DESIGNING OF SHAFTS

The shafts may be designed on the basis of -

- Strength
- Stiffness

Cases of designing of shafts, on the basis of Strength

(a) Shafts subjected to Twisting Moment only.

(b) Shafts subjected to Bending Moment only.

(c) Shafts subjected to Combined Twisting and Bending Moment only.

SHAFTS SUBJECTED TO TWISTING MOMENT ONLY

Determination of diameter of Shafts, by using the Torsion Equation;

$$T/J = \varsigma/r$$

Where;

T- Twisting moment acting upon shafts.

J - Polar moment of inertia of shafts.

ς – Torsional shear stress.

r – Distance from neutral axis to outermost fibre.

For round solid Shafts,Polar Moment of inertia,

$$J = \pi/32 * d^4$$

Now Torsion equation is written as,

$$T / [\pi/32 * d^4] = \varsigma / (d/2)$$

Thus;

$$T = \pi/16 * \varsigma * d^3$$

For hollow Shafts, Polar Moment of inertia ,
$J = \pi/32 [(D^4) – (d^4)]$

Where ,

D and d – outside and inside diameter,

Therefore, torsion equation;

$$T / \pi/32[D^4 – d^4] = \varsigma / (D/2)$$

Thus;

$$T = \pi/16 * \varsigma * D^3 * (1 – k^4).$$

SHAFTS SUBJECTED TO BENDING MOMENT ONLY

For Bending Moment only, maximum stress is given by BENDING EQUATION

$$M / I = \sigma / y$$

Where;

M→ Bending Moment .

I → Moment of inertia of cross sectional area of shaft.

σ → Bending stress,

y → Distance from neutral axis to outer most fibre.

For round solid shafts, moment of inertia,

$$I = \pi/64 * d^4$$

Therefore Bending equation is;

$$M / [\pi/64 * d^4] = \sigma / (d/2)$$

Thus;

$$\boxed{M = \pi/32 * \sigma * d^4}$$

For hollow Shafts, Moment of Inertia, $I = \pi / 64 [D^4 – d^4]$

Putting the value, in Bending equation;

We have;

$$M / [\pi/64 \{ D^4 (1- k^4) \}] = \sigma / (D/2)$$

Thus;

$$M = \pi/32 * \sigma * D^3 * (1 – k^4)$$

$k \rightarrow D/d$

SHAFTS SUBJECTED TO COMBINED TWISTING AND BENDING MOMENT

In this case, the shafts must be designed on the basis of two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. Two theories are as follows -

1. Maximum Shear Stress Theory or Guest's theory
2. Maximum Normal Stress Theory or Rankine's theory

According to maximum Shear Stress theory, maximum shear stress in shaft;

$$\varsigma_{max} = (\sigma^2 + 4\,\varsigma^2)\text{^}0.5$$

Putting- $\varsigma = 16T/\pi d^3$ and $\boldsymbol{\sigma = 32M/\pi d^3}$

In equation, on solving we get,

$$\pi/16 * \varsigma_{max} * d^3 = [M^2 + T^2]\text{^}0.5$$

The expression [M + T]^0.5 is known as

equivalent twisting moment

Represented by T_e

Now according to Maximum normal shear stress, the maximum normal stress in the shaft;

$$\sigma_{max} = 0.5^* \sigma + 0.5[\sigma^2 + 4\varsigma^2]^\wedge 0.5$$

Again put the values of **σ,** ς in above equation

We get,

$$\sigma_{max} {}^* d^3 * \pi/32 = ½ [M +\{ M^2 + T^2 \}^{½}]$$

The expression ½ [M +{ M^2 + T^2 }½] is known as *equivalent bending moment*

Represented by *M_e*

THANK YOU