Average Values

The average voltage (or current) of a periodic waveform whether it is a sine wave, square wave or triangular waveform is defined as: "the quotient of the area under the waveform with respect to time". In other words, the averaging of all the instantaneous values along time axis with time being one full period, (T).

Average value of periodic instantaneous power p(t)

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} p(t)dt$$

Two-wire sinusoidal case

$$v(t) = V\sin(\omega_o t + \delta), \quad i(t) = I\sin(\omega_o t + \theta)$$

$$p(t) = v(t) \bullet i(t) = V\sin(\omega_o t + \delta) \bullet I\sin(\omega_o t + \theta)$$

zero average

$$p(t) = VI\left[\frac{\cos(\delta - \theta) - \cancel{\cos(2\omega_o t + \delta + \theta)}}{2}\right]$$

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} p(t)dt = \frac{VI}{2}\cos(\delta - \theta) = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}}\cos(\delta - \theta)$$

$$\boxed{P_{avg} = V_{rms} I_{rms} \boxed{\cos(\delta - \theta)}}$$

Displacement power factor

Average power

Effective Values

The effective value of an alternating current is that number of amperes which will produce heat at the same average rate as that number of amperes of steady direct current flowing through a given resistance. It is the square root of the mean of the instantaneous current values squared, and is also known as the root-mean-square or **R.M.S** value. A.C. ammeters and voltmeters are calibrated to read effective values. For sine curves, the effective value is approximately 70 per cent of the peak or maximum value.

Root-mean squared value of a periodic waveform with period T

$$V_{rms}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v^2(t)dt$$

The average value of the squared voltage

Compare to the average power expression

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} p(t)dt$$

compare

Apply v(t) to a resistor

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} p(t)dt = \frac{1}{T}\int_{t_o}^{t_o+T} \left[\frac{v^2(t)}{R}\right]dt = \frac{1}{RT}\int_{t_o}^{t_o+T} v^2(t)dt$$

$$P_{avg} = \frac{V_{rms}^2}{R}$$

rms is based on a power concept, describing the equivalent voltage that will produce a given average power to a resistor

Root-mean squared value of a periodic waveform with period T

$$V_{rms}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v^2(t)dt$$

For the sinusoidal case

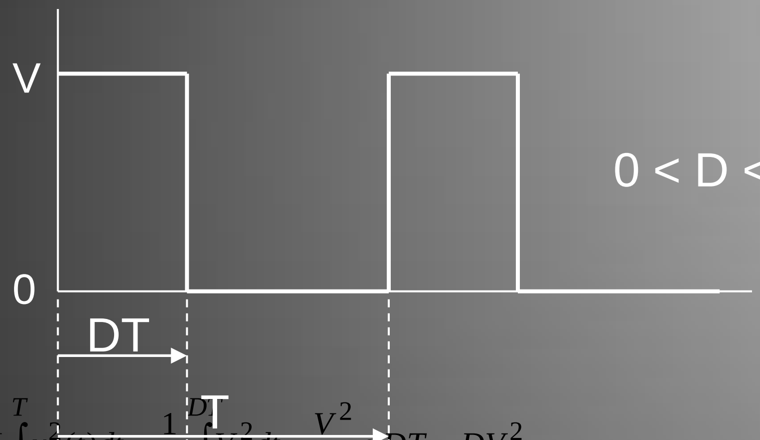
$$V_{rms}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} V^2 \sin^2(\omega_o t+\delta)dt \qquad v(t) = V\sin(\omega_o t+\delta),$$

$$V_{rms}^2 = \frac{V^2}{2T}\int_{t_o}^{t_o+T}\left[1-\cos 2(\omega_o t+\delta)\right]dt = \frac{V^2}{2T}\left[t-\frac{\sin 2(\omega_o t+\delta)}{2\omega_o}\right]_{t_o}^{t_o+T}$$

$$V_{rms} = \frac{V}{\sqrt{2}} \qquad V_{rms}^2 = \frac{V^2}{2},$$

RMS of some common periodic waveforms

Duty cycle controller



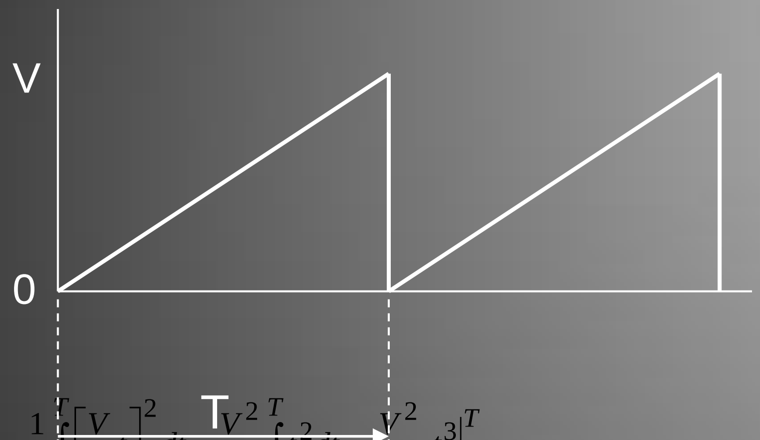
$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)dt = \frac{1}{T}\int_0^{DT} V^2 dt = \frac{V^2}{T} \bullet DT = DV^2$$

By inspection, this is the average value of the squared waveform

$$V_{rms} = V\sqrt{D}$$

RMS of common periodic waveforms, cont.

Sawtooth

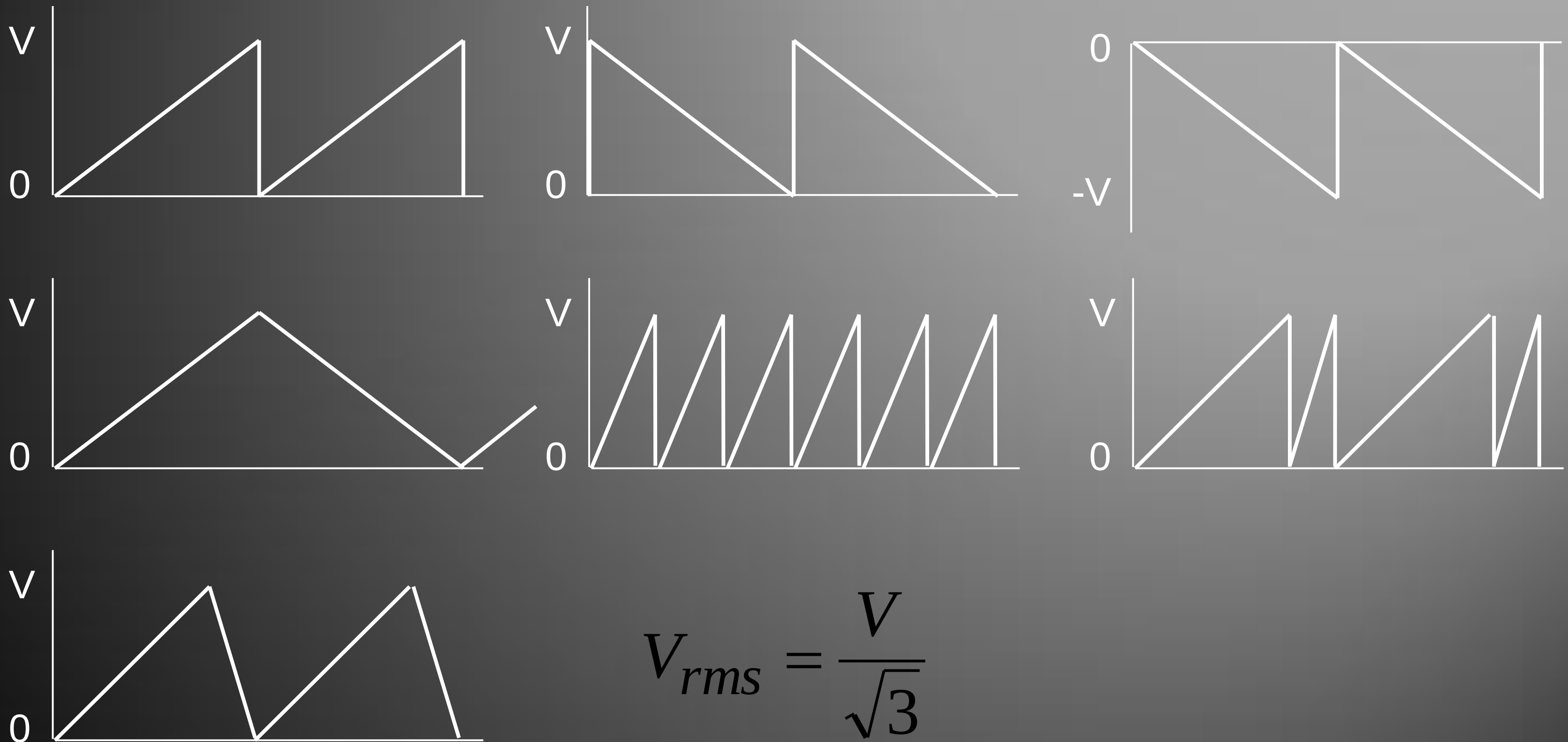


$$V_{rms}^2 = \frac{1}{T}\int_0^T \left[\frac{V}{T}t\right]^2 dt = \frac{V^2}{T^3}\int_0^T t^2\, dt = \frac{V^2}{3T^3}t^3\Big|_0^T$$

$$V_{rms} = \frac{V}{\sqrt{3}}$$

RMS of common periodic waveforms, cont.

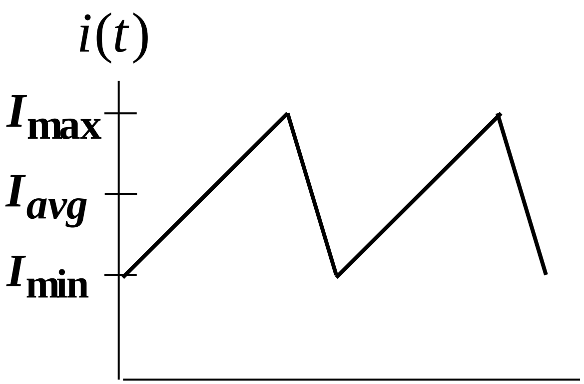
Using the power concept, it is easy to reason that the following waveforms would all produce the same average power to a resistor, and thus their rms values are identical and equal to the previous example

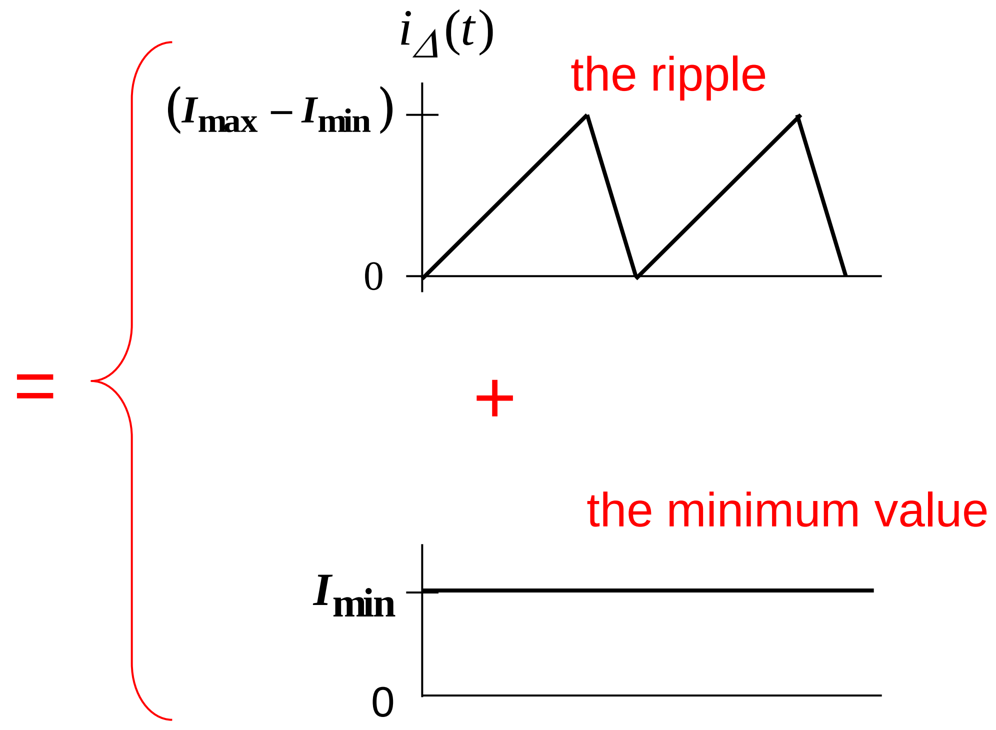


$$V_{rms} = \frac{V}{\sqrt{3}}$$

RMS of common periodic waveforms, cont.

Now, consider a useful example, based upon a waveform that is often seen in DC-DC converter currents. Decompose the waveform into its ripple, plus its minimum value.





$$I_{avg} = \frac{(I_{max} + I_{min})}{2}$$

RMS of common periodic waveforms, cont.

$$I_{rms}^2 = Avg\left\{\left(i_{\Delta}(t) + I_{\min}\right)^2\right\}$$

$$I_{rms}^2 = Avg\left\{i_{\Delta}^2(t) + 2i_{\Delta}(t) \bullet I_{\min} + I_{\min}^2\right\}$$

$$I_{rms}^2 = Avg\left\{i_{\Delta}^2(t)\right\} + 2I_{\min} \bullet Avg\left\{ i_{\Delta}(t)\right\} + I_{\min}^2$$

$$I_{rms}^2 = \frac{\left(I_{\max} - I_{\min}\right)^2}{3} + 2I_{\min} \bullet \frac{\left(I_{\max} - I_{\min}\right)}{2} + I_{\min}^2$$

Define $I_{PP} = I_{\max} - I_{\min}$

$$I_{rms}^2 = \frac{I_{PP}^2}{3} + I_{\min} I_{PP} + I_{\min}^2$$

Practice Problem

The periodic waveform shown is applied to a 100Ω resistor. What value of α yields 50W average power to the resistor?