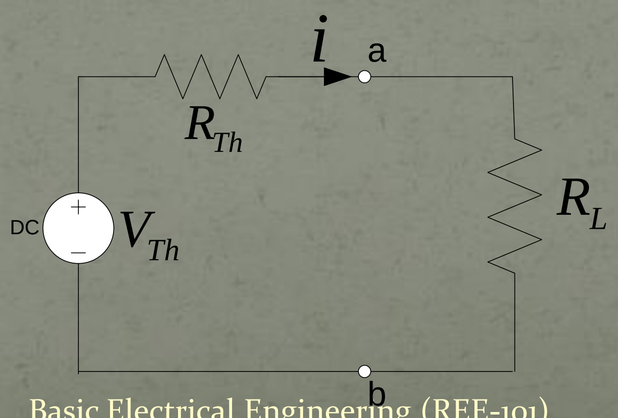
Circuit Theorems

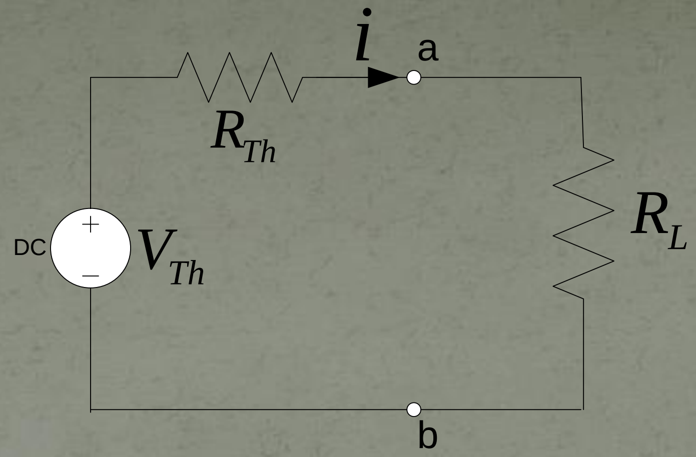
- Linear Circuits and Superposition
- Thevenin's Theorem
- Norton's Theorem
- **Maximum Power Transfer**

Maximum Power Transfer

In all practical cases, energy sources have non-zero internal resistance. Thus, there are losses inherent in any real source. Also, in most cases the aim of an energy source is to provide power to a load. Given a circuit with a known internal resistance, what is the resistance of the load that will result in the maximum power being delivered to the load?

Consider the source to be modeled by its Thevenin equivalent.





The power delivered to the load (absorbed by R_L) is

$$p = i^2 R_L = \left[V_{Th} / \left(R_{Th} + R_L \right) \right]^2 R_L$$

This power is maximum when $\partial p / \partial R_L = 0$

$$\frac{\partial p}{\partial R_L} = V_{Th}{}^2 \left[\left(R_{Th} + R_L \right)^{-2} - 2 R_L \left(R_{Th} + R_L \right)^{-3} \right] = 0$$

$$\frac{dp}{dR_L} = V_{Th}^{\ 2}\left[\left(R_{Th}+R_L\right)^{-2} - 2R_L\left(R_{Th}+R_L\right)^{-3}\right] = 0$$

$$R_{Th} + R_L = 2R_L$$

$$R_L = R_{Th}$$

Thus, maximum power transfer takes place when the resistance of the load equals the Thevenin resistance R_{Th}. Note also that

$$p_{\max} = \left[V_{Th}/\left(R_{Th}+R_L\right)\right]^2 R_L\Big|_{R_L=R_{Th}}$$

$$p_{\max} = \left[V_{Th}/\left(2R_{Th}\right)\right]^2 R_{Th} = V_{Th}^{\ 2}/4R_{Th}$$

Thus, at best, one-half of the power is dissipated in the internal resistance and one-half in the load.