Basic Electric Circuits

Concept of linearity and linear network

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Linearity and Superposition: Linearity.

Basically, a mathematical equation is said to be linear if the following properties hold.

- **homogenity**
- **additivity**

What does this mean? We first look at the property of homogenity.

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Linearity : **Homogeneity.**

Homogenity requires that if the input (excitation) of a system (equation) is multiplied by a constant, then the output should be obtained by multiplying by the same constant to obtain the correct solution.

Sometimes equations that we think are linear, turn out not be be linear because they fail the homogenity property. We next consider such an example.

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Linearity : Homogeneity (scaling).

Illustration: Does homogenity hold for the following equation?

Given,

$$y = 4x$$

If x = 1, y = 4. If we double x to x = 2 and substitute this value into Eq 9.1 we get y = 8.

Now for homogenity to hold, scaling should hold for y. that is, y has a value of 4 when x = 1. If we increase x by a factor of 2 when we should be able to multiply y by the same factor and get the same answer and when we substitute into the right side of the equation for x = 2.

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Linearity : Homogeneity (scaling).

Illustration: Does homogenity hold for the following equation?

Given,

$$y = 4x + 2$$

If x = 1, then y = 6. If we double x to x=2, then y = 10.

Now, since we doubled x we should be able to double the value that y had when x = 1 and get y = 10. In this case we get y = (2)(6) = 12, which obviously is not 10, so homogenity does not hold.

We conclude that Eq 9.2 is not a linear equation. In some ways that goes against the gain of what we have been taught about linear equations.

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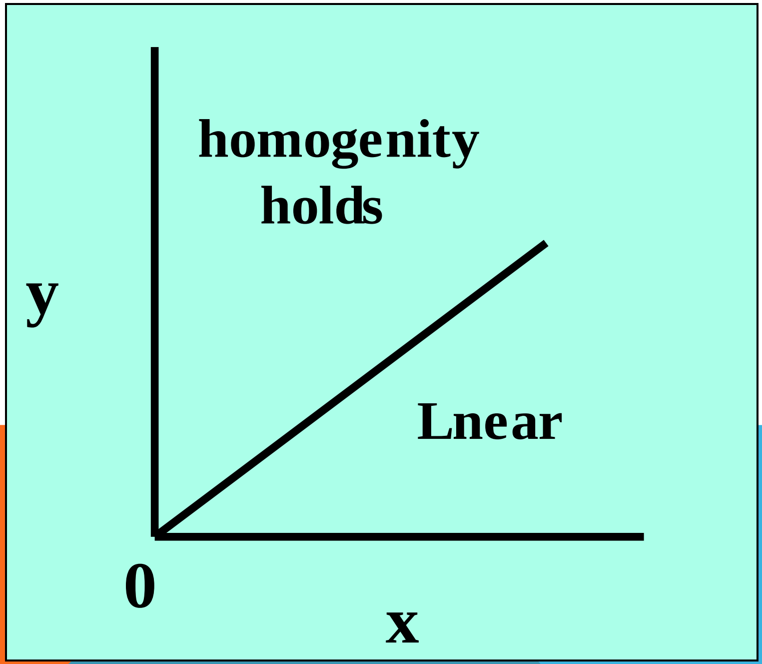
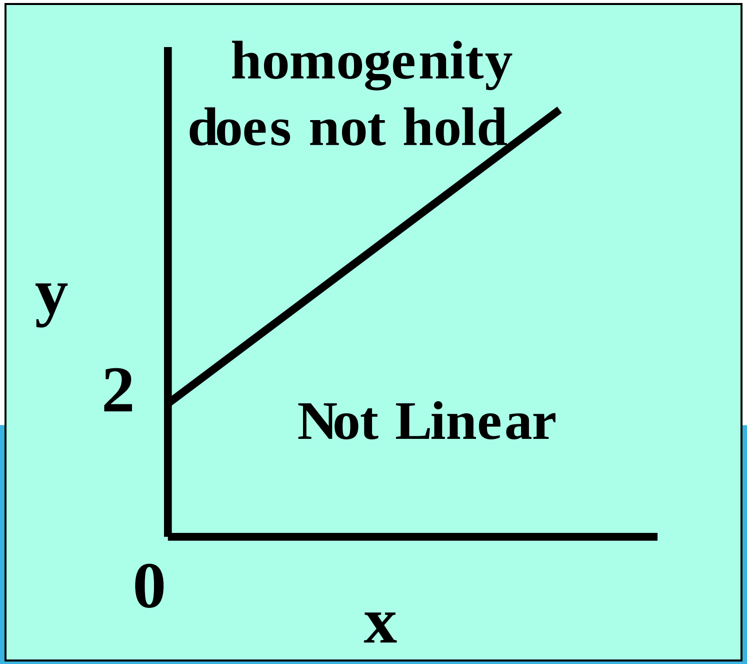
Linearity : **Homogeneity (scaling).**

Many of us were brought-up to think that if plotting an equation yields a straight line, then the equation is linear. From the following illustrations we have;





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Linearity : **Additivity Property.**

The additivity property is equivalent to the statement that the response of a system to a sum of inputs is the same as the responses of the system when each input is applied separately and the individual responses summed (added together).

This can be explained by considering the following illustrations.

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Linearity : Additivity Property.

Illustration: Given, $y = 4x$.

Let $x = x_1$, then $y_1 = 4x_1$
Let $x = x_2$, then $y_2 = 4x_2$

Then $y = y_1 + y_2 = 4x_1 + 4x_2$
Also, we note,

$$y = f(x_1 + x_2) = 4(x_1 + x_2) = 4x_1 + 4x_2$$

Since Equations (9.3) and (9.4) are identical, the additivity property holds.

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Linearity : Additivity Property.

Illustration: Given, $y = 4x + 2$.

Let $x = x_1$, then $y_1 = 4x_1 + 2$
Let $x = x_2$, then $y_2 = 4x_2 + 2$

Then $y = y_1 + y_2 = 4x_1+2 + 4x_2+2 = 4(x_1+x_2) + 4$
Also, we note,

$$y = f(x_1 + x_2) = 4(x_1 + x_2) + 2$$

Since Equations (9.5) and (9.6) are not identical, the additivity property does not hold.

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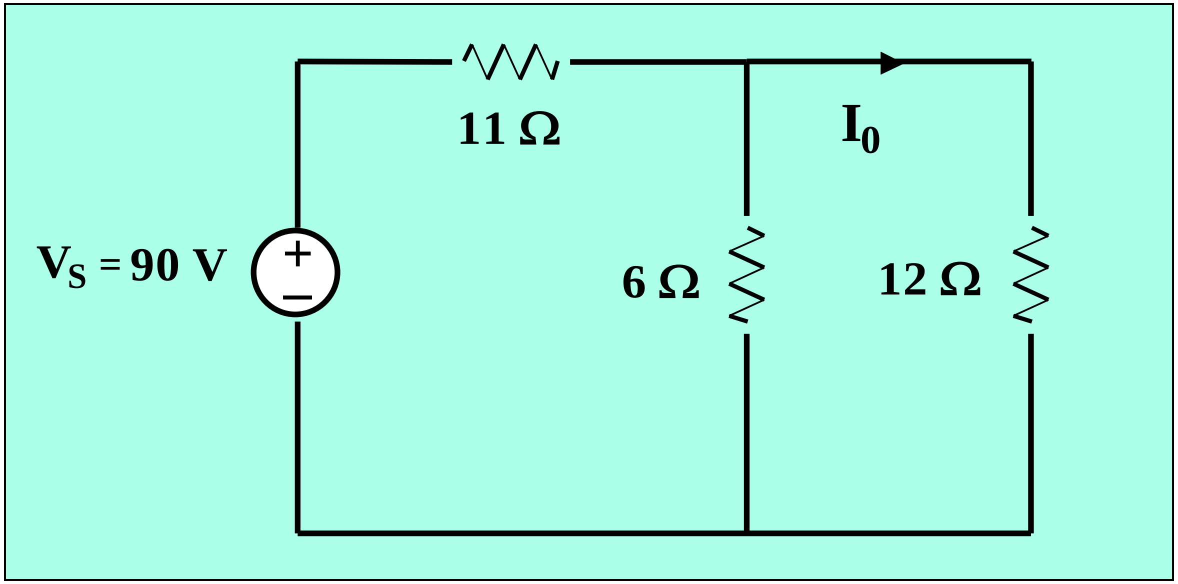
Linearity Given the circuit shown in Figure Use the concept of linearity (homogeneity or scaling) to find the current I_0.



Circuit for Example

Assume I_0 = 1 A. Work back to find that this gives V_S = 45 V. But since V_S = 90 V this means the true I_0 = 2 A.

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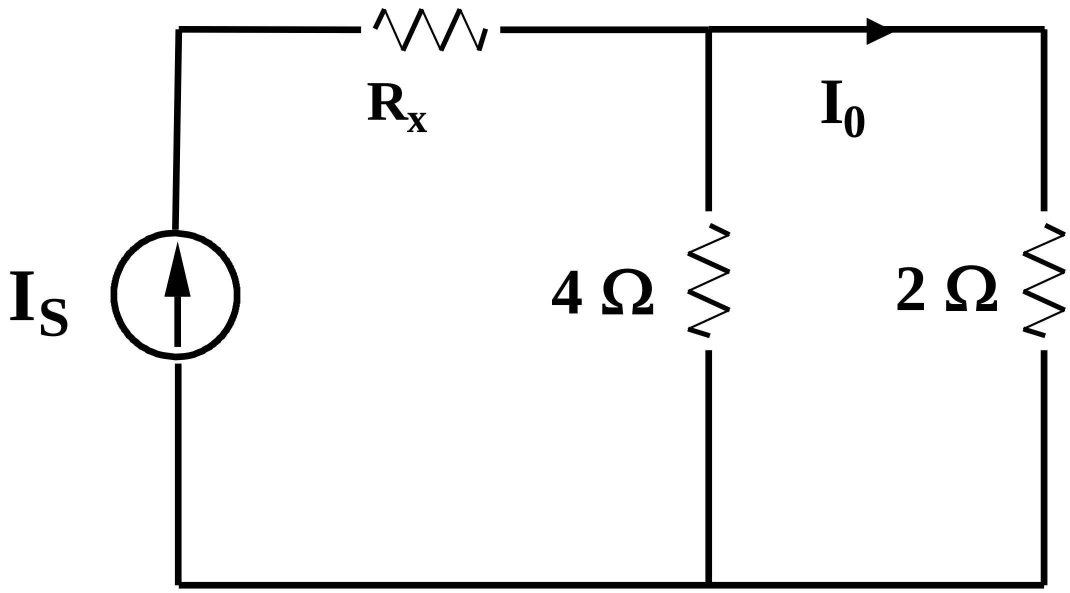
Linearity : In the circuit shown below it is known that $I_0 = 4$ A when $I_S = 6$ A. Find I_0 when $I_S = 18$ A.



Since $I_{S\,NEW} = 3xI_{S\,OLD}$ we conclude $I_{0\,NEW} = 3xI_{0\,OLD}$.
Thus, $I_{0\,NEW} = 3x4 = 12$ A.

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Linearity : Question.

For Example, one might ask, "how do we know the circuit is linear?" That is a good question. To answer, we assume a circuit of the same form and determine if we get a linear equation between the output current and the input current. What must be shown for the circuit below?

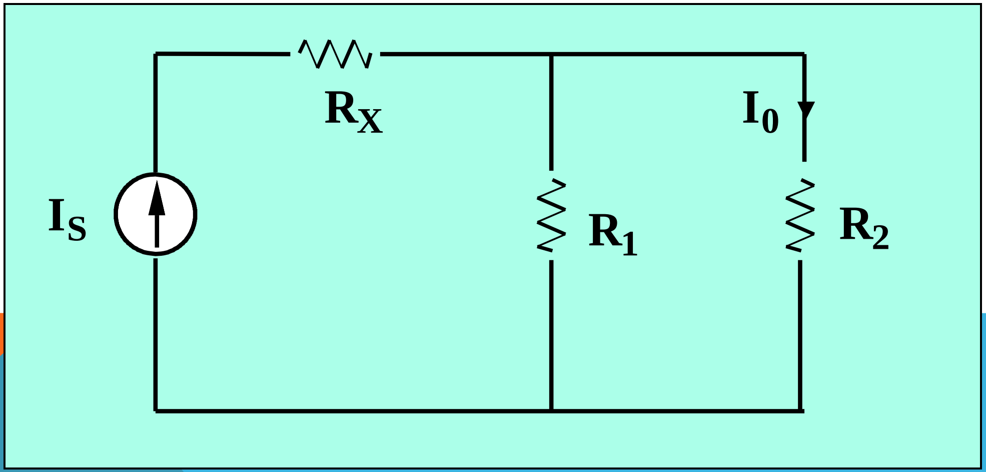

Figure 9.3: Circuit for investigating linearity.

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Linearity : **Question, continued.**

We use the current splitting rule (current division) to write the following equation.

$$I_0 = \frac{(I_S)(R_1)}{(R_1 + R_2)} = (K)(I_s)$$

The equation is of the same form of y = mx, which we saw was linear. Therefore, if R_1 and R_2 are constants then the circuit is linear.

Unilateral circuits

In unilateral circuits, the property of circuit changes with the change of direction of supply voltage or current. In other words, unilateral circuit allows the current to flow only in one direction. Diode rectifier is the best example of unilateral circuit because it does not perform the rectification in both direction of supply.

Bi-lateral circuits

In bilateral circuits, the property of circuit does not change with the change of direction of supply voltage or current. In other words, bilateral circuit allows the current to flow in both directions. Transmission line is the best example of bilateral circuit because, if you give supply from any direction, the circuit properties remain constant