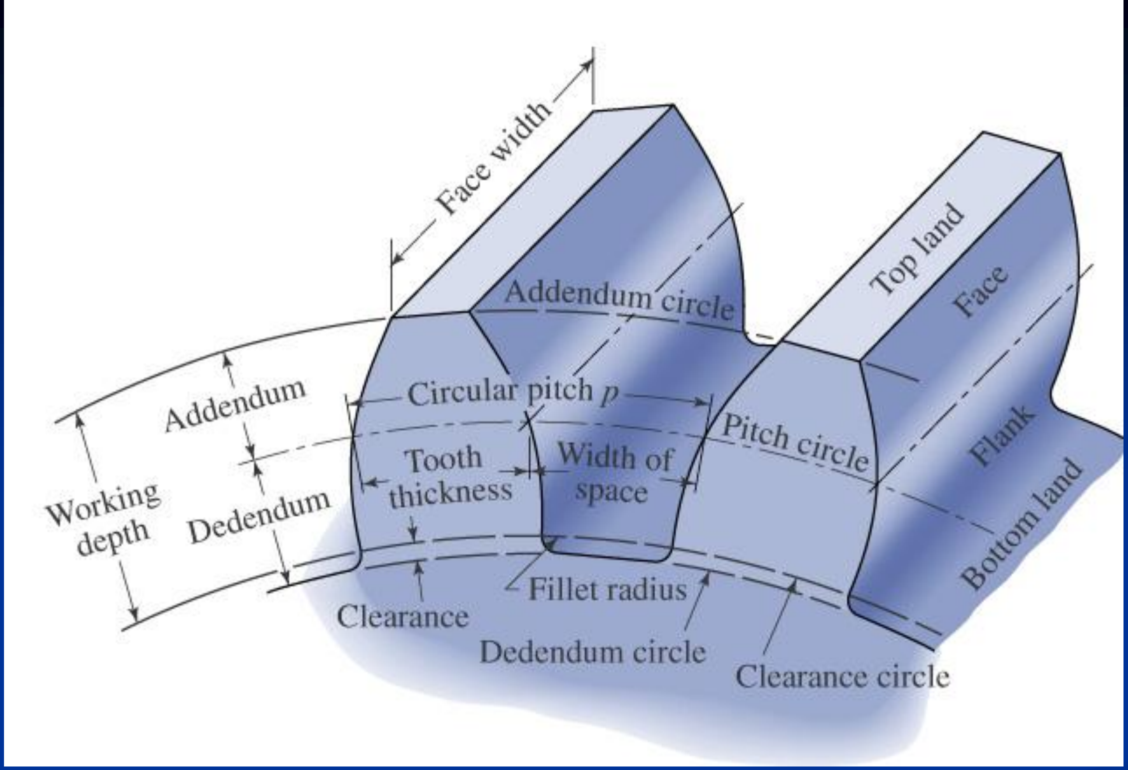
- **Addendum circle**:

- **Root (or dedendum) circle**:

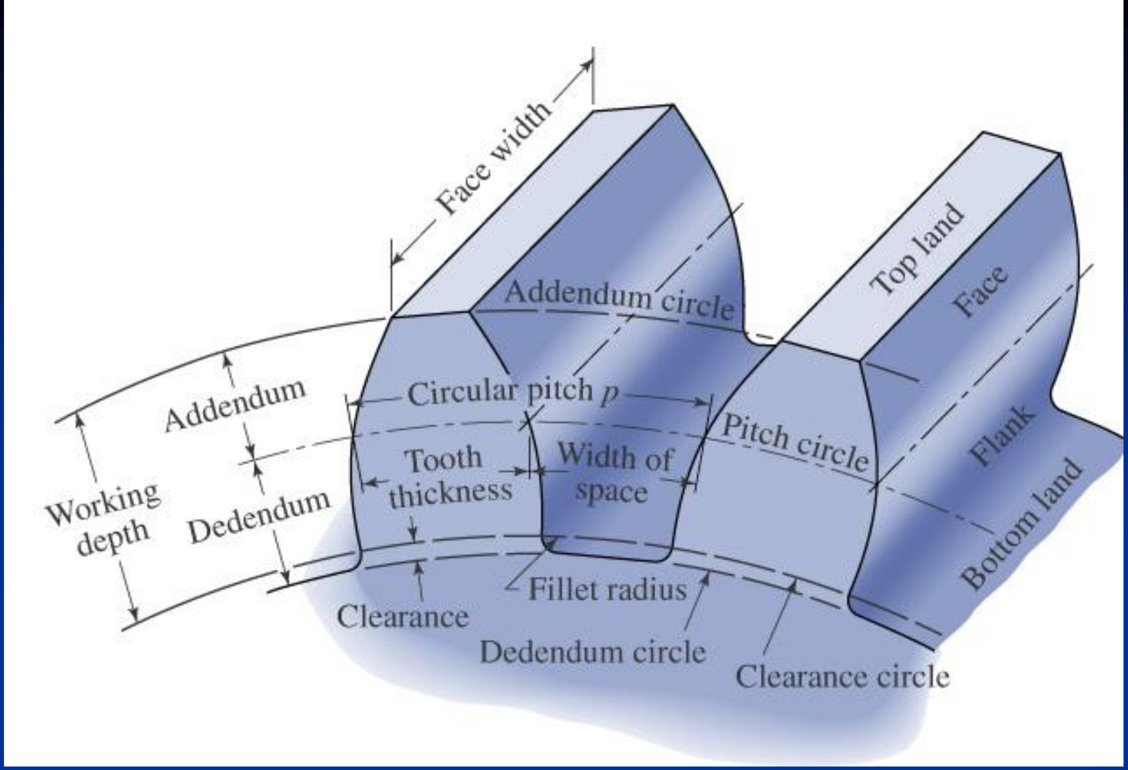
- **Clearance**: The difference between the dedendum of one gear and the addendum of the mating gear.



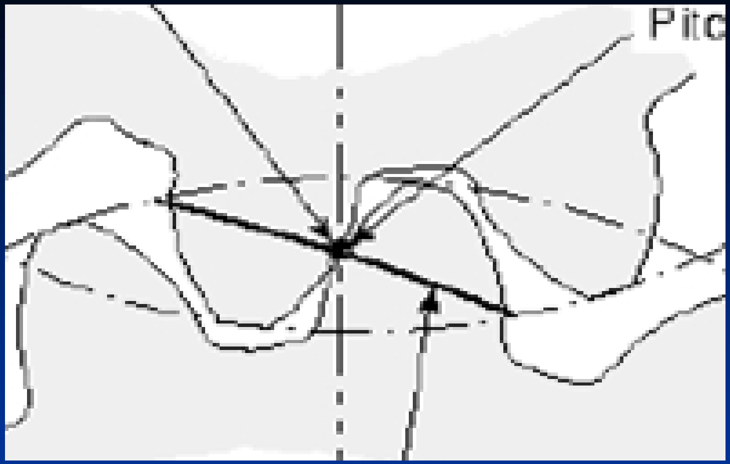
- **Full depth of teeth:**
- **Working depth of teeth:**



- **Face of a tooth**: That part of the tooth surface lying outside the pitch surface
- **Flank of a tooth**: The part of the tooth surface lying inside the pitch surface.
- Fillet ; Top Land; Face width



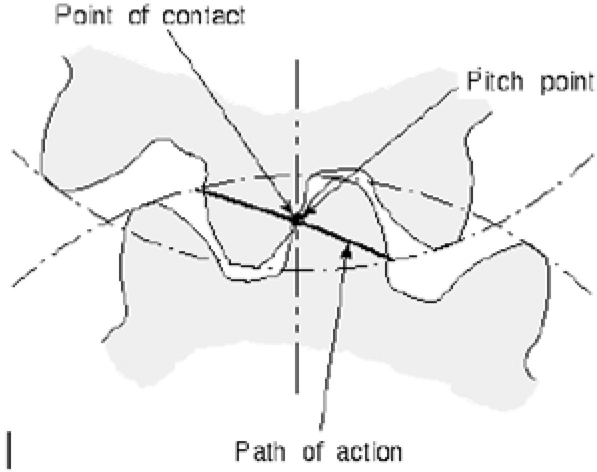
- **Circular thickness** (also called the **tooth thickness**): The thickness of the tooth measured on the pitch circle. It is the length of an arc and not the length of a straight line.
- **Tooth space**: pitch diameter The distance between adjacent teeth measured on the pitch circle.
- **Backlash**: The difference between the circle thickness of one gear and the tooth space of the mating gear.



- When 2 gears are meshed, there is a certain amount of built in “play” between them called backlash.
- When 2 gears are not meshed properly i.e. within specification you get too much backlash called slop OR too little backlash and they are jammed together and this creates friction.

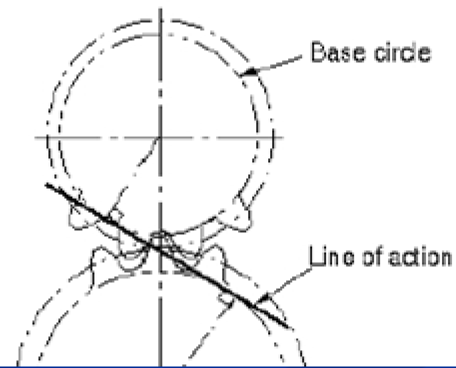
Pressure Line (Line of Action)

Point of contact: any point at which two tooth profiles touch each other.

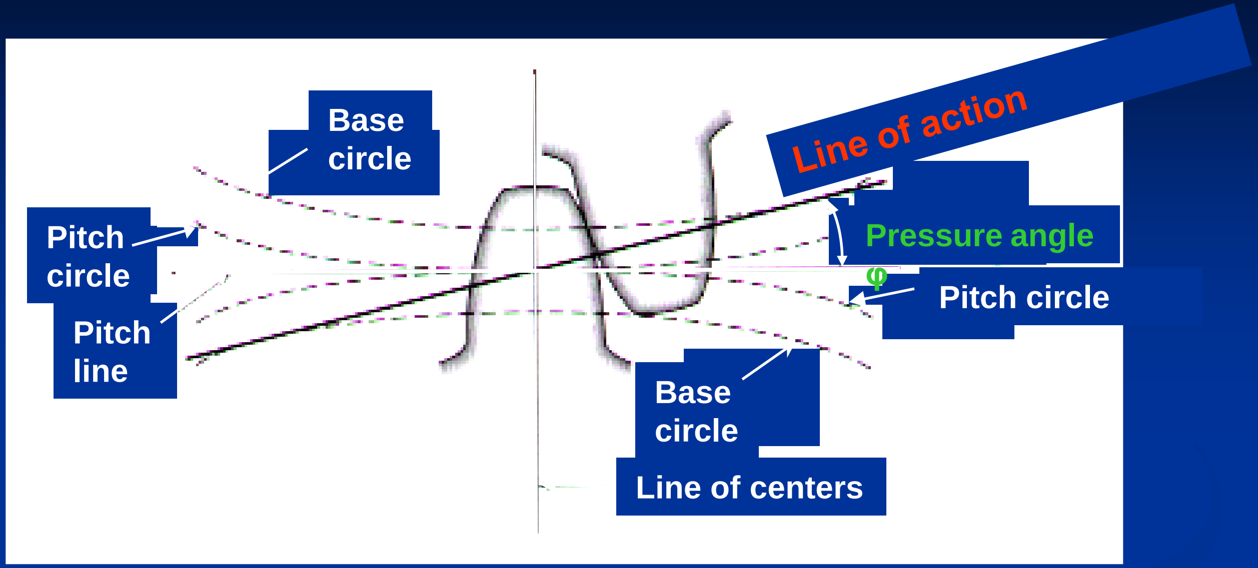


Pressure Line OR

Line of action: The line of action is the path of action for involute gears. It is the straight line passing through the pitch point and tangent to both base circles.



Pressure angle



Standard pressure angles, 14.5^o (old), 20^o, and 25^o

Types of Profile of Gear Teeth

Conjugate Teeth:

Any arbirary profile of teeth ; Profile of another teeth is obtained by law of gearing

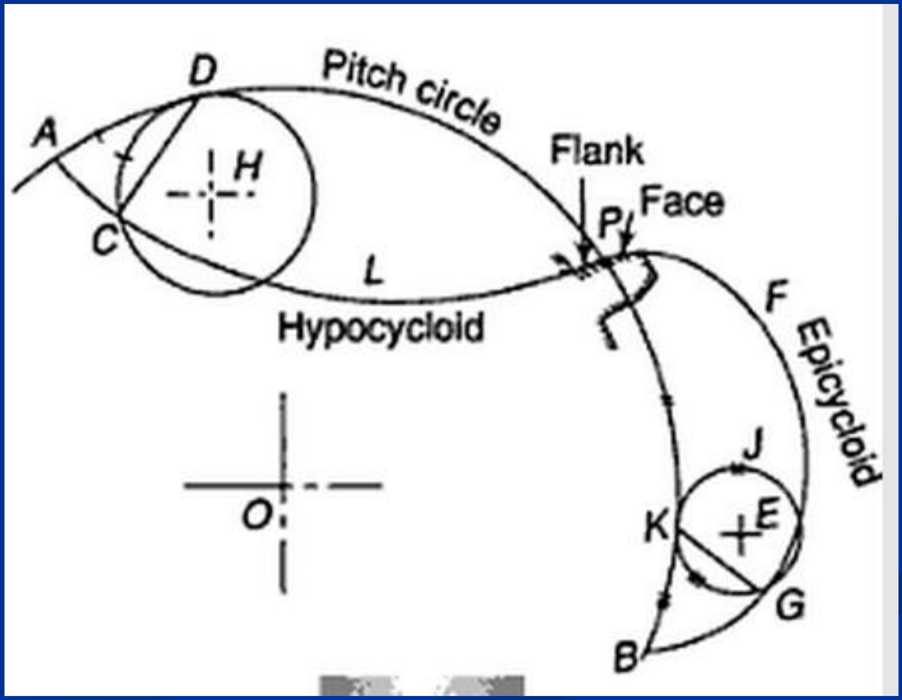
1 Cycloidal Profile Teeth

2 Involute Profile Teeth

Cycloid Profile Teeth

Epicycloid: Circles rolls outside the pitch circle

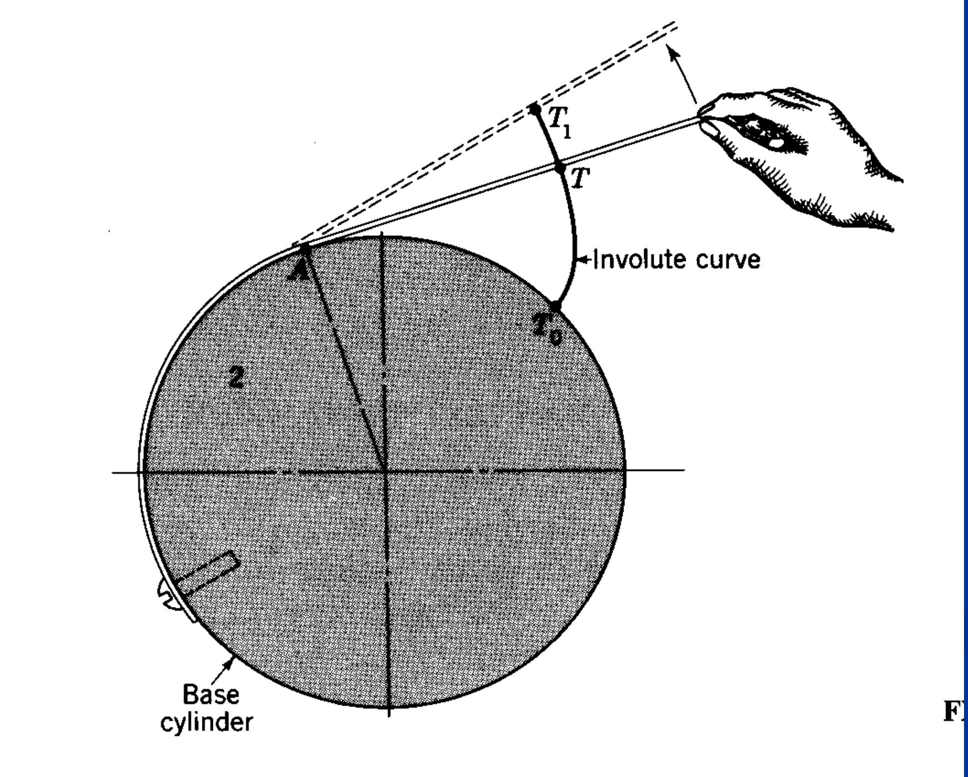
Hypocycloid: Circles rolls inside the pitch circle

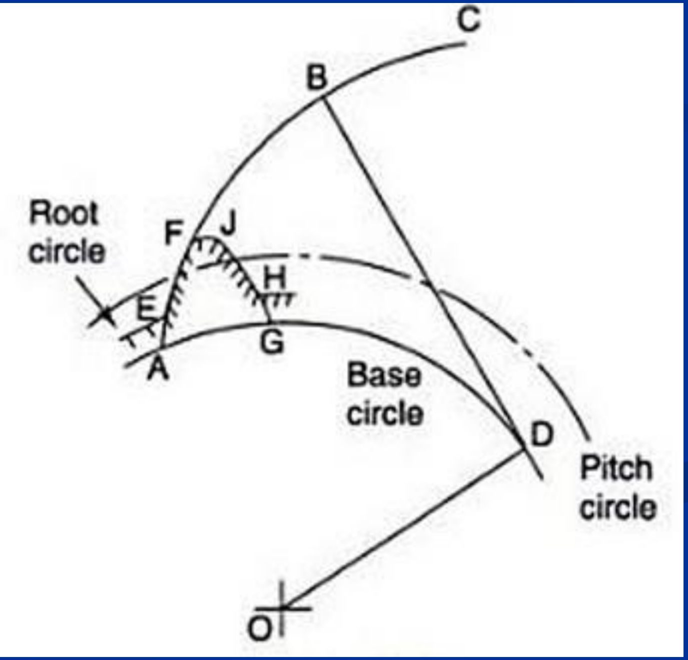


Involute profile of Teeth

Involute: Locus of point on a straight line which rolls without slipping on the circumference of a circle

At any point on involute profile , normal is always tangent to the base circle. Hence a common tangent to the base circle is a common normal to the the two involute of tooth profile of gears in mesh.





Comparison

- **Involute**
  - **Pressure Angle:**

  Constant throughout the engagement
  - Ease of Manufacture-Easy
  - Centre Distance: Do not require exact center distance
  - Intereference: May Occur
  - Strength :Less
  - Wear: More wear and tear
  - Operation: smooth

- **Cycloidal**
  - Varies from commencement to end
  - Difficult
  - requires exact centre distance
  - No intereference
  - More(wider flank)
  - Less wear and tear
  - Less smooth(φ varies)

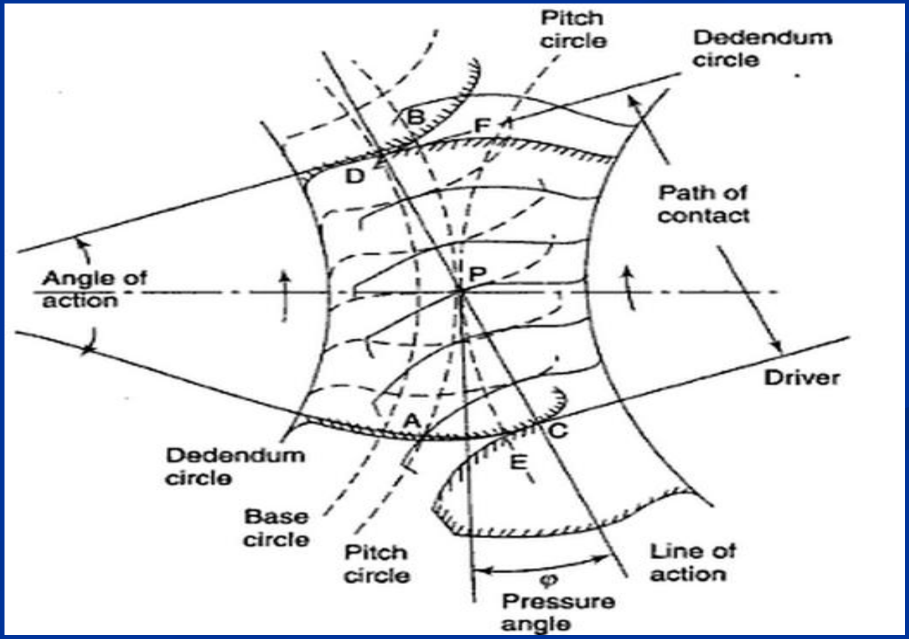
Path of Contact:Locus of point of contact of two mating teeth from the beginning of engagement to the end of engagement

Path of approach: Path of contact from the beginning of engagement to pitch poi

Path of recess: Path of contact from the pitch point to end of engagement

Arc of contact: Locus of point on the pitch circle from the beginning to end of engagement of two mating teeth

Arc of approach
Arc of Recess

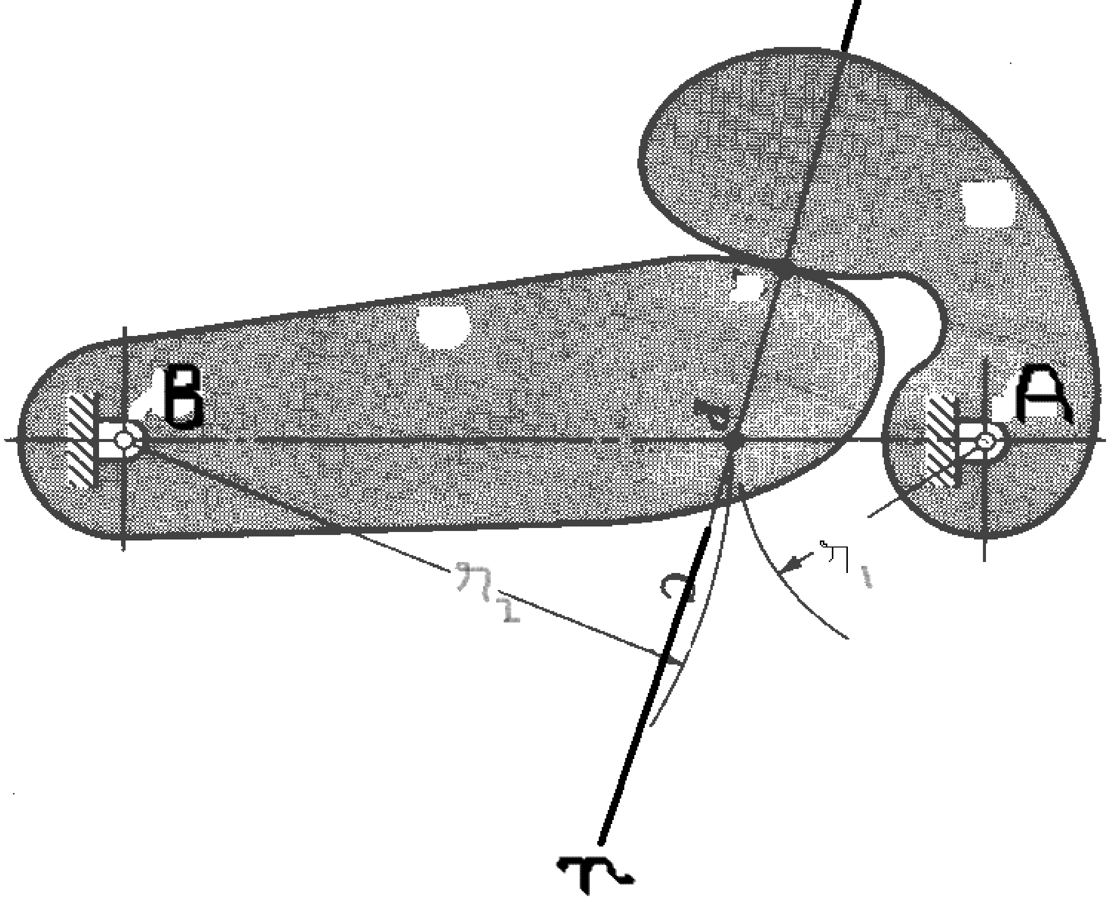


Law of Gearing

Fundamental law of gearing:

velocity ratio must be constant as gears rotate

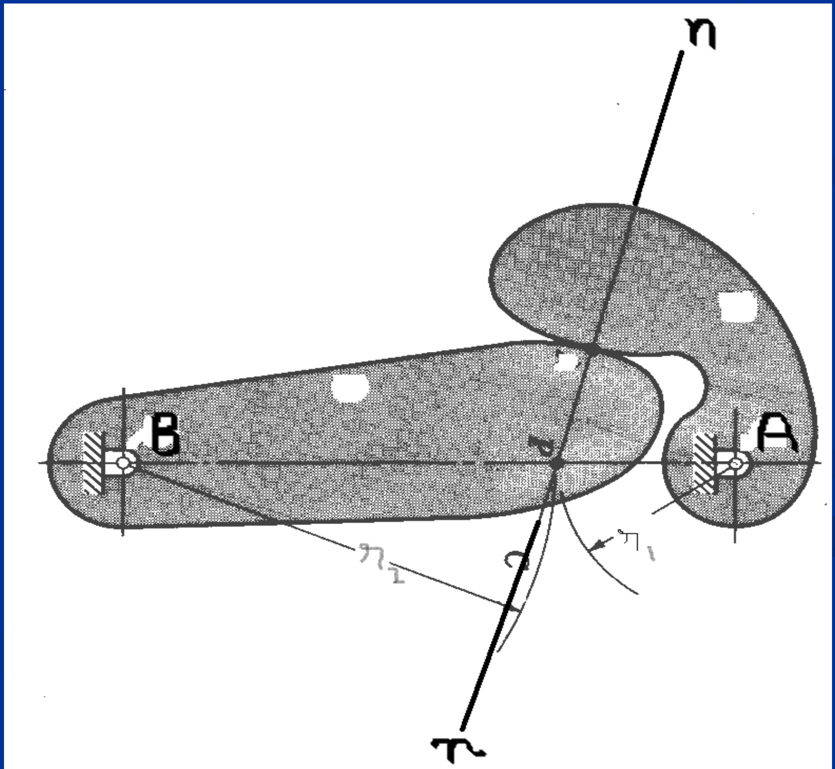
$$\frac{\omega 2}{\omega 1} = \frac{AP}{BP}$$



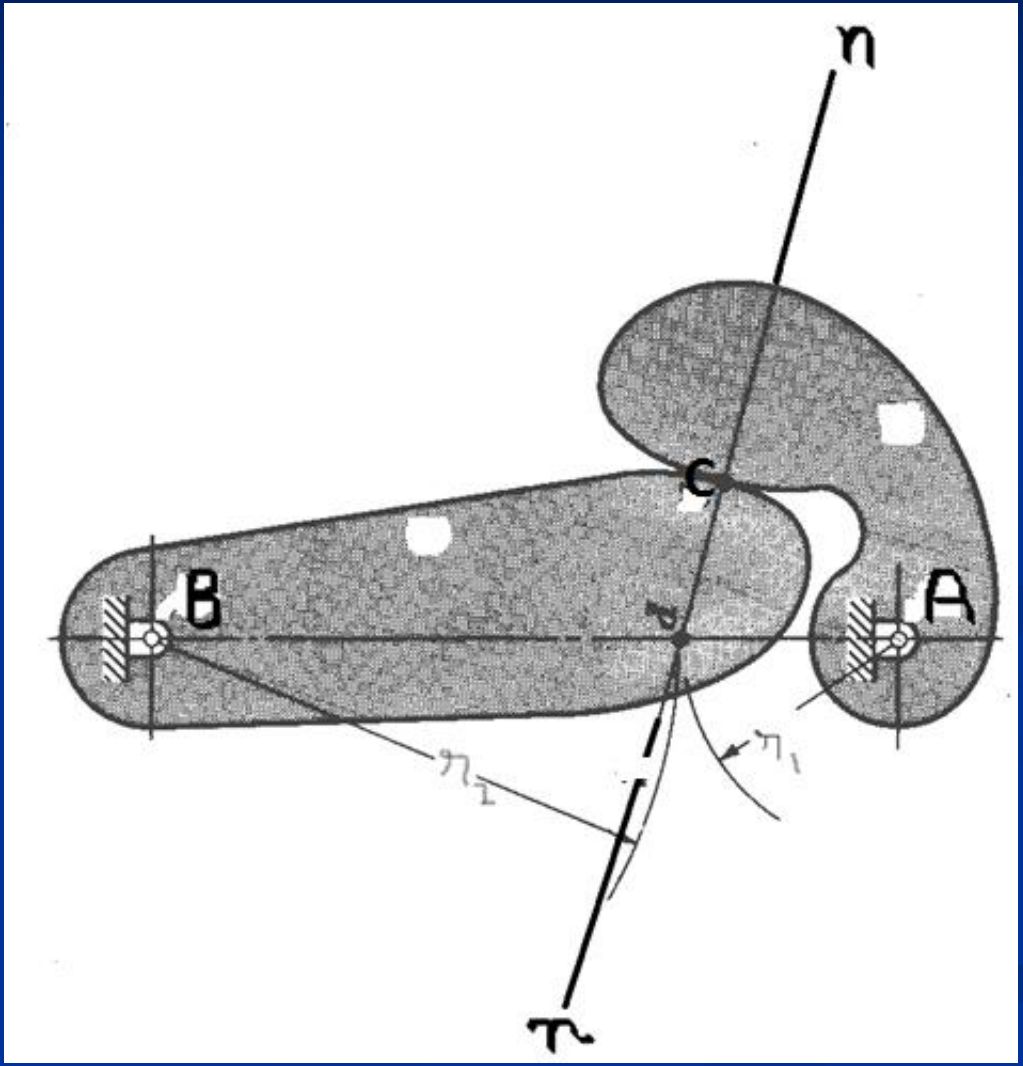
Fundamental law of gearing:

velocity ratio must be constant as gears rotate

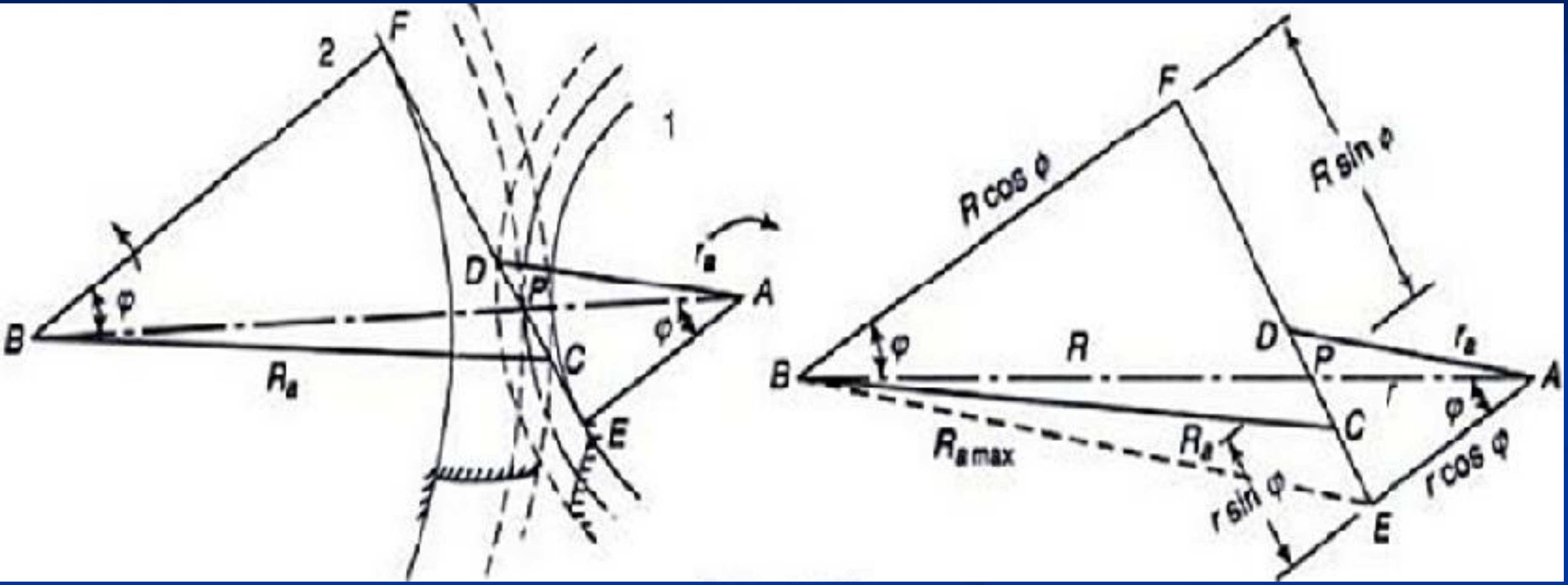
The common normal of the tooth profiles at all points within the mesh must always pass through a fixed point on the line of the centers called *pitch point*. Then the gearset's velocity ratio will be constant through the mesh and be equal to the ratio of the gear radii.



Velocity of sliding



Length of Path of contact



Path of contact= Path of approach + Path of Recess

CP + PD

$(\sqrt{R_a^2 - R^2 Cos^2 Ø}$ -$R\ Sin\ Ø$)+ ($\sqrt{r_a^2 - r^2 Cos^2 Ø}$ -r $Sin\ Ø$)

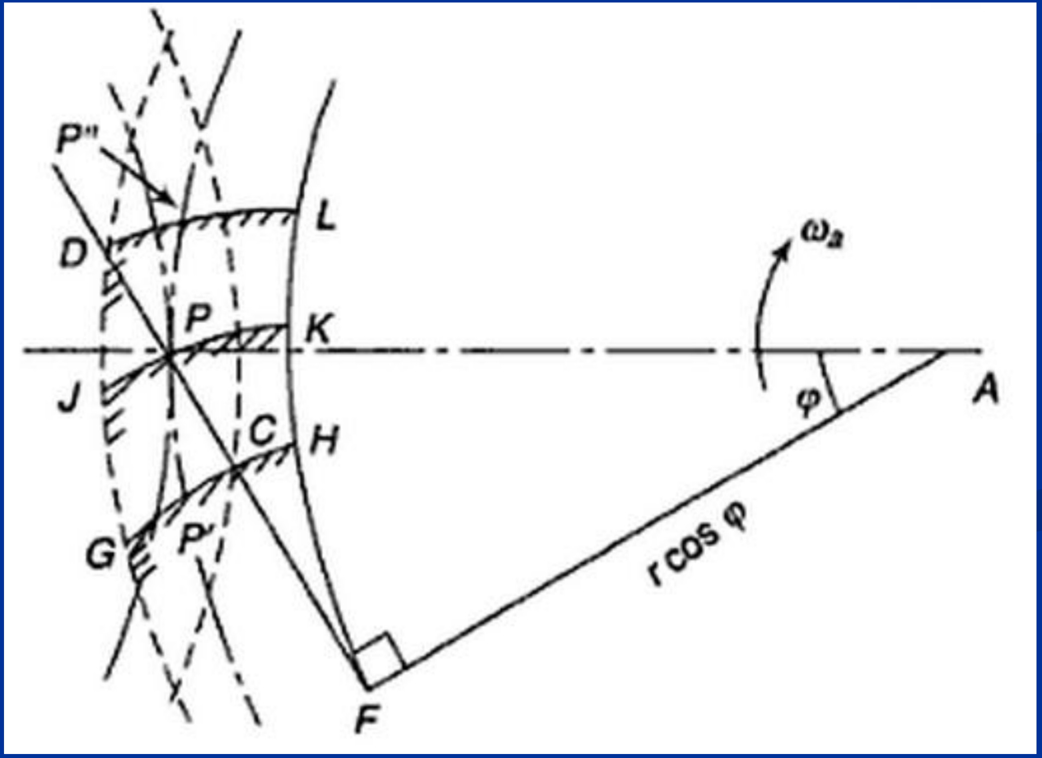
Length of Arc of contact

= Arc of Approach + Arc of Recess
= Arc P' P + Arc P P'

Arc of Approach = Tangential velocity of P' x time of approach

Arc P'P = $\omega_a . r \; X \; t_a$

= Path of Approach / Cos φ

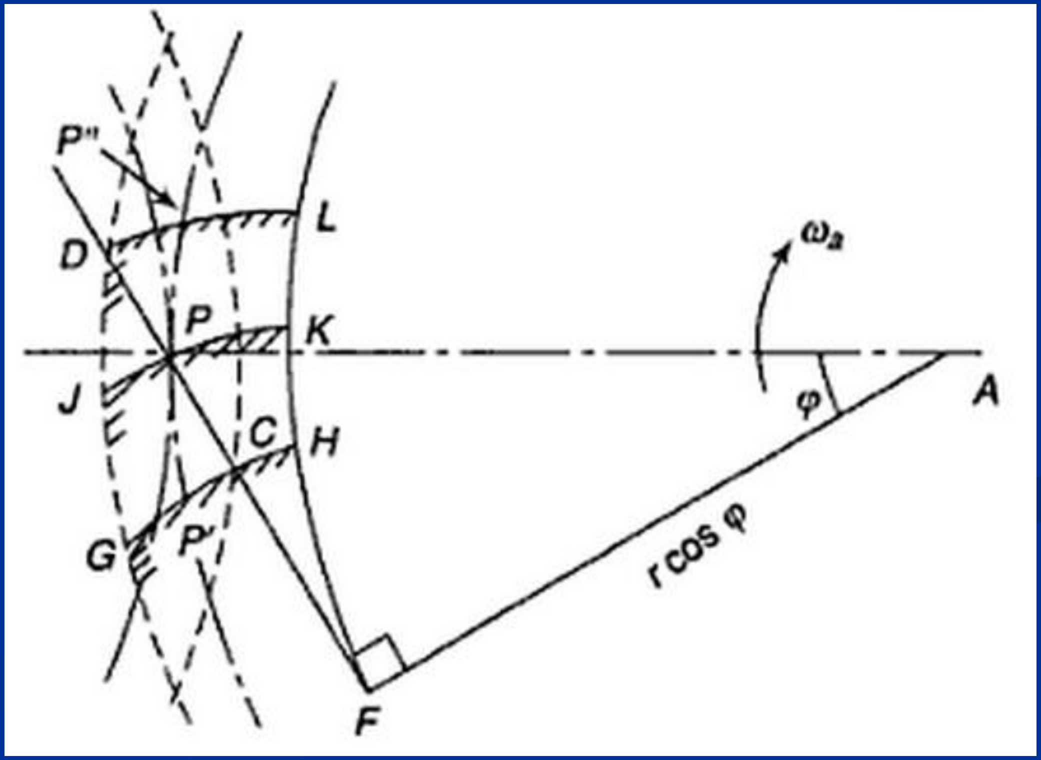


Number of pairs of teeth in contact(Contact Ratio)

= Length of Arc of Contact

Circular Pitch

Contact Ratio must be greater than 1



Interference in Involute Gears And Undercutting

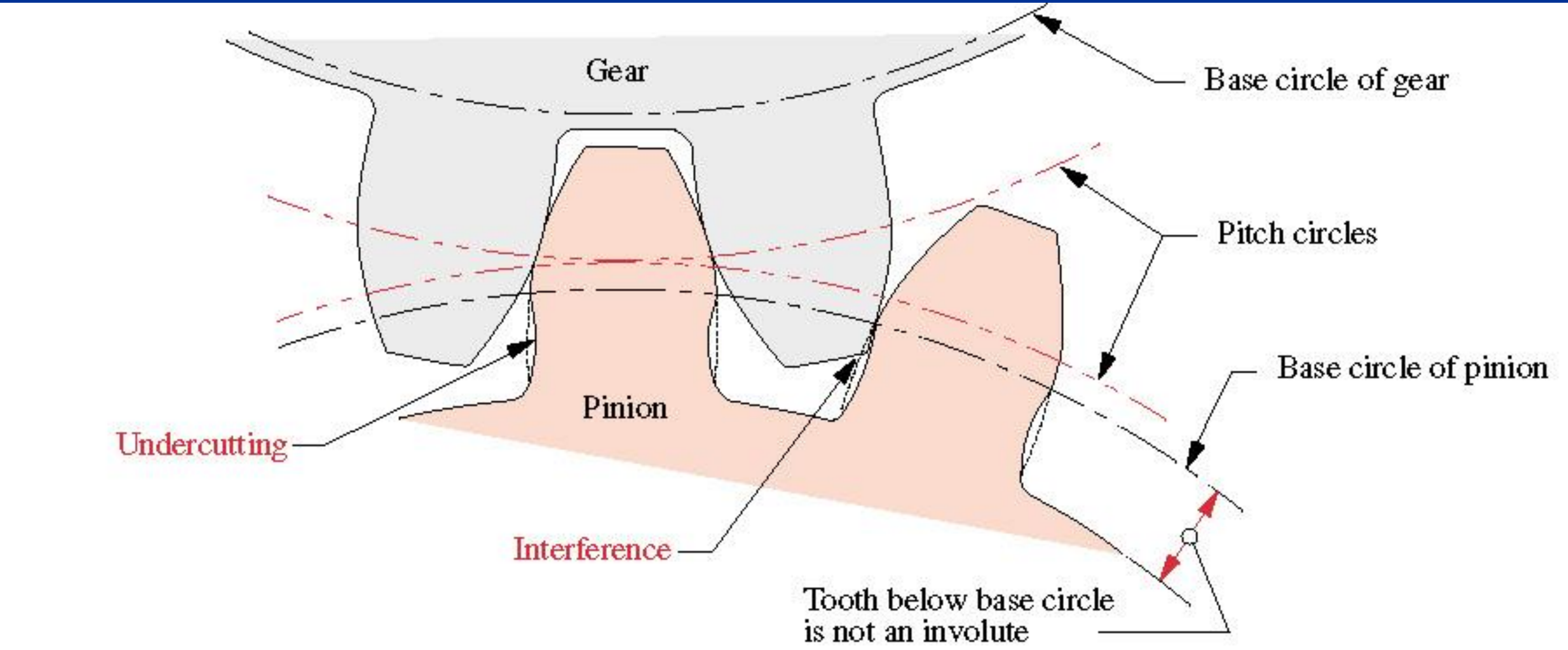


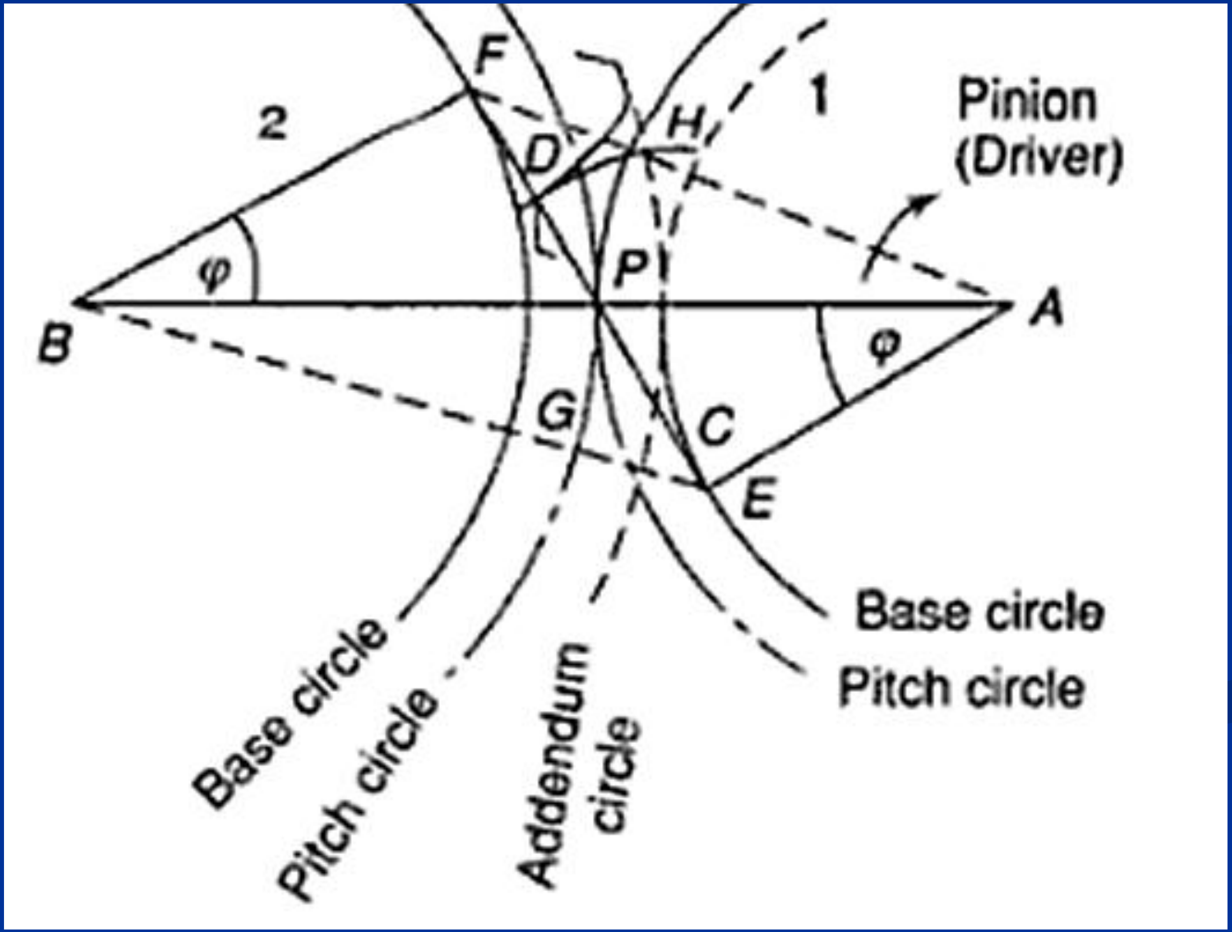
FIGURE 9-12

Interference and undercutting of teeth below the base circle

Interference in Involute Gears

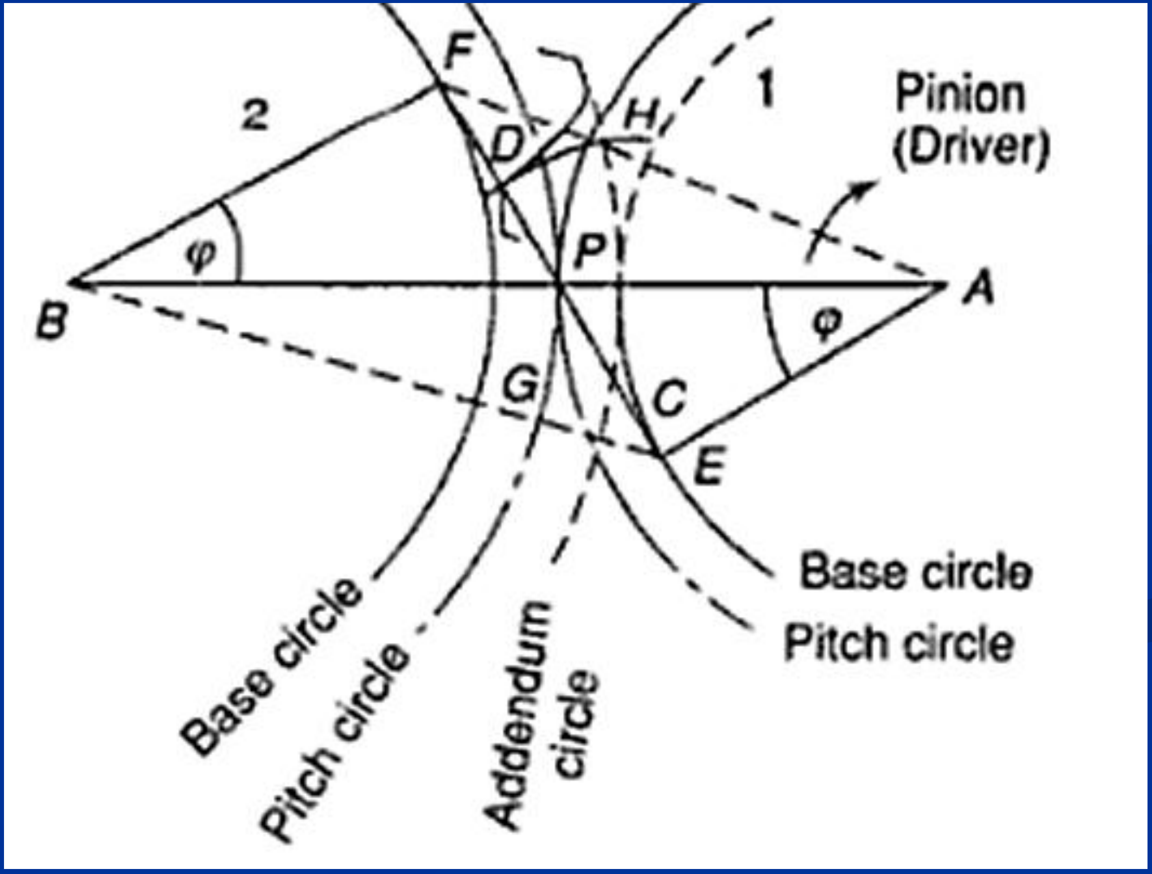
- Point E and F are interference points.
- Generally GE < HF , Therefore GE will decide about the interference
- Since an involute can exist only outside the base circle ,

therefore any profile of teeth inside the base circle result in interference



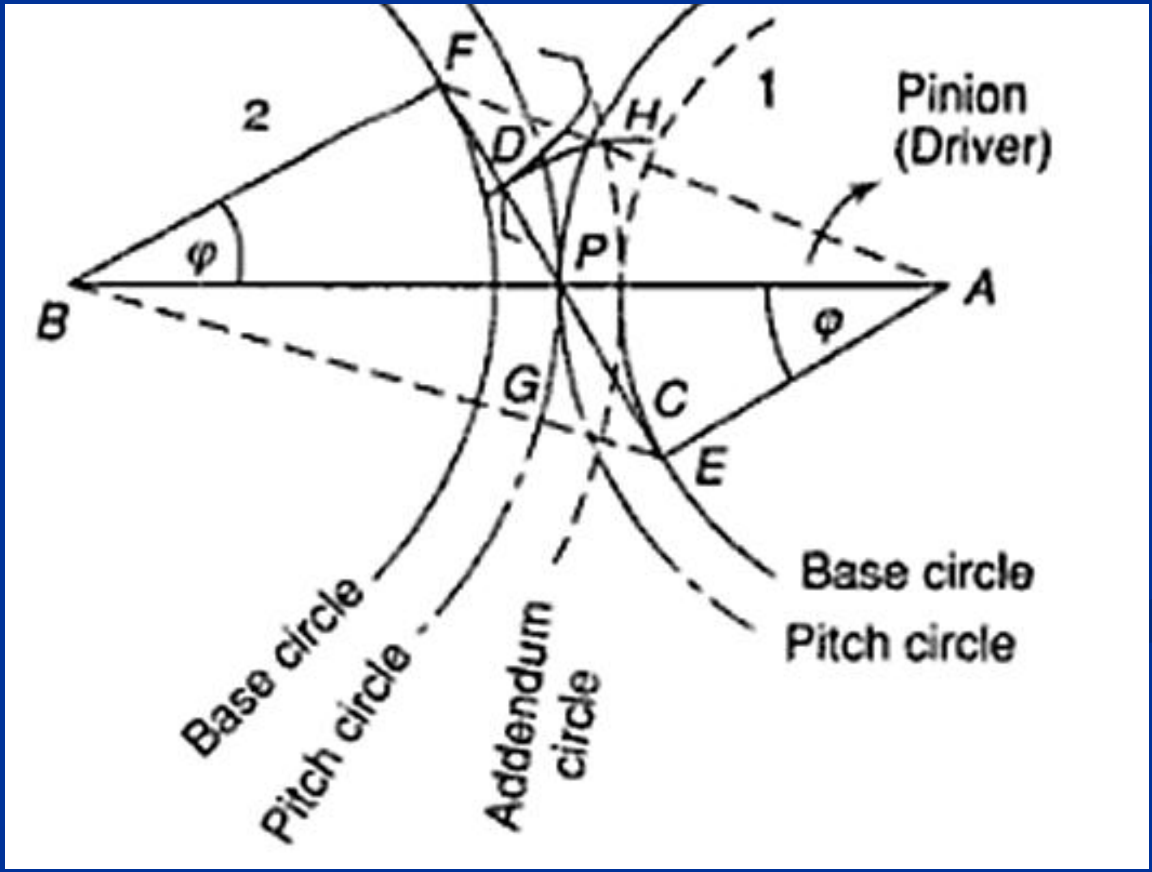
Interference in Involute Gears

For minimum number of teeth to avoid interference ,
the common tangent to base circle cuts the addendum circles
between E and F , Which are interference points



Interference in Involute Gears

$$T_{min} \geq \frac{2a_w}{\left\{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)Sin^2\emptyset}\right\} - 1} \qquad t_{min} \geq \frac{2a_p}{\left\{\sqrt{1 + G(G+2)Sin^2\emptyset}\right\} - 1}$$



Gear Review

- What is a gear?
- Name 4 different types of gears.
- What is torque and rpm?
- What is diametral pitch & how is it shown?
- List 10 places where gears are used.

Gear Train

- Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called ***gear train or train of toothed wheels.***

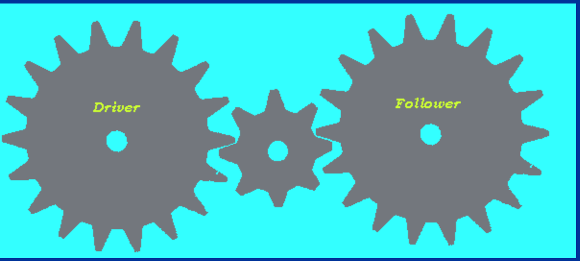
- The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts.

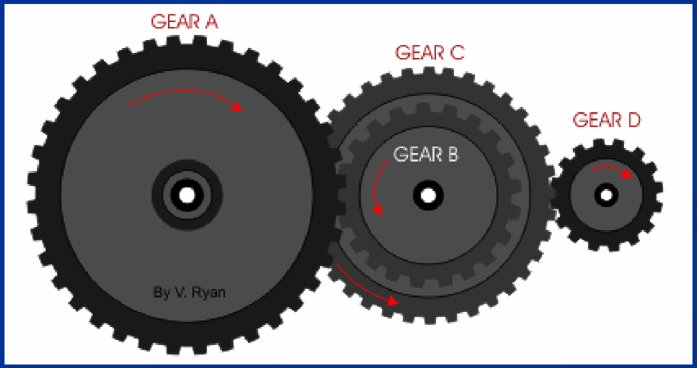
- A gear train may consist of spur, bevel or spiral gears.

Types of Gear Trains

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Epicyclic gear train

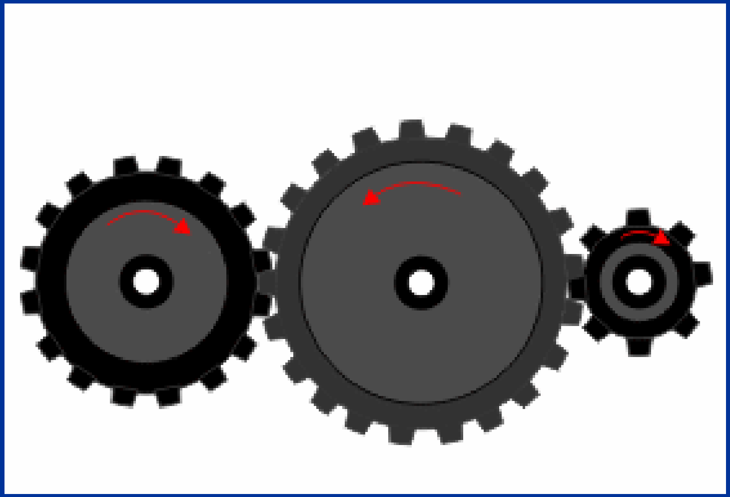
- In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other.
- But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.





Simple Gear Train

- ***Multiple gears can be connected together to form a gear train.***



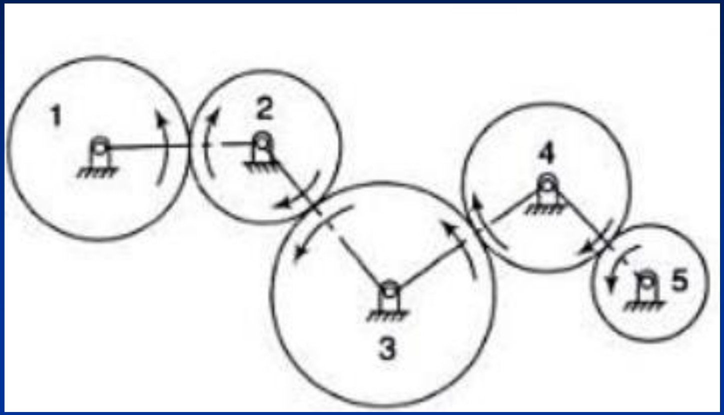
Each shaft carries only one gear wheel.

Intermediate gears are known as Idler Gears.

It has no affect on the gear ratio.

Velocity ratio = Speed of driver / Speed of driven(follower)

Train Value = 1/ Velocity ratio



Velocity ratio = N1/ N2

Train Value = N2/N1

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} \qquad \left[\text{Also } \frac{\omega_2}{\omega_1} = \frac{2\pi N_2}{2\pi N_1} = \frac{N_2}{N_1}\right]$$

$$\frac{N_3}{N_2} = \frac{T_2}{T_3}, \frac{N_4}{N_3} = \frac{T_3}{T_4} \text{ and } \frac{N_5}{N_4} = \frac{T_4}{T_5}$$

$$\frac{N_2}{N_1} \times \frac{N_3}{N_2} \times \frac{N_4}{N_3} \times \frac{N_5}{N_4} = \frac{T_1}{T_2} \times \frac{T_2}{T_3} \times \frac{T_3}{T_4} \times \frac{T_4}{T_5}$$

$$\text{Train value } \frac{N_5}{N_1} = \frac{T_1}{T_5} = \frac{\text{number of teeth on driving gear}}{\text{number of teeth on driven gear}}$$

Compound Gear Train

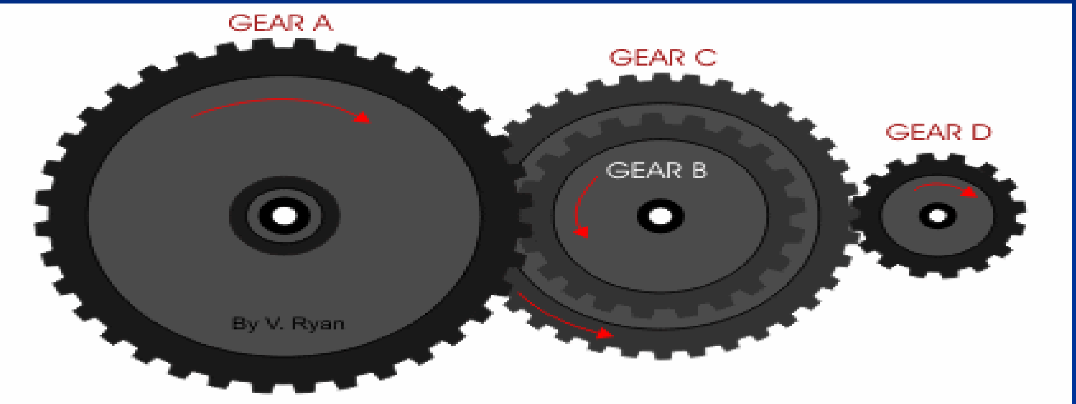
- When there are more than one gear on a shaft, it is called a

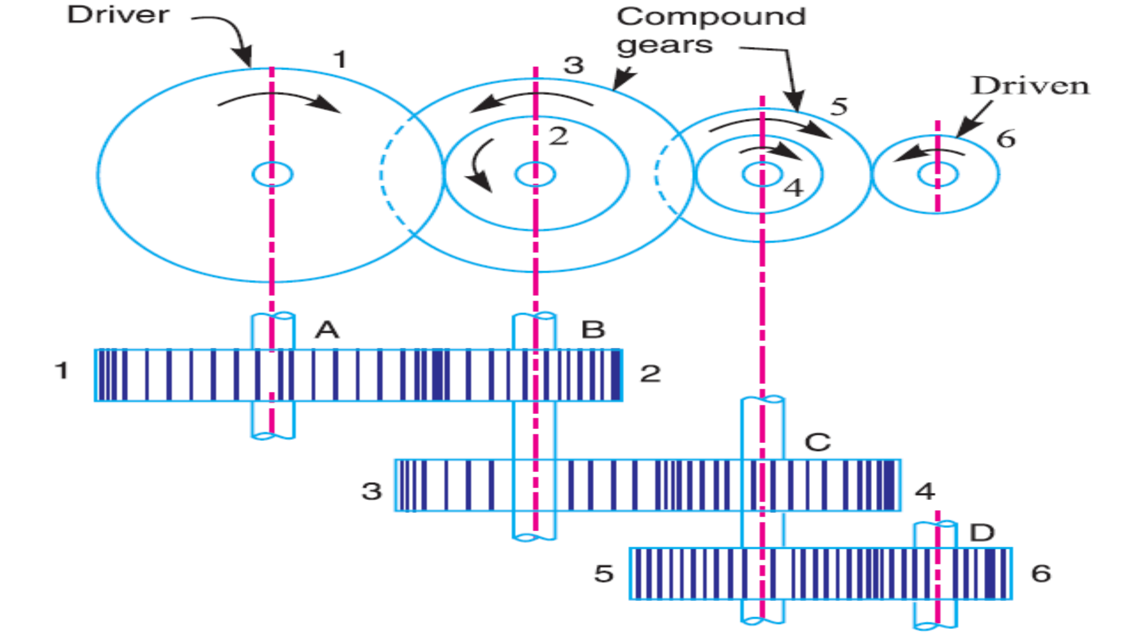
compound gear train

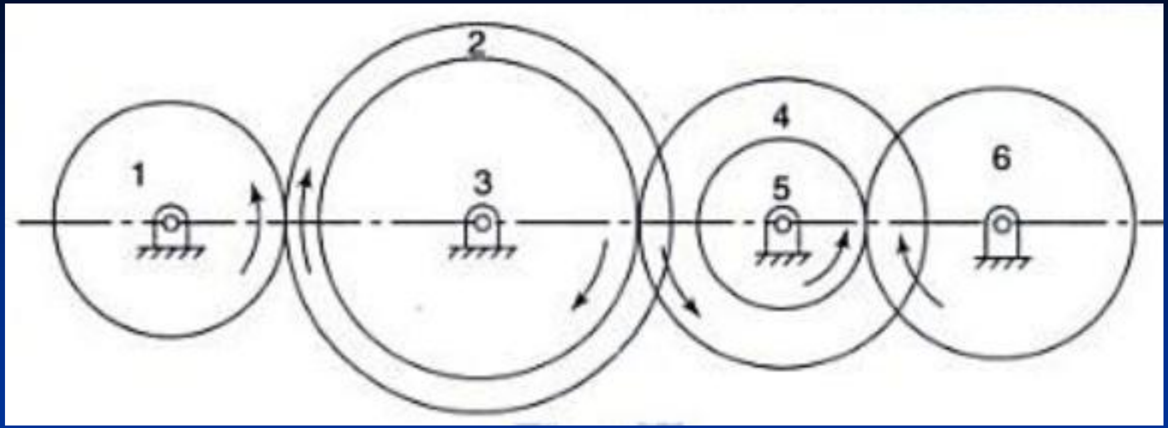
- For large velocities ratios, compound gear train arrangement is preferred.
- ***Gear Ratio > 7***

Compound Gear Train

- When there are more than one gear on a shaft, as shown in Fig. , it is called a *compound train of gear.*







$$\frac{N_2}{N_1} = \frac{T_1}{T_2}, \quad \frac{N_4}{N_3} = \frac{T_3}{T_4} \text{ and } \frac{N_6}{N_5} = \frac{T_5}{T_6}$$

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} \times \frac{N_6}{N_5} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

$$\frac{N_2}{N_1} \times \frac{N_4}{N_2} \times \frac{N_6}{N_4} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

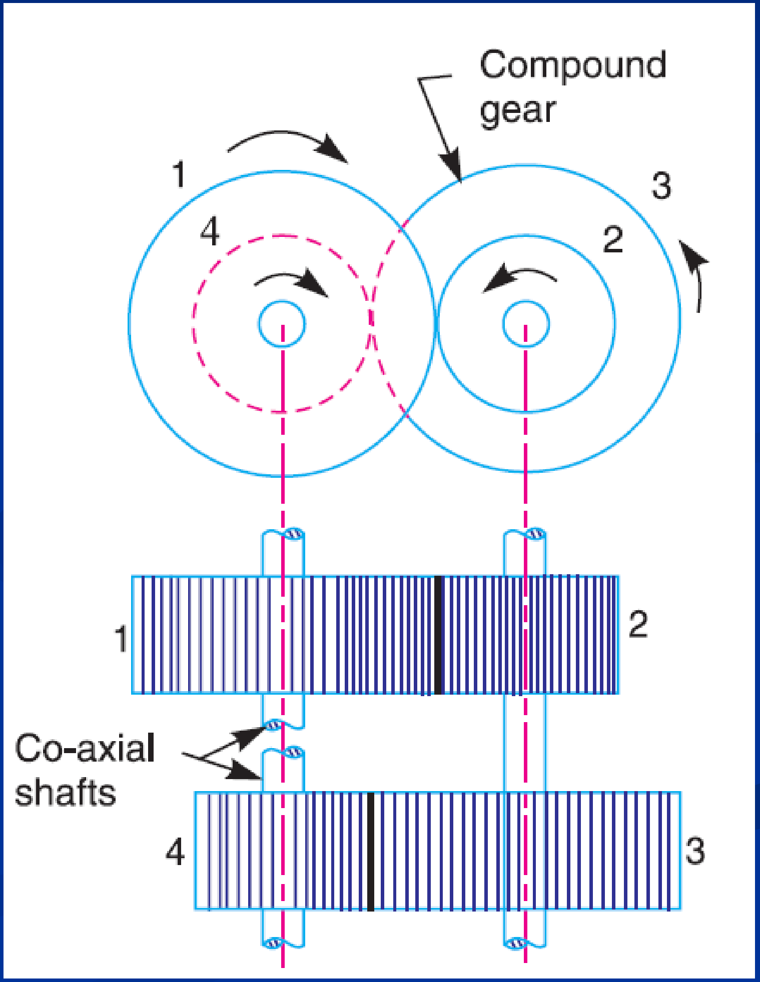
$$\frac{N_6}{N_1} = \frac{T_1}{T_2} \frac{T_3}{T_4} \frac{T_5}{T_6}$$

$$\text{Train value} = \frac{\text{product of number of teeth on driving gears}}{\text{product of number of teeth on driven gears}}$$

Reverted Gear Train

- When the axes of the first driver and the last driven are co-axial, then the gear train is known as ***reverted gear* train.**

- In a reverted gear train, the motion of the first gear and the last gear is ***same.***

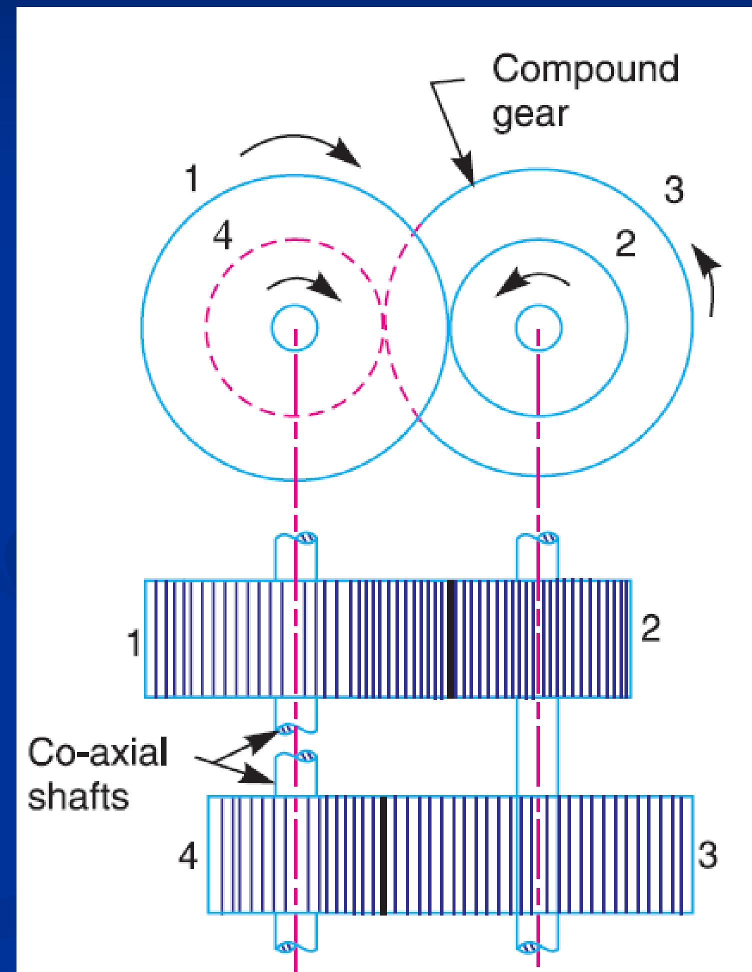


$$\frac{N_4}{N_1} = \frac{\text{product of number of teeth on driving gears}}{\text{product of number of teeth on driven gears}}$$

$$\frac{T_1}{T_2}\,\frac{T_3}{T_4} \qquad (11.3)$$

o, if r is the pitch circle radius of a gear,

$$r_1 + r_2 = r_3 + r_4$$



$$\frac{mT_1}{2} + \frac{mT_2}{2} = \frac{mT_3}{2} + \frac{mT_4}{2}$$

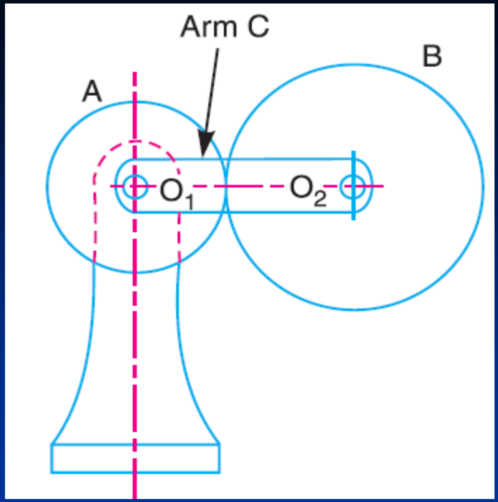
T1 +T2 =T3 + T4

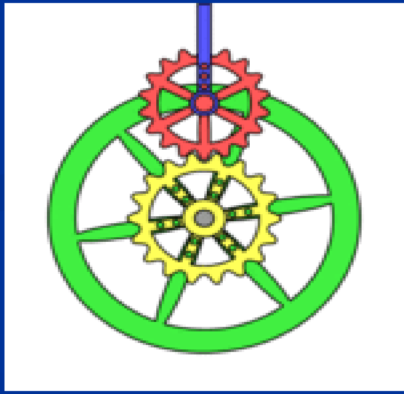
Epicyclic Gear Train

- Epicyclic means one gear revolving upon and around another. The design involves planet and sun gears as one orbits the other like a planet around the sun.

Epicyclic Gear Train

- In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis.
- Gear *A and the arm C have a common axis at* O1 about which they can rotate.





•Useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space.

•The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

Velocity ratio of epicyclic gear train

- **velocity ratio;**

velocity ratio of epicyclic gear train is the ratio of the speed of the driver to the speed of the driven or follower.

- The following two methods may be used for finding out the velocity ratio of an epicyclic gear train.

1. **Tabular method**
2. **Algebraic method**

1. Tabular method

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear *A* rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_A}{T_B}$

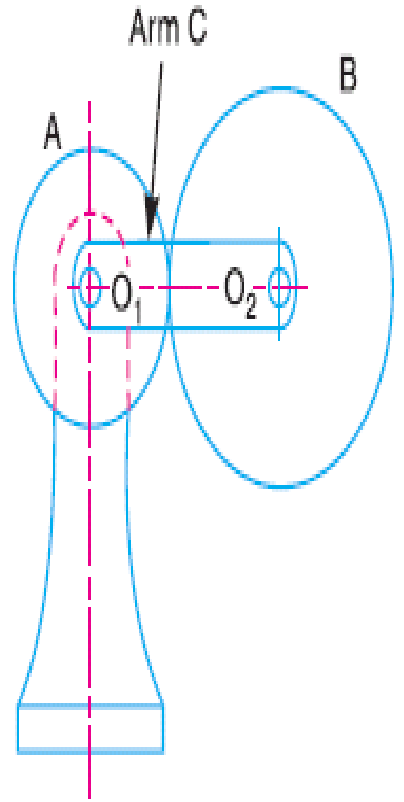


Fig. 13.6. Epicyclic gear train.

2. Algebraic method

In this method, the motion of each element of the epicyclic train relative to the arm is set down in the form of equations. The number of equations depends upon the number of elements in the gear train. Let the arm C be fixed in an epicyclic gear train as shown in Fig

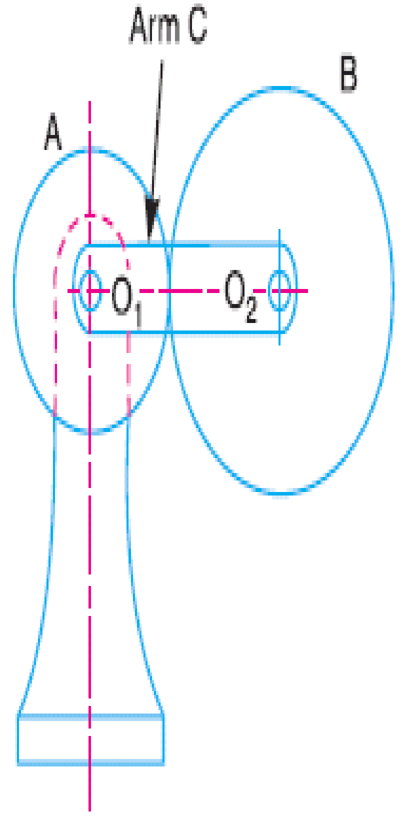


Fig. 13.6. Epicyclic gear train.

Therefore,

Seed of the gear A relative to the arm C= N(A)-N(C)

Speed of the gear B relative to the arm C= N(B) –N(C)

Since the gears A and B are meshing directly, therefore they will revolve in *opposite* directions.

$$\therefore \quad \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

Since the arm C is fixed, therefore its speed, $N_C = 0$.

$$\therefore \quad \frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

If the gear A is fixed, then $N_A = 0$.

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \quad \text{or} \quad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$

Note : The tabular method is easier and hence mostly used in solving problems on epicyclic gear train.

ADVANTAGES of Simple Gear Train

- to connect gears where a large center distance is required

- to obtain desired direction of motion of the driven gear (CW or CCW)

- to obtain high speed ratio

ADVANTAGES of Compound Gear Train

- A much larger speed reduction from the first shaft to the last shaft can be obtained with small gear.
- If a simple gear trains used to give a large speed reduction, the last gear has to be very large.

Advantages of Riverted Gear Train

- The reverted gear trains are used in automotive transmissions, lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).

YouTube Gear Animations:

- Speed Reducers:
- http://www.youtube.com/watch?v=7LReoWPg_pM&feature=related
-
- http://www.youtube.com/watch?v=1_jbZVBXjWc&feature=related
-
- Automotive Differential: http://www.youtube.com/watch?v=iBLE0_Sjqw4&feature=related
- Manual Transmission: http://www.youtube.com/watch?v=MBmLJCeGu7o&feature=related
- Gear Cutting:
- http://www.youtube.com/watch?v=fps0OR1eF_s&feature=related
- http://www.youtube.com/watch?v=xF9CjluRFJ4&feature=related