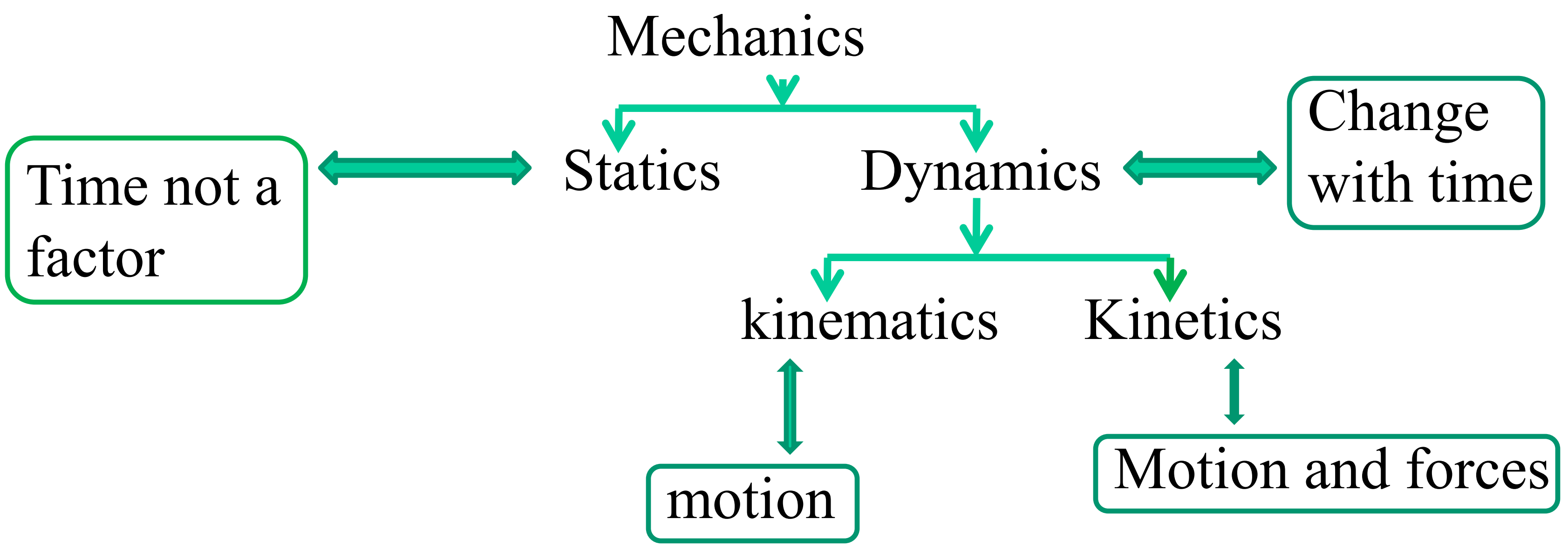
Machine design—what is it?

Subset of Mechanical design…which is
Subset of Engineering design…which is
Subset of Design….which is
Subset of the topic of Problem Solving

What is a machine? …a combination of resistant bodies arranged so that by their means the mechanical forces of nature can be compelled to do work accompanied by certain determinate motions.

Big picture


Mechanics
Time not a factor
Statics
Dynamics
Change with time
kinematics
Kinetics
motion
Motion and forces

The Design Process

- Recognize need/define problem
- Create a solution/design
- Prepare model/prototype/solution
- Test and evaluate
- Communicate design

Important to review the fundamentals of….

- Statics
- Dynamics
- Materials/material properties
    - elasticity
    - homogeneity
    - isotropy
    - mass and area parameters

Lets begin our brief review

$T = I\alpha$ rotary motion equivalent of F=MA

I= mass moment of inertia $\rightarrow \int M*r^2\, dM$

not to be confused with the area moment of inertia which we will discuss later.

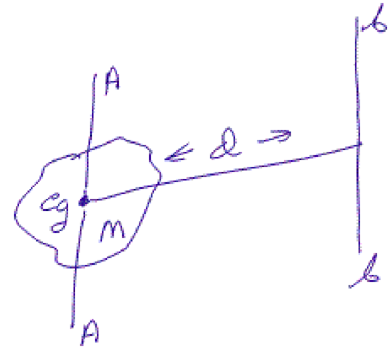
Remember the parallel axis theorem

If Icg is a mass moment of inertia about some axis "aa" thru the centroid (cg) of a body then the moment of inertia about an axis "bb" which is parallel to "aa" and some distance "d" away is given by:

$$I_{bb} = I_{cg} + (d^2) * M$$

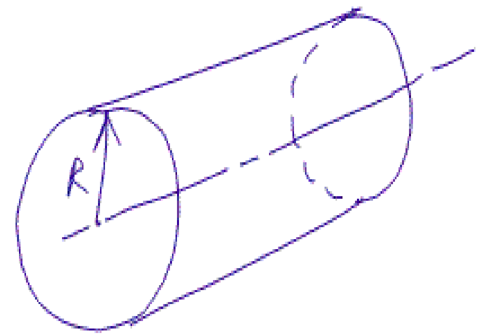
where M is the mass

Note: **This same theorem also works for area moments of inertia in the same way**

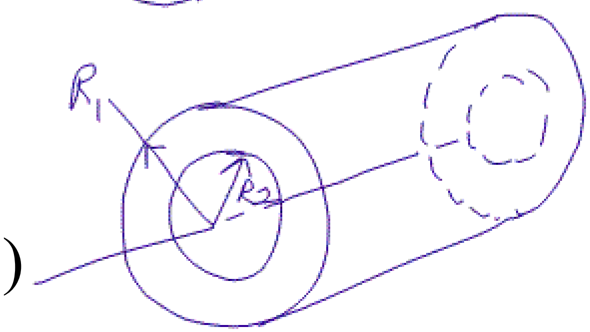


More generally $I=M k^2$ where k is called the radius of gyration which can be thought of as the radius where all the mass could concentrated (relative to the axis of interest) to give the same moment of inertia I that the body with distributed mass has.

For a solid cylinder
$I= M(k^2) = \frac{1}{2} M (R^2)$ where
R= radius
M= mass
K= radius of gyration



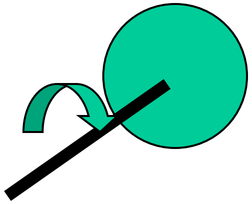
For a hollow cylinder
$I = M(k^2) = \frac{1}{2} M(R1^2 + R2^2)$



Note: this intuitively seems like it should be $(R1^2 – R2^2)$ but that is not the case. Deriving this is a good review of basic calculus.

Short problem…take a few minutes to work on this.

A round flywheel 20 inches diameter has a mass of 100 kgm and is subjected to an input torque of 1000Nm. What is the rate of angular acceleration?



Solution synthesis:

$T = I\alpha$ therefore $\alpha = T/I$

But $I = (1/2)\, M\, r^2 =$

.5*100kgm*(10/39.37 m)^2

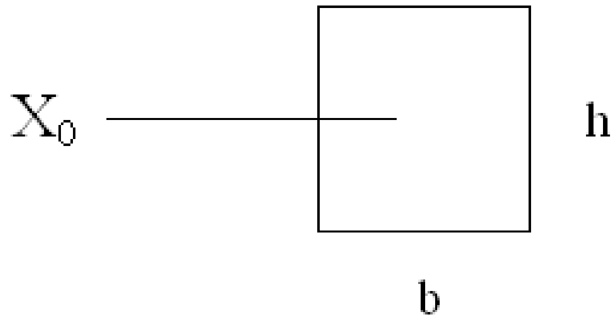
=3.225 kg-m^2

α = 1000 kg-m^2/s^2/3.225 kg-m^2

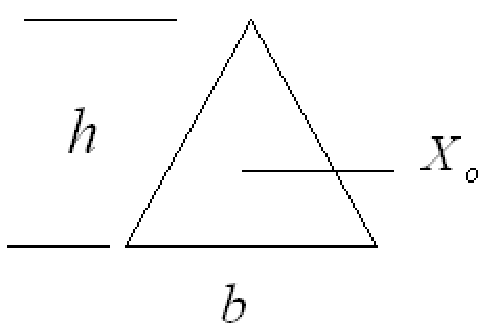
α= 310 rad/s^2

(Note: Must remember $1N = 1Kg{*}m/s^2$ also remember that radian is dimensionless…length/length)

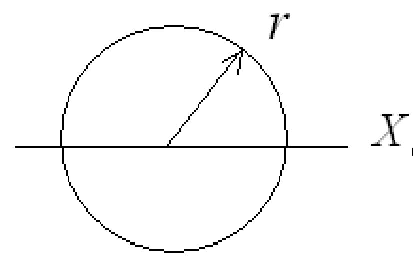
Area Moment of inertia for some shapes



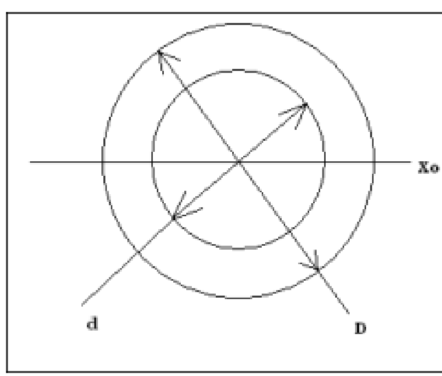
$$I_{x0} = bh^3/12$$



$$I_{xo} = \frac{bh^3}{36}$$



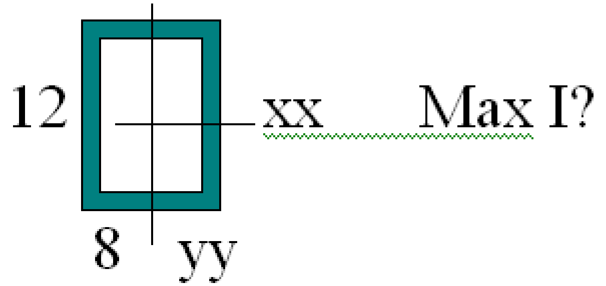
$$I_{xo} = \frac{\pi r^4}{4}$$



$$I_{xo} = \frac{\pi\left(D^4 - d^4\right)}{64}$$

Review problem #135

A hollow structural tube has outside dimensions of 8 inches x 12 inches. The tube thickness is .5 inches on all sides. What is the maximum (area) moment of inertia (in^4)?



Solution synthesis: We know I= $bh^3/12$ for a rectangular section.
Solution execution: We must decide on the max I. Will Ixx or Iyy be larger. For Ixx, b=8 and h=12. But for Iyy, b=12 and h=8 so it is obvious that Ixx will be larger. Since the tube is hollow we must subtract out the contribution of the material that does not exist….the rectangular air space on the inside.
Hence the solution is:
$I=boho^3/12 – bihi^3/12$
Where bo=8, ho=12, bi=7, hi=11
This gives I=(8)(12^3)/12 – (7)(11^3)/12
I=1152-776.4=375.6….the answer (d)

Factors of safety

$$N = \left(\frac{\text{allowable stress (or load) of material}}{\text{Working or design or actual stress}} \right)$$

More generally

$$N = \left(\frac{\text{load which will cause failure}}{\text{Load which exits}} \right)$$

Often safety factor is a policy question. Here are some rules Of thumb.

Recommended N	materials	loads	environ. Cond.
1.25 – 1.5	very reliable	certain	controlled
1.5-2	well known	det. Easily	fairly const.
2-2.5	avg.	Can be det.	Ordinary
2.5-3	less tried	“”	“”
3-4	untried matl’s	“”	“”
3-4	well known	uncertain	uncertain

Design relationships for elastic design

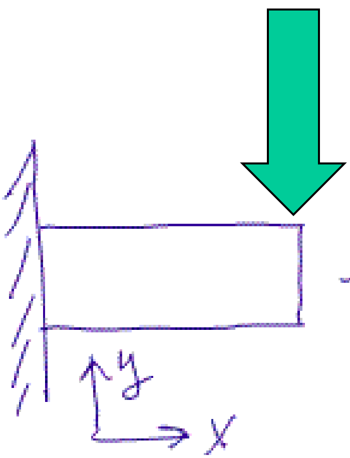
Axial loading

τ_{max} = Sy/N = F/A

Where F= axial force

A = cross sectional area

Transverse shear



τ_{max} = Ssy/N = VQ/(I*b)

Where V = vertical shear

$Q = \int_0^c y \, dA$

τ_{max} = max at the neutral axis

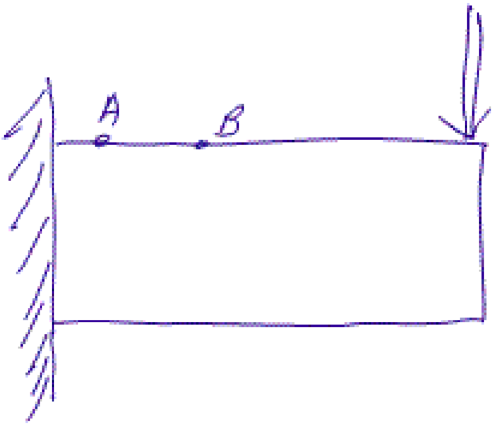
$\tau_{max} = \frac{3V}{2A}$ 

$= \frac{4V}{3A}$ 

$= \frac{2V}{A}$ 

$= \frac{V}{A_{WEB}}$ FOR I, W, S, [SECTIONS

Design relationships for elastic design



Bending

$$\sigma_{max} = S_y/N = \frac{Mc}{I} = M/S$$

Where

σ_{max} = max allowable design stress

Sy = yield stress of material, tensile

N = safety factor

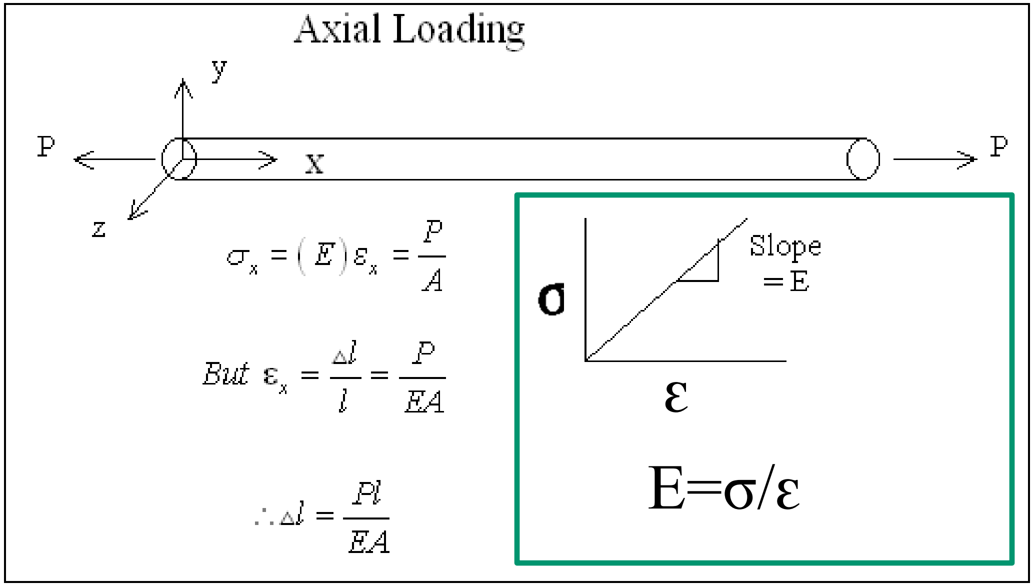
M = bending moment

C = distance from neutral surface to outer fiber

I = area moment of inertia about neutral axis

S= I/C referred to as the section modulus

Hooke's law/stresses/strains



Problem: a round metal rod 1" dia is 10 ft long. A tensile load of 10000 lbf is applied and it is determined that the rod elongated about 0.140 inches. What type of material is the bar likely made of ? How much did the diameter of the rod change when the load was applied ?

Plan:

We will apply Hooke's law to determine what the modulus of elasticity E is. Then we should also be able to apply the same law to determine the change in diameter of the rod.

We recall Hooke's law as follows

$$\varepsilon_x = \sigma_x/E - \nu\,\sigma_y/E - \nu\,\sigma_z/E$$

$$\varepsilon_y = -\nu\,\sigma_x/E + \sigma_y/E - \nu\,\sigma_z/E$$

$$\varepsilon_z = -\nu\,\sigma_x/E - \nu\,\sigma_y/E + \sigma_z/E$$

Where ν is Poisson's Ratio

ν is about 0.3 for most metals

For tensile load $\sigma_y = \sigma_z = 0$

σ_x for the tensile load = $P/A = 10000/(\pi * 1^2/4) = 12732$psi

Since $\sigma = E\varepsilon$ we know that $E = \sigma/\varepsilon$ both of which we now know. $E = 12732/(.140/120) = 10.9 \times 10^6$ psi ⇨ Alum.

Now the change in dia….Δ Dia = (ε_y)(Dia)

$\varepsilon_y = -\nu\, \sigma_X/E = -\nu\varepsilon_X = -.3\ (.14/120)$

$\varepsilon_y = -.00035$ in/in

$\therefore \Delta$ Dia = (ε_y) (Dia) = (.00035) (1)

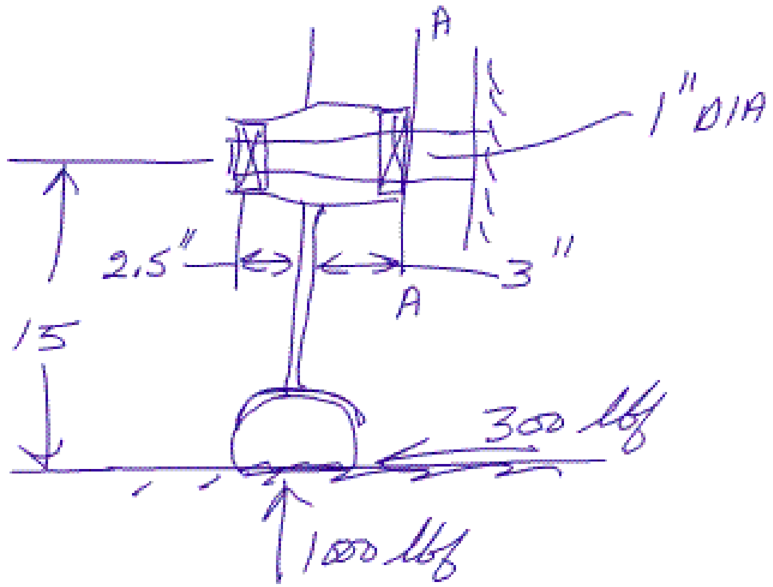
Δ Dia = -.00035 inches

Loads and stresses example

Under certain conditions a wheel and axle is subjected to the loading shown in the sketch below.

a) What are the loads acting on the axle at section A-A?

b) What maximum direct stresses are developed at that section?



Plan:

- Sum forces and moments
- Compute bending moment
- Compute bending stress
- Compute tensile or compressive stress

Execution:

Summing Fx we determine the axial tensile load at A-A=300lbf
Summing Fy direct shear load = 1000 lbf
Summing moments about the A-A section at the neutral axis
We find the bending moment= 1000*3 + 300*15= 7500 lb-in

$$\sigma\ Bending = \frac{mc}{I}\quad Where\ I = \frac{\pi d^4}{64} = \frac{\pi r^4}{4}$$

$$= \frac{mr}{\left(\pi \frac{r^4}{4}\right)}\quad Where\ \ r = c$$

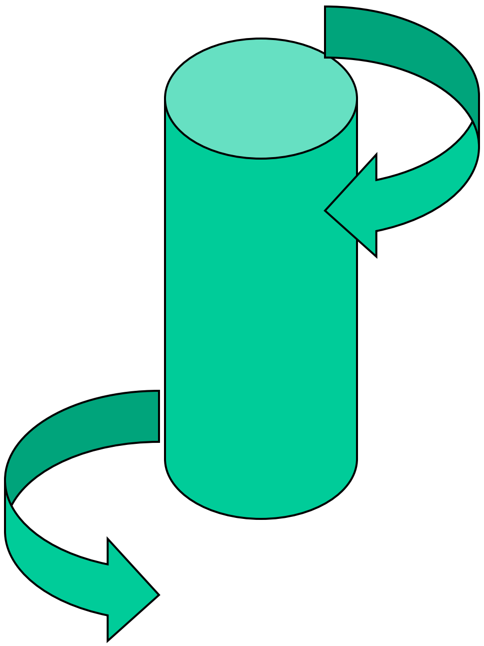
$$= \frac{4m}{\pi r^3} = \frac{4(7500)}{\pi(.5)^3} = 76394\ PSI$$

$$\sigma\ tensile = \frac{P}{A} = \frac{300}{A} = \frac{300}{\left(\frac{\pi d^2}{4}\right)} = 382\ PSI$$

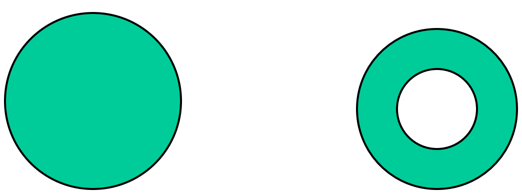
$$\therefore Max\ Stress\ at\ A - A = 76394 + 382 = 76776\ PSI$$

Design relationships for elastic design

Torsion



τ_{max} = Ssy/N = T*r/J
Where T= torque applied
r = radius
J= polar moment of inertia (area)



J= (pi)(r^4)/2 = (pi)(d^4)/32 J= (pi)(D^4 – d^4)/ 32

Combined stress

In a two dimensional stress field (where σ_3 & $\tau_3 = 0$) the principal stresses on the principal planes are given by:

$$\sigma_1 \text{ and } \sigma_2 = (\sigma_x + \sigma_y)/2 \pm [((\sigma_x - \sigma_y)/2)^2 + \tau_{xy}^2]^{1/2}$$

$$\tau_{max} = \pm [((\sigma_x - \sigma_y)/2)^2 + \tau_{xy}^2]^{1/2}$$

or

$$\tau_{max} = \text{largest} \begin{bmatrix} |\sigma_1 - \sigma_2| /2 \\ \\ |\sigma_1| /2 \\ |\sigma_2| /2 \end{bmatrix}$$

Combined stress continued

In combined stresses problems involving *shaft design* we are generally dealing with only bending and torsion i.e., where $\tau_y = 0$

In this case

$$\sigma_1 \text{ and } \sigma_2 = \sigma_x/2 \pm [(\sigma_x/2)^2 + \tau_{xy}^2]^{1/2}$$

and

$$\tau_{max} = [(\sigma_x/2)^2 + \tau_{xy}^2]^{1/2}$$

Where $\sigma_x = Mc/I$ or $32 M/(\pi D^3)$ for solid circular shaft

And $\tau_{xy} = Tr/J = 16 T/(\pi D^3)$ for solid circular shaft

Theories of failure

1) Maximum normal stress

Based on failure in tension or compression applied to materials strong in shear, weak in tension or compression.

Static loading

a) Design based on yielding, keep:

$$\sigma_{MAX} \leq S_y/N \qquad \sigma_{MAX\ COMPRESSION} \leq \frac{S_y\ (COMPR)}{N}$$

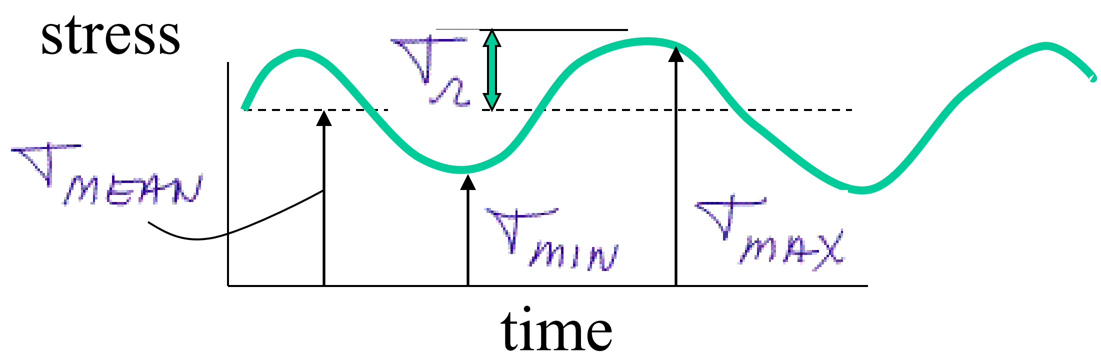
(for materials with different compressive and tensile strengths)

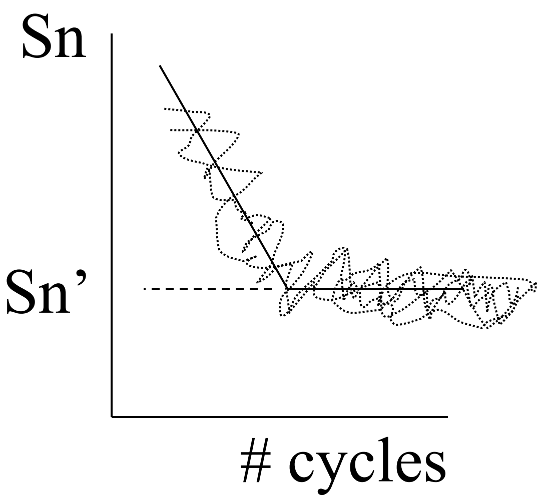
b) For *brittle materials* (no yield point) …design for:

$$\sigma_{MAX} \leq \frac{S_u}{N} \qquad \sigma_{MAX\ COMPR} \leq \frac{S_u\ (COMPR)}{N}$$

Theories of failure cont'd

Fatigue loading (fluctuating loads)



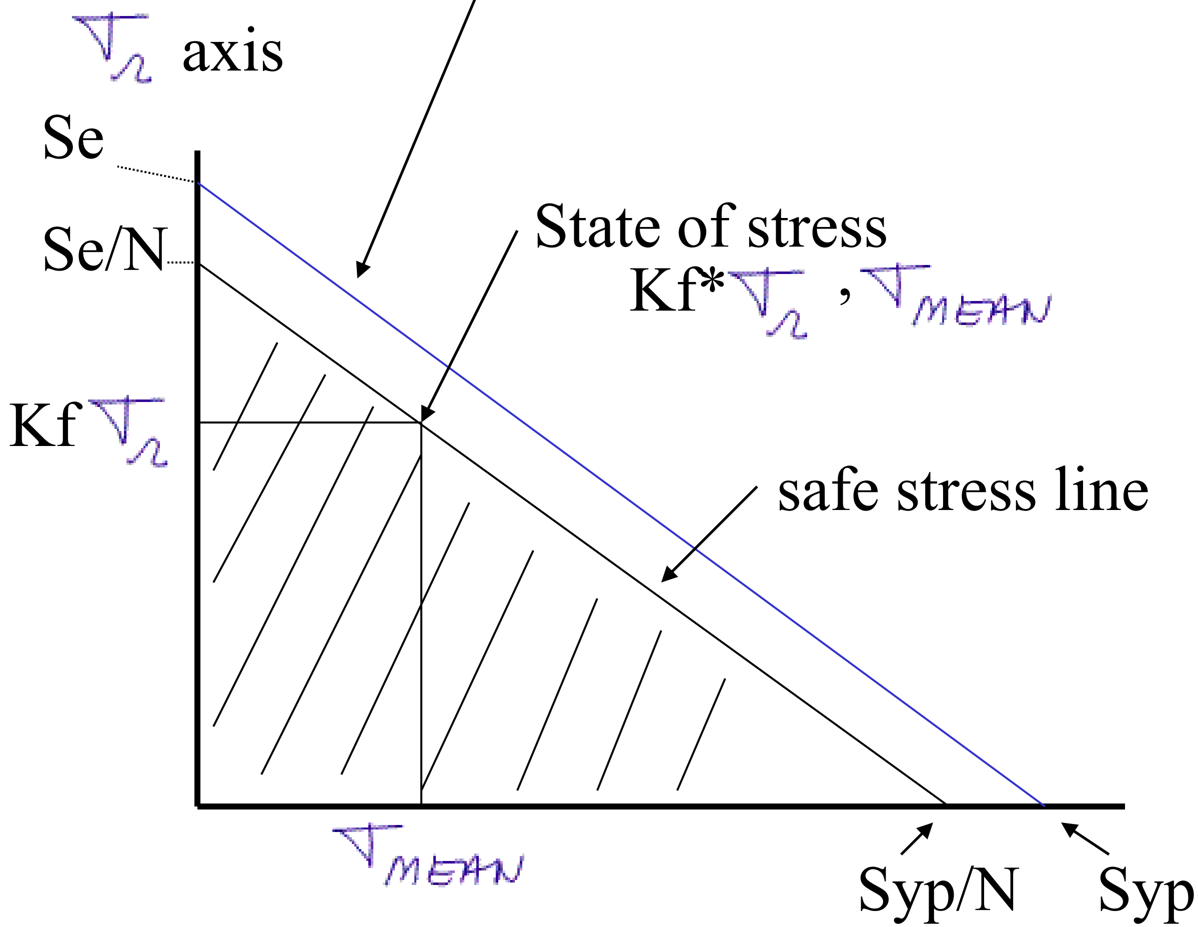


Se=Cf*Cr*Cs*Cw*Sn'

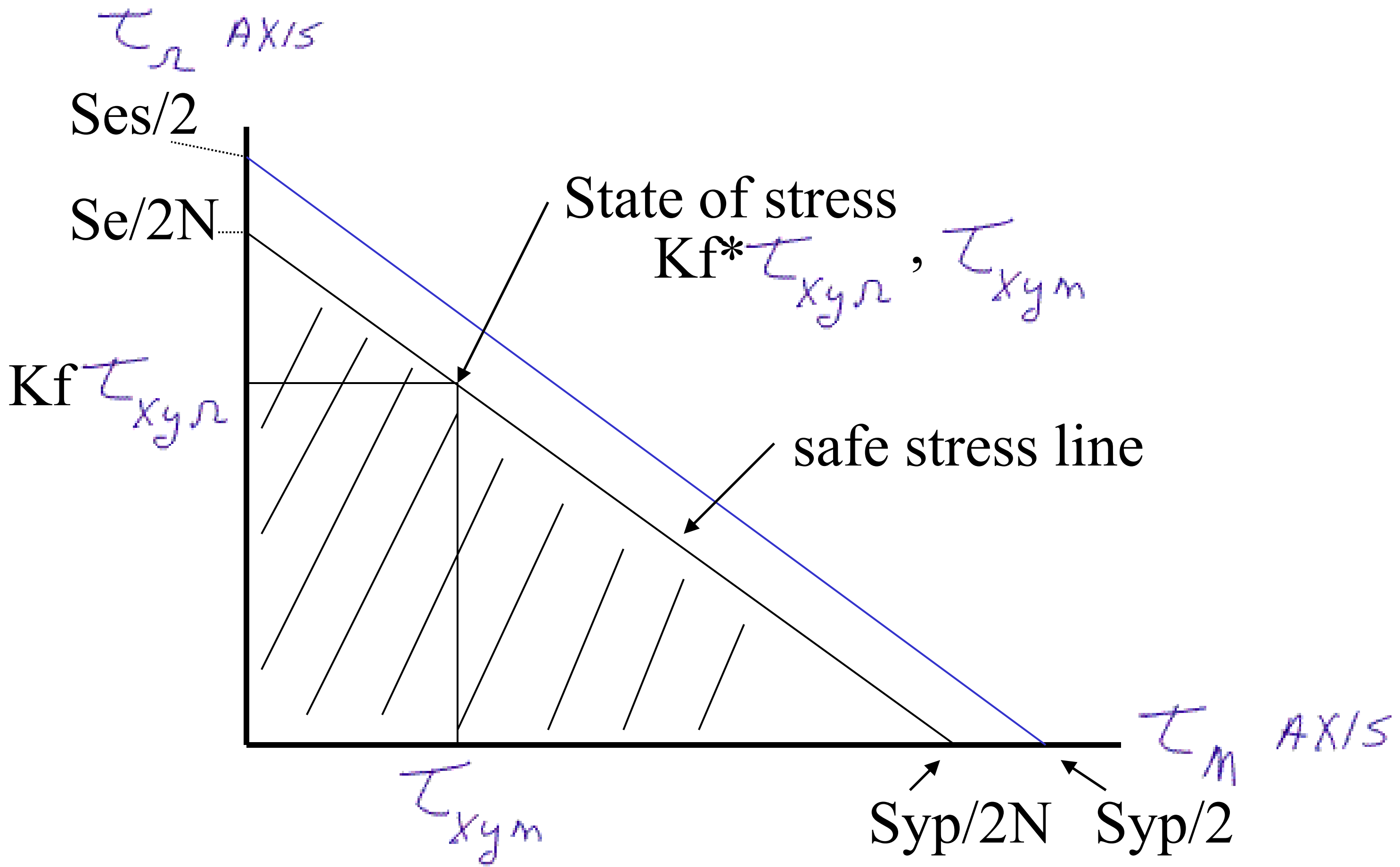
Where Sn' = endurance limit
Se = allowable working stress or modified endurance limit

Note: stress concentration factor Kf is not in this formula for Se. Kf is included later to be *part specific*

Soderberg failure line for fatigue



Maximum shear theory of failure



For design with ***ductile materials*** and it is conservative and on the premise: failure occurs when the maximum (spatial) shear stress exceeds the shear strength. Failure is by yielding.

Formulae for sizing a shaft carrying bending and torsion

For a hollow shaft…."Do"=outside dia, "Di" = inside dia

$$D_o^{\,3}\left(1-\left(\frac{D_i}{D_o}\right)^4\right)=\frac{32N}{\pi S_{yp}}\sqrt{\left(K_{sb}\frac{S_{yp}}{S_e}M\right)^2+(K_{st}T)^2}$$

For a solid shaft Di=0 and the equation becomes:

$$D_o^{\,3}=\frac{32N}{\pi S_{yp}}\sqrt{\left(K_{sb}\frac{S_{yp}}{S_e}M\right)^2+(K_{st}T)^2}$$

Where "Do" will be the smallest allowable diameter based on max shear theory. M is the bending moment and T is the torsion T is the mean torque assumed to be steady here…and M is the Bending moment which becomes the fluctuating load as the shaft Rotates.

Other shaft sizing considerations

Other criterion of shaft design may be requirements on torsional Rigidity (twist) and lateral rigidity (deflection)

Torsional rigidity

Theta = 584* T*L/(G*(Do^4-Di^4)) for hollow circ. shaft

Theta = 584* T*L/(G*(Do^4)) for solid circ. shaft

Where:

theta= angle of twist, degrees

L = length (carrying torque), in inches

T = torsional moment, lb-in

G = torsional (shear) modulus of elasticity

(11.5x10^6 psi, steels) (3.8x10^6 psi, Al alloys

D = shaft diameter, inches

Review problem #110

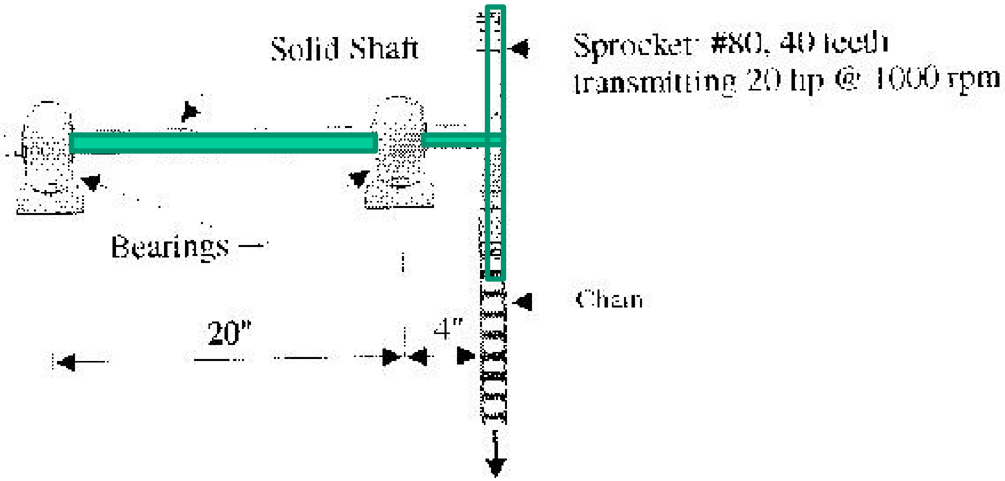
11. The unit below drives a grinding operation. Bearings are ball units in self-aligning pillows. Ignoring fatigue and using a factor of safety of 2.0 for the given chain drive, the appropriate shaft diameter in inches for cold drawn AISI 1018 steel (untreated) is most nearly:

(a) 1.00

(b) 1.25

(c) 1.50

(d) 2.00



Review problem #110

Solution synthesis: We are not asked to design for fatigue life so we are sizing the shaft as if a static load were applied. There are two types of loads in the shaft 1) a bending moment due to the overhang at the right bearing and 2) the torsion load determined by the power transmitted.

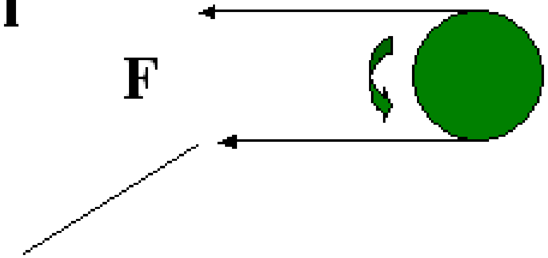
We must compute the torque to get the chain force applied to the sprocket (shaft). We know Hp=TN/63025 where N is RPM and T is in lb-in. The gives T=Hp(63025)/N=20(63025)/1000= 1260.5 lbf-in.

We now must know the pitch radius of the sprocket…all we are told is that the we have a 40 tooth sprocket for a #80 roller chain. How do we compute the pitch diameter?

#80 chain has a pitch of 8(1/8) inches=1 in. So the circumference of the sprocket must be (40teeth)(1in pitch)= 40 inches pitch circum.

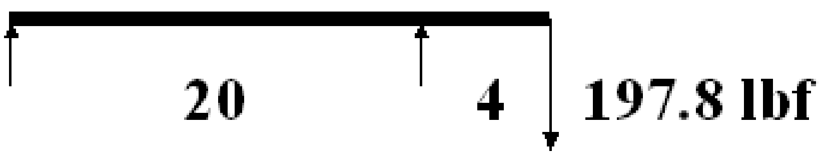
Thus the pitch dia Pd= Circum/π
Pd=40/π = 12.74”
Hence the pitch radius R=Pd/2=6.37”

The force in the chain results in the torque, T=F*R ⇒ F=1260/6.37=197.8 lbf



Slack side force = zero

From the FBD of the shaft



Bending moment= (4)(197.8)= 791.2

Torsion moment= 1260 lbf-in

We are now ready to compute the required size of shaft. For this we find a shaft sizing equation like the following:

$$D_o^3 = \frac{32N}{\pi S_{yp}} \sqrt{\left(K_{sb} \frac{S_{yp}}{S_e} M\right)^2 + (K_{st} T)^2}$$

Since we are ignoring fatigue let Se=Syp

The shock factors Ksb and Kst must be chosen for the application. The author of this problem used 3.0 for both. This would indicate fairly severe shock loads anticipated.

We must also be able to determine the yield stress for 1018 cd steel. For this we must consult a reference book. The following table is found in Mark's Handbook for Mech. Engrs. (9th ed)

Table 6.2.15 Average Mechanical Properites of Cold-Drawn Steel

AISI No.	Tensile strength MPa	Tensile strength 1,000 lb/in²	Yield strength MPa	Yield strength 1,000 lb/in²	Elong in 50 (2 in
1010	462	67	379	55.0	25
1015	490	71	416	60.3	22
1020	517	75	439	63.7	20
1025	552	80	469	68.0	18
1030	600	87	509	73.9	17
1035	634	92	539	78.2	17
1040	669	97	568	82.4	16
1045	703	102	598	86.7	15
1117	552	80	469	68.0	19
1118	569	82.5	483	70.1	18
1137	724	105	615	89.2	16
1141	772	112	656	95.2	14

Sizes 16 to 50 mm (⅝ to 2 in) diam, test specimens 50 × 13 mm (2 × 0.505 in).
SOURCE: ASM "Metals Handbook."

We note that 1018 is not listed but if we interpolate between 1020 & 1015 we could get an approximate Syp=62000psi.

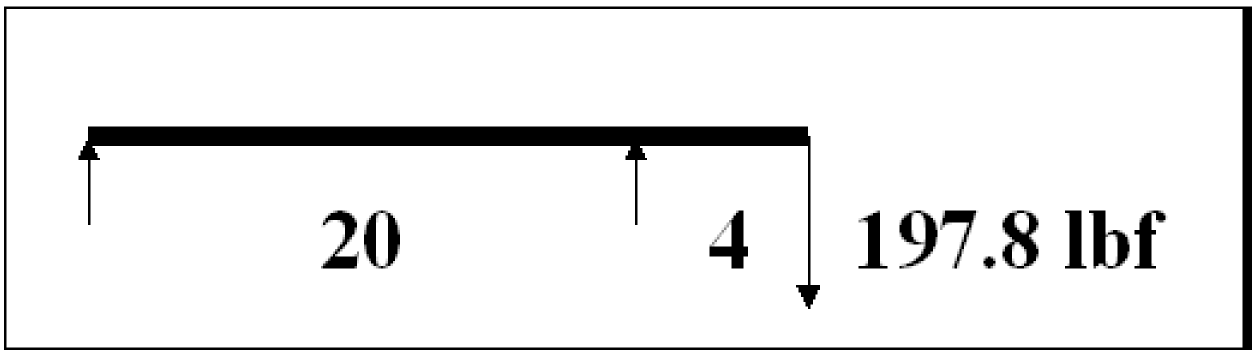
Thus applying the sizing equation we get:

$$D_o^3 = \frac{(32)(2)}{(\pi)(62000)}\sqrt{(3*791)^2+(3*1260)^2}$$

Thus $D_o^3=1.467$

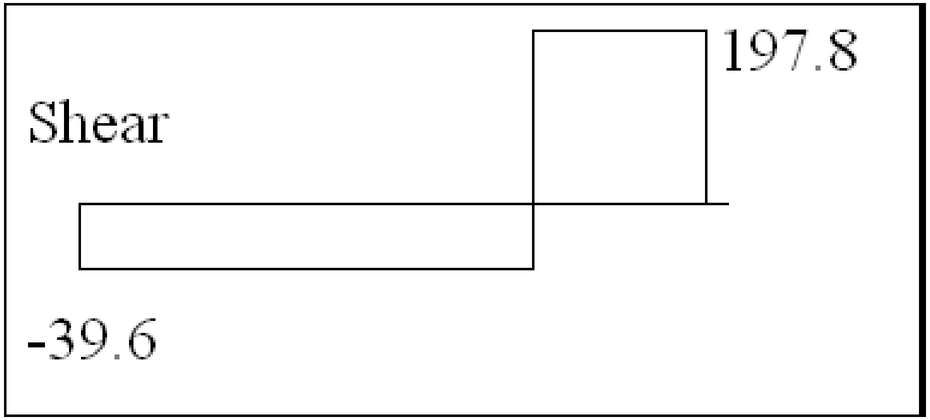
$D_o=1.14 \Rightarrow$ closest ans.=1.25”

Practice question: If you were asked to sketch the shear and moment diagrams for the shaft in Problem #110 above could you do it? Here is the FBD again:

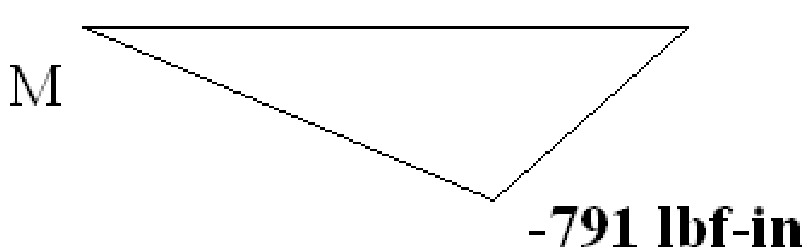


Take a couple of minutes to see if you can do this…

Solution: Depending on the wording of such a problem we may need to determine the bearing reaction loads if we are asked to label the magnitudes of shear and moment at the load points. So in this case we solve for the bearing reaction loads Rl and Rr (left and right) By noting that $\sum Fy=0$ thus $Rl+Rr-197.8=0$ and to solve for these two unknowns we need another equation…we know also that $\sum Moments=0$ so summing about the left bearing….$20Rr-(197.8)(24)=0$ gives $Rr=237.36$ lbf hence $Rl=-237.36+197.8=-39.6$ lbf

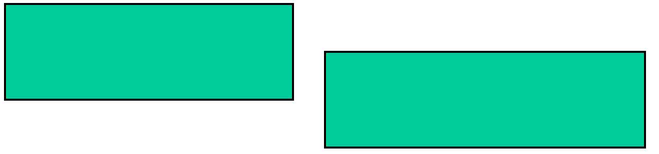


Moment is the integral of the shear

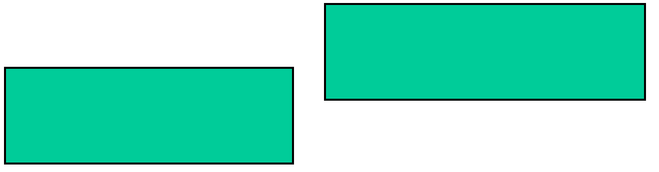


Shear and Moment sign conv.

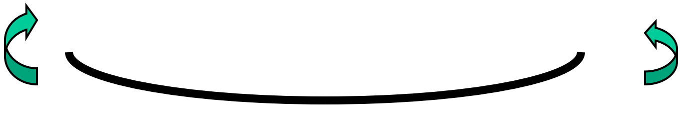
- Positive shear



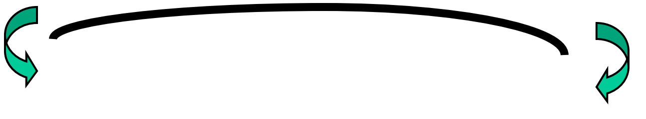
- Negative shear



- Positive moment

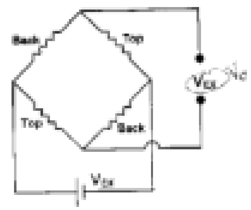


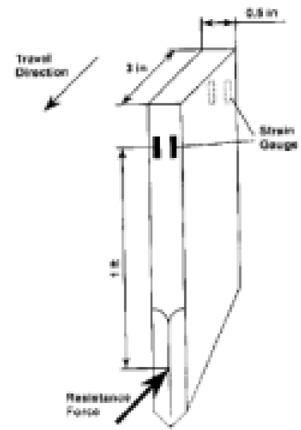
- Negative moment



Review Problem 513

(a) 7.2
(b) 43.3
(c) 86.5
(d) 519.0





Given: Full Bridge hookup

GF=2.08

Vex = 2.5V

Implement is beam 3” deep x .5” wide

Beam is steel

Load applied 1 ft = 12 in from gages

For load, the output Vo=120μV

What is the load (resistance force) in pounds?

Synthesis:----what do we know? (note error in diagram of bridge, output is Vo not Vex)

For full bridge circuit $V_o/V_{ex} = GF * \varepsilon$

$E = 30 \times 10^6$ PSI for steel

$\sigma = E*\varepsilon$

For beam we are measuring strain due to bending moment

$\sigma = M c/I$ for bending where

M = bending moment

c = distance neutral axis to surface

I = area moment of inertia of beam cross section

Solution Plan: compute $\sigma = f(F) \Rightarrow \varepsilon = f(F) \Rightarrow Vo = f(F)$ then solve for F.

$M = 12F$, $c = 1.5$, $I = bh^3/12 = (.5)(3)^3/12 = 13.5/12$

$\therefore \sigma = (12F)(1.5)/(13.5/12) = (12)(12)(1.5)(F)/13.5$
$= (30x10^6)(\varepsilon)$

$\therefore \varepsilon = (0.533x10^{-6})(F)$ but $\varepsilon = Vo/(Vex*GF) = 23.1\ \mu\varepsilon$

Thus $(0.533x10^{-6})(F) = 23.1 \text{ x } 10^{-6}$

$F = 43.27$ lbf

Problem: If the above implement problem had been given this same Vo for a half-bridge circuit what would have been the force acting on the implement?

Solution: For a half bridge Vo/Vex = -GF*ε/2

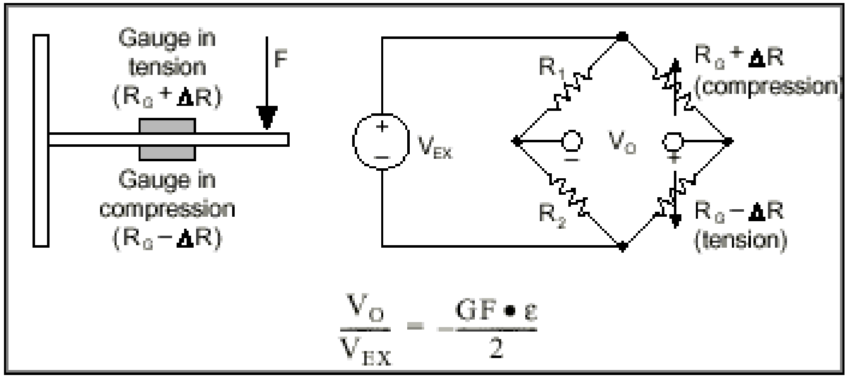


Figure 6. Half-Bridge Circuit

Here is an excellent discussion of strain gage basics:
http://zone.ni.com/devzone/cda/tut/p/id/3642

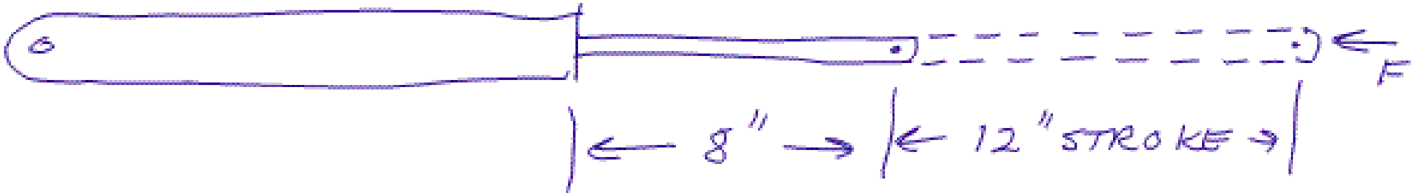
Thus for the same Vo ⇒ ε must be twice a large
So if ε is twice as large the load is must be twice as large.

Column buckling

A hydraulic actuator is needed to provide these forces: minimum force in contraction…4000 lb. Maximum force in extension (push) …8000lb. The rod is made of steel with a tension or compression yield strength of 40,000psi. Assume a hydraulic system pressure of 2000psi.

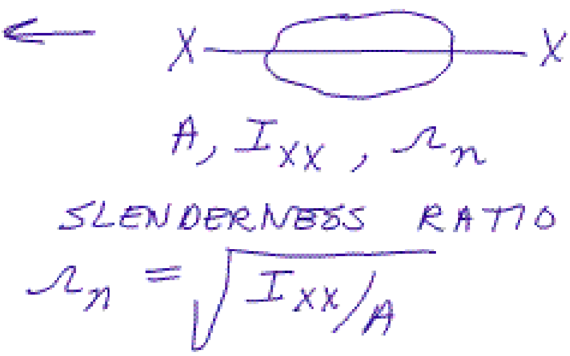
a. What nominal (nearest 1/16”) diameter rod is required for a safety factor of 5 and what nominal bore?
b. What size piston is needed?

We sketch the cylinder as shown here:



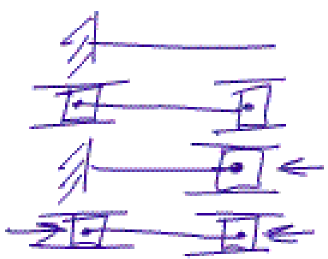
With 8000 lbs of push capability we must be concerned about possible buckling of the rod in its most vulnerable position which would be at full extension to 20" length. We will not worry about the cylinder itself buckling and concern ourselves with the rod.

- What do you recall about solving a buckling problem?
- Lets review a few basics

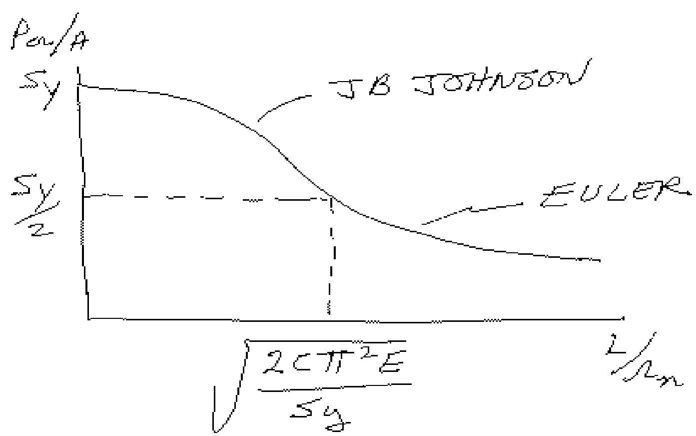


- Is the rod considered to be long or short column?
- What are the end conditions?
- We must design for Pallowable=8000lbs=Pcr/N
- But N the safety factor =5 so Pcr=40000lbs

Recall from buckling theory:



END CONDITION	C THEORETICAL	C RECOMMENDED
FREE – FIXED	1/4	1/4
PINNED – PINNED	1	1
PINNED – FIXED	2	1.2
FIXED – FIXED	4	1.2



SUMMARY

EULER

$$\frac{P_{cr}}{A} = \begin{cases} \dfrac{c\pi^2 EI}{AL^2} = \dfrac{c\pi^2 E}{(L/r_n)^2} & \text{for } L/r_n \geq \sqrt{\dfrac{2c\pi^2 E}{S_y}} \\ \text{JB JOHNSON} & \\ S_y - \dfrac{1}{cE}\left(\dfrac{S_y}{2\pi}\right)^2\left(\dfrac{L}{r_n}\right)^2 & \text{for } L/r_n < \sqrt{\dfrac{2c\pi^2 E}{S_y}} \end{cases}$$

Plan:

- In a typical problem we would determine if the column is long or short then apply the Euler or Johnson equ. accordingly but in our case here we are designing the size of the column and the size information is not given so what do we do?
- Piston diameter must be determined based on forces required and the system pressure and the rod size.

Execution:

Since we are trying to compute rod diameter we could size the rod to be a short or a long column keeping in mind that the Euler formula applies to long columns where the stress is less than Sy/2 and where the slenderness ratio L/rn is greater than the critical value given by the table above. Lets use Euler and design it as a long column.

$$\frac{Pcr}{A} = \frac{C\pi\, EI}{AL^2}$$ Assume C = ¼ For "Fixed – Free"

$$Pcr = \frac{c\pi^2\, EI}{L^2} = \frac{\left(\frac{1}{4}\right)(\pi^2)(30 \times 10^6)I}{20^2} = 40000$$

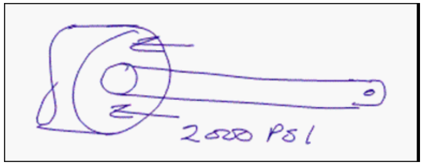
$$\therefore I = (40{,}000)(20)^2(4)/\left(\pi^2 * 30 \times 10^6\right) = 0.216$$

BUT $I = \frac{\pi d^4}{64} = .216$

$$d^4 = \frac{(.216)(64)}{\pi} = 4.40$$

$$d^2 = 2.09$$

d=1.448 $\Rightarrow 1.5\ inches$

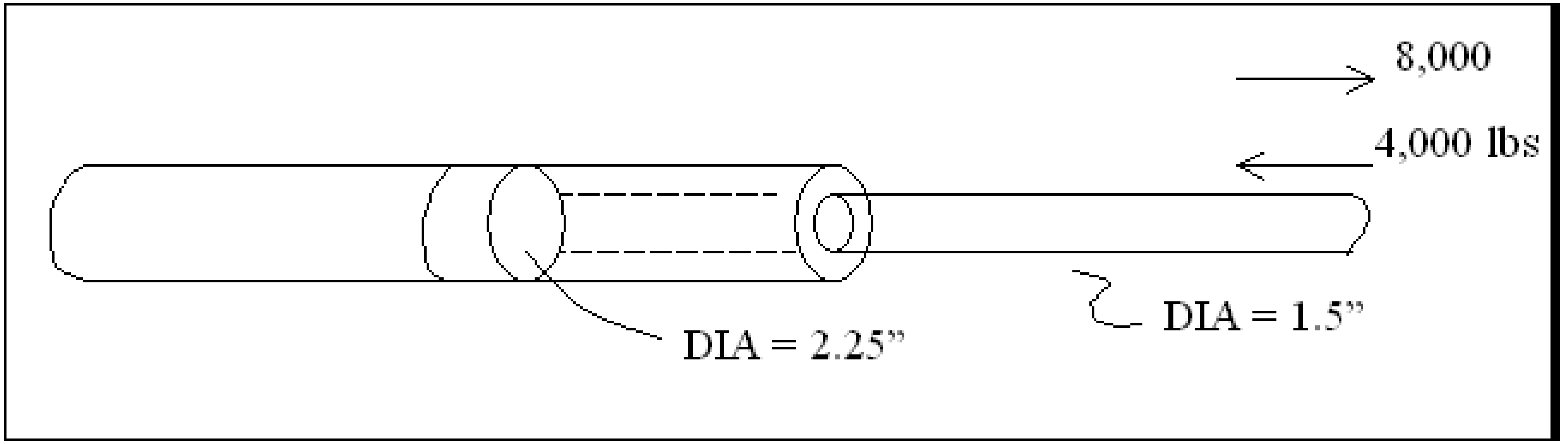


For a force in tension =4000lbs

(Piston area)(2000psi)=4000
Piston area =2.0 in^2 effective area
But we must remember that in contraction the rod is occupying
Part of the cylinder area.
Area of the rod = (Pi)(d^2)/4=3.14*(1.5^2)/4=1.767 in^2

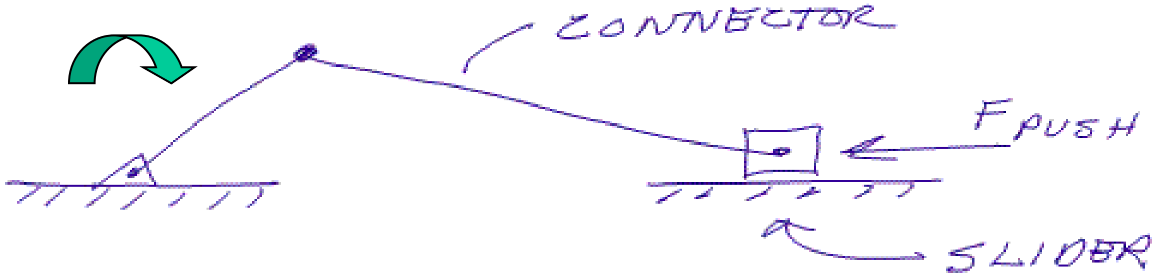
Thus the total bore area must be 2.0 + 1.767=3.767 in^2
Hence (pi)*(D^2)/4=3.767
D^2=4.796
D=2.19in → 2.25 in dia piston

Can a piston 2.25 in dia generate 8000lbs push with a 2000psi
Hydraulic pressure?
Force push=P*Area= 2000*(pi)*(2.25^2)/4= 7952lbs so OK.


8,000
4,000 lbs
DIA = 2.25"
DIA = 1.5"

Lets work a follow-on example

Assume you want to check the connector in a slider crank mechanism which is to generate a force at the slider



Lets assume you have chosen the following:
Connector length 12”
Cross-section ¼ x 1 inch, area = ¼ inch sq.
Mat’l Al, E= 10.6x 10^6 psi
Max load in connector will be 500 lbf
Lets assume we need a safety factor N=2

Problem definition: we need Pallowable>= 500 lbf
For safety N=2, will the chosen design have adequate buckling strength?

Plan:

Compute the slenderness ratio and decide if the connector column Is “long” or “short” then apply either Euler or JB Johnson to compute Pcr.

If Pcr/N=Pcr/2=Pallowable>=500 lbf then the proposed size is OK

Execution:

Buckling will occur about yy if we assume a pinned-pinned joint about both axes at each end.

$$\therefore Iyy = \frac{(1)\left(\frac{1}{4}\right)^3}{12} = \frac{1}{(64)(12)} = \frac{1}{768}\ in^4$$

$$A = \frac{1}{4}\ in^2$$

$$r_{ny} = \sqrt{\frac{Iyy}{A}} = \sqrt{4/768} = .0722$$

$$\text{Slenderness Ratio}\ \frac{L}{r_{ny}} = \frac{12}{.0722} = 166.2$$

Now evaluate the critical slenderness ratio:
where C=1 for pinned-pinned and Sy=24000psi
for say Al 2011 T6 alloy

$$\sqrt{\frac{2C\pi^2E}{Sy}} = \sqrt{\frac{(2)(1)(\pi^2)(10.6\times10^6)}{24,000}} = 93.3$$

$$\therefore \quad \frac{L}{r_{ny}} = 166 > 93.3 \Rightarrow \quad Long\ Column$$

$$Use\ Euler$$

$$Pcr = \frac{C\pi^2EA}{\left(\frac{L}{r_{ny}}\right)^2} = \frac{(1)(\pi^2)(10.6\times10^6)\left(\frac{1}{4}\right)}{(166)^2} = 948lbf$$

$$\therefore \quad P_{allowable} = \frac{P_{cr}}{N} = \frac{948}{2} = 474 \, lbs$$

Which means our safety factor is less than 2, actually it is

$$\frac{948}{N} = 500$$

$$N = \frac{948}{500} = 1.89$$

We can decide as a designer to go with this or we can increase the thickness to get N=2

If we go back and write the slenderness ratio in terms of the Thickness we should be able to compute the thickness req'd For the 500 lbf (N=2) allowable load requirement.

$$Iyy = \frac{(1)(b)^3}{12} = \frac{b^3}{12}$$

$$A = (1)(b) = b$$

$$r_{xy} = \sqrt{\frac{b^3}{12(b)}} = \sqrt{\frac{b^2}{12}} = \frac{b}{3.464}$$

Now the connector will still be "long" so plugging Euler: We need Pcr>=1000 lbf (ie, so that Pcr/2 >= 500 lbf)

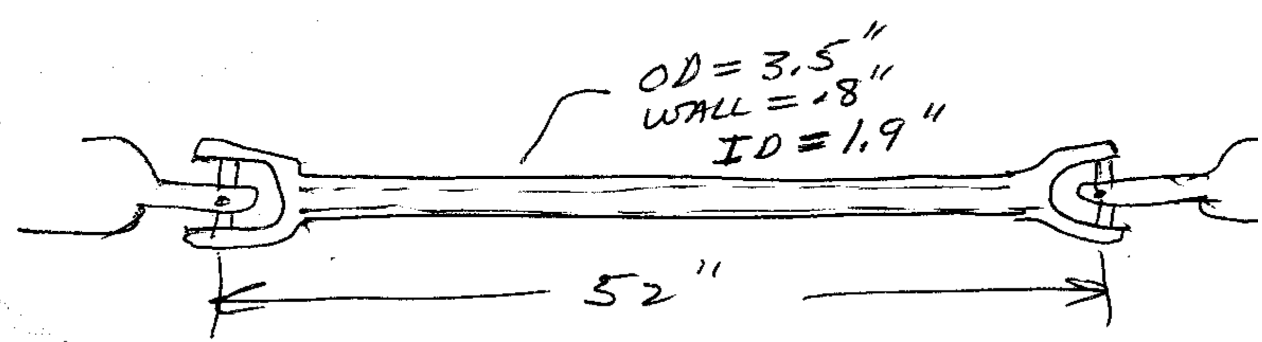
$$\therefore 1{,}000 = \frac{(1)(\pi^2)(10.6\times 10^6)(b)}{\left(\frac{12}{\sqrt{\frac{b^2}{12}}}\right)^2} = \frac{(\pi^2)(10.6\times 10^6)b^3}{(144)(12)}$$

$$\therefore b^3 = .01653$$

$$\boxed{b = .255''}$$

Example Shaft Problem

Problem statement: The drive shaft in the sketch below is made of mild steel tube (3.5" OD x 0.80 wall) welded to universal joint, yokes and a splined shaft as shown. It is driven by an engine developing 250 hp at 2000 rpm what is the stress in the shaft tube? If the shaft is considered to have uniform properties, end to end, what is the critical speed of the shaft?



Plan: this is a torsion problem with a hollow shaft. The stress in the shaft will be due to shearing stress. We will need to apply the formula for shear stress for a hollow shaft. For the critical speed question we are then dealing with a vibration issue…at what frequency (rpm) will the shaft be inclined to go into a resonant condition…what do we know about this? Spring rate?, static deflection? The Rayleigh-Ritz formula? Etc,…since the shaft has only distributed mass we could break it into segments and apply the Rayleigh-Ritz but that would be a lot of work for the time constraint…so that is not likely what is expected…the simplest thing we can so do is compute the max static deflection and use that to compute the approximate frequency.

Note: Rayleigh-Ritz says:

The first critical freq (rpm) = 187.7 $\left[\frac{\sum_{i=1}^{J} w_i\, y_i}{\sum_{i=1}^{J} w_i\, y_i^2}\right]^{\frac{1}{2}}$

Solution execution:
Stress in the shaft due to torsional shearing stress

$\tau = \frac{TC}{J}$ Where J is the Polar Moment for A Hollow Shaft

T = Torque

C = $\frac{D_0}{2}$ Radius to Outermost Fiber

For Hollow Shaft $J = \frac{\pi}{32}\left(D_o{}^4 - D_i{}^4\right)$

$$= \frac{\pi}{32} D_0{}^4 \left(1 - \left(\frac{D_i}{D_o}\right)^4\right)$$

$$C = \frac{D_0}{2}$$

$$\therefore \quad \tau = \frac{T(D_0/2)}{\frac{\pi}{32} D_0{}^4 \left(1 - \left(\frac{D_i}{D_o}\right)^4\right)} = \frac{16\,T}{\pi\, D_o{}^3 \left(1 - \left(\frac{D_i}{D_o}\right)^4\right)}$$

But What is the Torque?

Recall hp = $\frac{TN}{63025}$

$$\therefore \quad T = \frac{(250)(63025)}{2000} = 7878 \text{ lbf-in}$$

$$\tau = \frac{(16)(7878)}{\pi\,(3.5)^3 \left(1 - \left(\frac{1.9}{3.5}\right)^4\right)} = \frac{126050}{134(1 - .087)} = 1030\ PSI$$

2.) To find the critical speed we can compute max static deflection in the shaft and apply

$$W_n = \sqrt{g/\delta_{st}} \quad \text{rad/s}$$

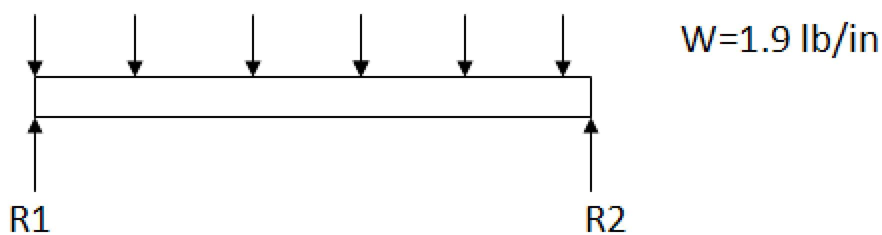
Where Wn is the undamped natural frequency, and δst is the static deflection of a spring-mass system

Assume shaft is uniform along its length

Weight / inch = (ρ) (AREA) = (.28 lb/in^3) $\left({}^{\pi}/_{4}\,(3.5^2 - 1.9^2)\right)$

$$= 1.9\,\frac{lb}{in}$$

Hence we have a beam problem



How do we find the max static deflection in a uniformly loaded shaft? Well it would be nice to have a set of beam tables giving formulae for beams loaded in different ways. If we had such a table we could look up a uniformly loaded beam, simply supported, we should find deflection y(x).

$$y = \frac{wx}{24EI}\left(2\,\ell\,x^2 - x^3 - \ell^5\right)$$

Where w is the load/unit length, or in this case weight/unit length

Thus deflection at mid-beam, x= $\ell/2$ gives,

$$y = \frac{w\ell^4}{24EI}\left(-\frac{5}{16}\right)$$ Where

$$I = \frac{\pi}{64}\left(D_o{}^4 - D_i{}^4\right) = 6.726$$

$$E = 30\,x\,10^6$$

$$w = 1.9\ lb/in$$

$$\ell = 52\ in$$

Which gives

$$\boxed{y = -.000896\ in\quad @\ x = \ell/2}$$

$$w_{c} = \sqrt{g/\delta_{ST}} = \sqrt{386/.000896}$$

$$= 656.4\ \frac{rad}{S} = (656.4)\left(\frac{60\ S/min}{2\pi\ rad/REV}\right)$$

w_c**=6270 RPM**

Some Engineering Basics

The following information helps you solve technical problems frequently encountered in designing and selecting motion control components and systems.

Torque

$$T = FR \tag{1}$$

Where:
T = Torque, lb-ft
F = Force, lb
R = Radius, or distance that the force is from the pivotal point, ft

Linear to rotary motion

$$N = \frac{V}{0.262D} \tag{2}$$

Where:
N = Speed of shaft rotation, rpm
V = Velocity of material, fpm
D = Diameter of pulley or sprocket, in.

Horsepower

Rotating objects:

$$P = \frac{TN}{5,250} \qquad (3)$$

Where:
P = Power, hp
T = Torque, lb-ft
N = Shaft speed, rpm

Objects in linear motion:

$$P = \frac{FV}{33,000} \qquad (4)$$

Where:
P = Power, hp
F = Force, lb
V = Velocity, fpm

Pumps:

$$P = \frac{QHS}{3,960\mu} \qquad (5)$$

Where:
P = Power, hp
Q = Flow rate, gpm
H = Head, ft
S = Specific gravity of fluid
μ = Pump efficiency

Fans and blowers:

$$P = \frac{Qp}{229\mu} \qquad (6)$$

Where:
P = Power, hp
Q = Flow rate, cfm
p = Pressure, psi
μ = Efficiency

Accelerating torque and force

Of rotating objects

$$T = \frac{\left(WK^2\right)\Delta N}{308t} \qquad (7)$$

Where:
T = Torque required, lb-ft
WK^2 = Total inertia of load to be accelerated, lb-ft^2. (See Formulas 9, 10, 11, and 12.)
ΔN = Change in speed, rpm
t = Time to accelerate load, sec

Objects in linear motion:

$$F = \frac{W\Delta V}{1{,}933t} \qquad (8)$$

Where:
F = Force required, lb
W = Weight, lb
ΔV = Change in velocity, fpm
t = Time to accelerate load, sec

Moment of inertia

Solid cylinder rotating about its own axis:

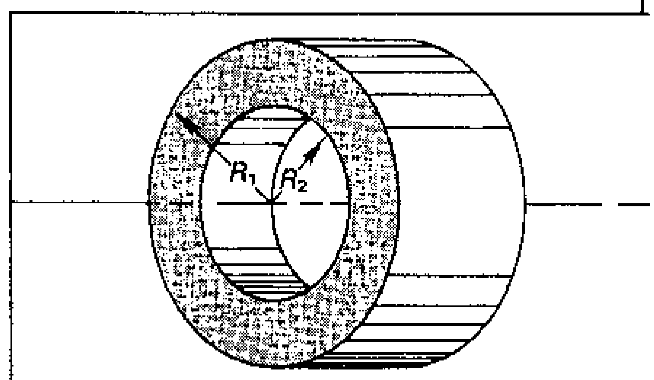
$$WK^2 = (1/2)WR^2 \qquad (9)$$

Where:
WK^2 = Moment of inertia, lb-ft^2
W = Weight of object, lb
R = Radius of cylinder, ft

Hollow cylinder rotating about its own axis:

$$WK^2 = \frac{W\left(R_1^2 + R_2^2\right)}{2} \qquad (10)$$

Where:
WK^2 = Moment of inertia, lb-ft^2
W = Weight of object, lb
R_1 = Outside radius, ft
R_2 = Inside radius, ft



Material in li
a continuous fi
rotational spee
veyor system:

$$WK_L^2 = W\left(\right.$$

Where:
WK_L^2 = Linear i
W = Weight of r
V = Linear velo
N = Rotational

Reflected in
through a speed
— gear, chain, o

$$WK_R^2 =$$

Where:
WK_R^2 = Reflect
WK_L^2 = Load in
R_r = Reduction

Duty cycle calculation

The RMS (root mean square) value of a load is one of the quantities often used to size PT components.

$$L_{RMS} = \sqrt{\frac{L_1^2 t_1 + L_2^2 t_2 + ... + L_n^2 t_n}{t_1 + t_2 + ... + t_n}} \qquad (13)$$

Where:

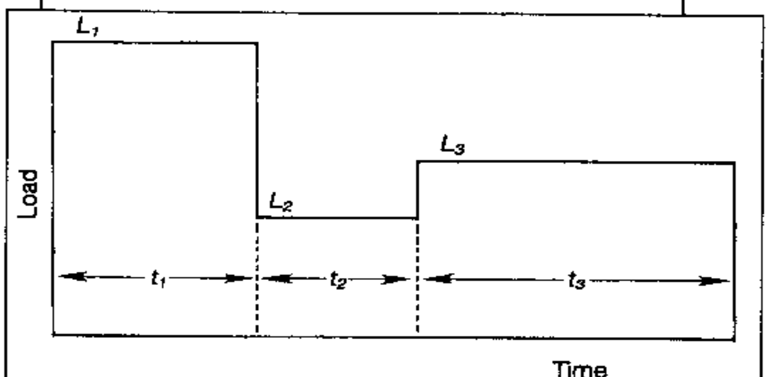
L_{RMS} = RMS value of the load which can be in any unit, hp, amp, etc.

L_1 = Load during time of period 1

L_2 = Load during time of period 2, etc.

t_1 = Duration of time for period 1

t_2 = Duration of time for period 2, etc.



Modulus of elasticity

$$E = \frac{PL}{A\Delta d} \qquad (14)$$

Where:

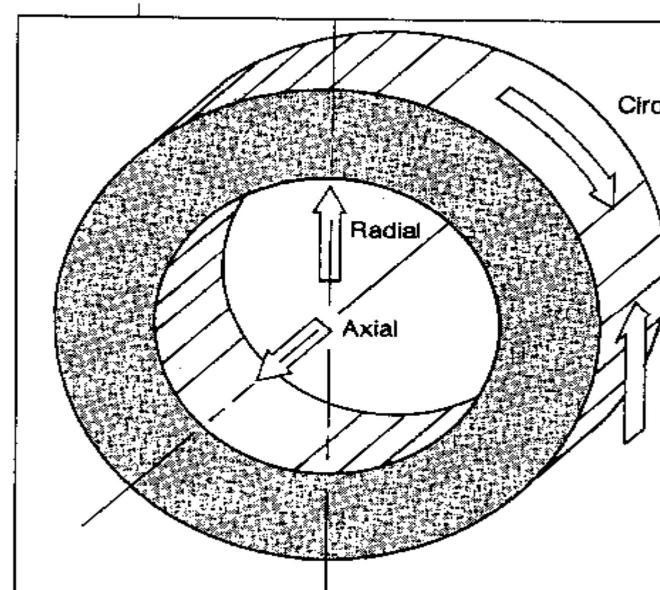
E = Modulus of elasticity, lb/in.^{2}

P = Axial load, lb

L = Length of object, in.

A = Area of object, in.^{2}

Δd = Increase in length resulting from axial load, in.



General technical references

1. S.I. Heisler, *The Wiley Engineer's Desk Reference,* John S. Wiley & Sons, New York, 1984.

2. Hindehide, Zimmerman, *Machine Design Fundamentals,* John S. Wiley & Sons, New York, 1983.

3. K.M. Walker, *Applied Mechanics for Engineering Technology,* Third Edition, Reston Publishing Co. Inc., Reston, Va., 1984.

4. *ASM Handbook of Engine... Mathematics,* American Soci... Metals, Metals Park, Ohio, 198...

5. *The Smart Motion Cheat ...* Amechtron Inc., Denton, Texas ...

Mechanical properties of common materials

Material	Equivalent	Ultimate strength, psi			Yield point, tension (psi)	Modulus of elasticity, tension or compression (psi)	Modulus of elasticity, shear (psi)	Weight (lb per in.^{3})
		Tension	Compression*	Shear				
Steel, forged-rolled								
C, 0.10-0.20	SAE 1015	60,000	39,000	48,000	39,000	30,000,000	12,000,000	0.28
C, 0.20-0.30	SAE 1025	67,000	43,000	53,000	43,000	30,000,000	12,000,000	0.28
C, 0.30-0.40	SAE 1035	70,000	46,000	56,000	46,000	30,000,000	12,000,000	0.28
C, 0.60-0.80	...	125,000	65,000	75,000	65,000	30,000,000	12,000,000	0.28
Nickel	SAE 2330	115,000	...	92,000	...	30,000,000	12,000,000	0.28
Cast iron:								
Gray	ASTM 20	20,000	80,000	27,000	...	15,000,000	6,000,000	0.26
Gray	ASTM 35	35,000	125,000	44,000	...	...	...	0.26
Gray	ASTM 60	60,000	145,000	70,000	...	20,000,000	8,000,000	0.26
Malleable	SAE 32510	50,000	120,000	48,000	...	23,000,000	9,200,000	0.26
Wrought iron	...	48,000	25,000	38,000	25,000	27,000,000	...	0.28
Steel cast:								
Low C	...	60,000	...	...	...	...	...	0.28
Medium C	...	70,000	...	...	...	...	...	0.28
High C	...	80,000	45,000	...	45,000	...	...	0.28
Aluminum alloy:								
Structural, No. 350	...	16,000	5,000	11,000	5,000	10,000,000	3,750,000	0.10
Structural, No. 17ST	...	58,000	35,000	35,000	35,000	10,000,000	3,750,000	0.10
Brass:								
Cast	...	40,000	...	...	...	...	...	0.30
Annealed	...	54,000	18,000	...	18,000	...	...	0.30
Cold-drawn	...	96,700	49,000	...	49,000	15,500,000	6,200,000	0.30
Bronze:								
Cast	...	22,000	...	...	...	...	...	0.31
Cold-drawn	...	85,000	...	...	...	15,000,000	6,000,000	0.31
Brick, clay	ASTM	...	1,500	3,000	...	...	...	0.72
Concrete 1:2:4 (28 days)	...	...	2,000	...	...	3,000,000	...	0.087
Stone	...	...	8,000	...	...	...	...	0.092
Timber	...	300	4,840	860	550	...	1,280,000	0.015

*The ultimate strength in compression for ductile materials is usually taken as the yield point. The bearing value for pins and rivets may be much higher, and for structural steel is taken as 90,000 psi. Source: S.I. Heisler, *The Wiley Engineer's Desk Reference*, 1984. Used with permission of John Wiley & Sons, New York.

CONVERSION FACTORS

Unless otherwise stated, pounds are U.S. avoirdupois, feet are U.S. standard, and seconds are mean solar.

Multiply	By	To obtain
Length		
Angstrom units	3.937×10^{-9}	in.
cm	0.3937	in.
ft	0.30480	m
in. (U.S.)	2.5400058	cm
in. (British)	0.9999972	in. (U.S.)
m	10^{10}	Angstrom units
m	3.280833	ft
m	39.37	in.
m	1.09361	yd
m	6.2137×10^{-4}	miles (U.S. statute)
yd	0.91440	m
miles (U.S. statute)	5,280	ft
Area		
cir mils	7.854×10^{-7}	in.^{2}
cm^2	1.07639×10^{-3}	ft^2
cm^2	0.15499969	in.^{2}
ft^2	0.092903	m^2
ft^2	929.0341	cm^2
in.^{2}	6.4516258	cm^2

Volume

cm^3	3.531445×10^{-5}	ft^3
cm^3	2.6417×10^{-4}	gal (U.S.)
cm^3	0.033814	oz (U.S. fluid)
ft^3 (British)	0.9999916	ft^3
ft^3 (U.S.)	28.31625	L (liter)
m^3	264.17	gal (U.S.)
gal (British)	4,516.086	cm^3
gal (British)	1.20094	gal (U.S.)
gal (U.S.)	0.13368	ft^3 (U.S.)
gal (U.S.)	231	$in.^3$
gal (U.S.)	3.78533	L (liter)
gal (U.S.)	128	oz (U.S. fluid)
oz (U.S. fluid)	29.5737	cm^3
oz (U.S. fluid)	1.80469	$in.^3$
yd^3	0.76456	m^3
yd^3 (British)	0.76455	m^3

Plane angle

radian	57.29578	deg

Weight

Dynes	2.24809×10^{-6}	lb
kg	35.2740	oz (avoirdupois)
kg	2.20462	lb
kg	0.001	tons (metric)
kg	0.0011023	tons (short)
oz (avoirdupois)	28.349527	grams
tons (long)	1,106	kg
tons (long)	2,240	lb
tons (metric)	1,000	kg
tons (metric)	2,204.6	lb
tons (short)	2,000	lb

Torque

lb-in.	0.113	(Newton-meters) N-m
lb-ft	1.356	N-m
lb-ft	1.3558×10^7	dyne-cm
oz-in.	0.00706	N-m

Energy

lb-in.	0.113	W-sec
lb-in.	0.113	j (joule)
Btu	251.98	calories
Btu	1,055.06	j

Power

gram-cm/sec	9.80665×10^{-5}	W
hp	2,545.08	Btu (mean)/hr
hp	550	lb-ft/sec
hp	0.74570	kW
hp	5,250	lb-ft/rpm

Inertia

Mass inertia:		
lb-in.^{2}	2.93×10^{-4}	kg-m^2
oz-in.^{2}	1.83×10^{-5}	kg-m^2
kg-cm^2	10^{-4}	kg-m^2
Weight inertia:		
lb-in.-sec^2	1.13×10^{-4}	kg-m^2
in-oz-sec^2	7.06×10^{-3}	kg-m^2
lb-ft-sec^2	1.355	kg-m^2

Source: S.I. Heisler, *The Wiley Engineer's Desk Reference*, 1984. Used with permission of John Wiley & Sons, New York.

The Smart Motion Cheat Sheet, Brad Grant, Amechtron Inc., Denton, Texas.

Multiply	By	To obtain
Velocities		
feet/sec (fps)	0.68182	mph
meters per sec	2.23693	mph
rpm	0.10472	radians/sec
mph	44.7041	cm/sec
mph	1.4667	fps
Temperature		
deg C = 0.555 (deg F − 32)		
deg F = 1.8 (deg F) + 32		
Pressure		
atmosphere	14.696	psi
atmosphere	10,333	kg/m^2
lb/ft^2	4.88241	kg/m^2
psi	70.307	grams/cm^2
psi	703.07	kg/m^2
Force		
Newton	0.22481	lb
Newton	9.80	kg

UNITS / DIM. CHECKS

F=ma   Rectilinear Motion

∴ 1N =(1kgm)(1m/s^2)
1 lbf = (1lbm)(386 in/s^2) or 1 lbf = 1 lbm * (32.17 ft/s^2)
∴ 1 lbm = 1/386 (lbf*s^2/in)

1 kgf = (1 kgm)(9.806 m/s^2)
∴1 kgm = (1/9.806) kgf*s^2/m
∴ 1 N = (1/9.806) kgf (s^2/m)(1 m/s^2) = (1/9.806) kgf
but 1 kgf = 2.2 lbf

∴1N = 2.2/9.806 lbf = 0.2243 lbf

or 1 lbf = 4.457 N