Introduction To Resonant Circuits

Resonance in series & parallel RLC circuits

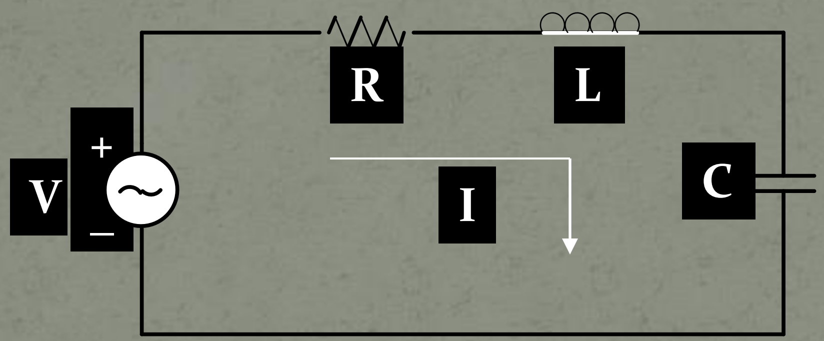
Resonance In Electric Circuits

- Any passive electric circuit will resonate if it has an inductor and capacitor.
- Resonance is characterized by the input voltage and current being in phase. The driving point impedance (or admittance) is completely real when this condition exists.
- In this presentation we will consider (a) series resonance, and (b) parallel resonance.

Series Resonance

Consider the series RLC circuit shown below.

$V = V_M \angle 0$



The input impedance is given by:

$$Z = R + j(wL - \frac{1}{wC})$$

The magnitude of the circuit current is;

$$I = |\bar{I}| = \frac{V_m}{\sqrt{R^2 + (wL - \frac{1}{wC})^2}}$$

Series Resonance

Resonance occurs when,

$$wL = \frac{1}{wC}$$

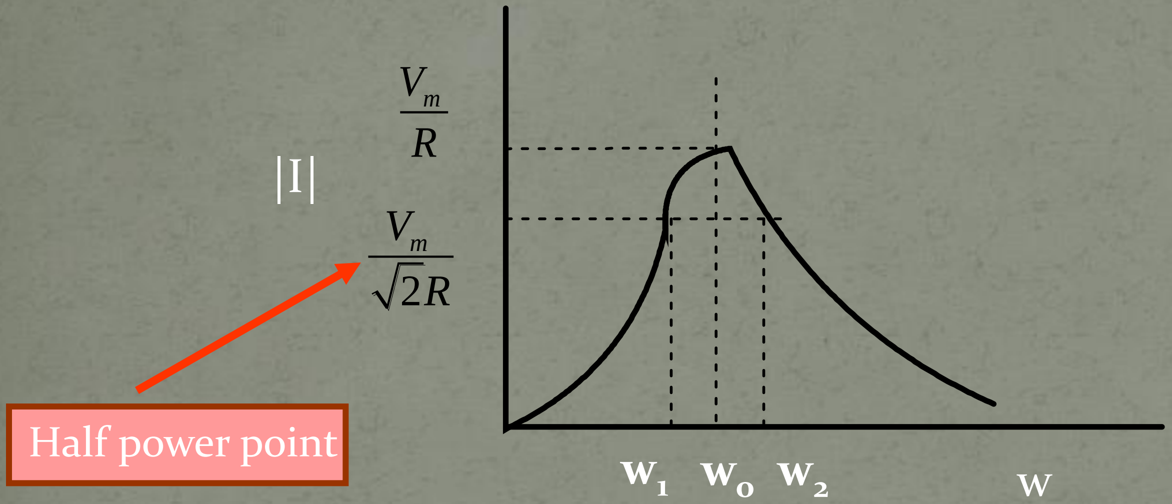
At resonance we designate w as w_o and write;

$$w_o = \frac{1}{\sqrt{LC}}$$

This is *an important equation* to remember. It applies to both series And parallel resonant circuits.

Series Resonance

The magnitude of the current response for the series resonance circuit is as shown below.



Bandwidth:

$BW = w_{BW} = w_2 - w_1$

Series Resonance

The peak power delivered to the circuit is;

$$P = \frac{V_m^{\ 2}}{R}$$

The so-called half-power is given when $I = \frac{V_m}{\sqrt{2}R}$.

We find the frequencies, w_1 and w_2, at which this half-power occurs by using;

$$\sqrt{2}R = \sqrt{R^2 + (wL - \frac{1}{wC})^2}$$

Series Resonance

After some insightful algebra one will find two frequencies at which the previous equation is satisfied, they are:

$$w_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

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$$w_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

The two half-power frequencies are related to the resonant frequency by

$$w_o = \sqrt{w_1 w_2}$$

Series Resonance

The bandwidth of the series resonant circuit is given by;

$$BW = w_b = w_2 - w_1 = \frac{R}{L}$$

We define the Q (quality factor) of the circuit as;

$$Q = \frac{w_o L}{R} = \frac{1}{w_o RC} = \frac{1}{R}\sqrt{\left(\frac{L}{C}\right)}$$

Using Q, we can write the bandwidth as;

$$BW = \frac{w_o}{Q}$$

These are all important relationships.

Series Resonance

An Observation:

If Q > 10, one can safely use the approximation;

$$w_1 = w_o - \frac{BW}{2} \qquad and \qquad w_2 = w_o + \frac{BW}{2}$$

These are useful approximations.

Series Resonance

An Observation:

By using Q = w_0L/R in the equations for w_1 and w_2 we have;

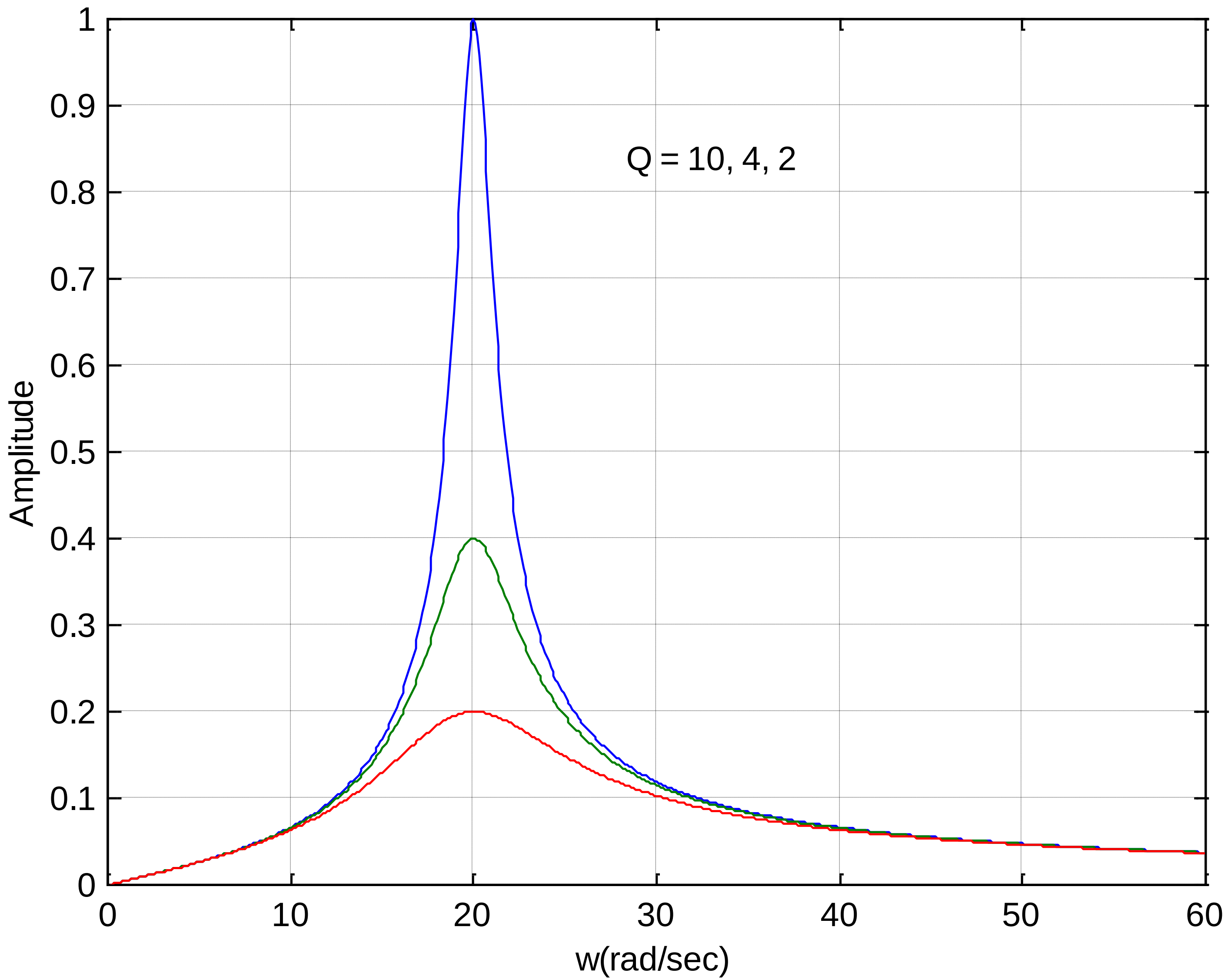
$$w_1 = w_o\left[\frac{-1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$$

and

$$w_2 = w_o\left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$$

Series Resonance

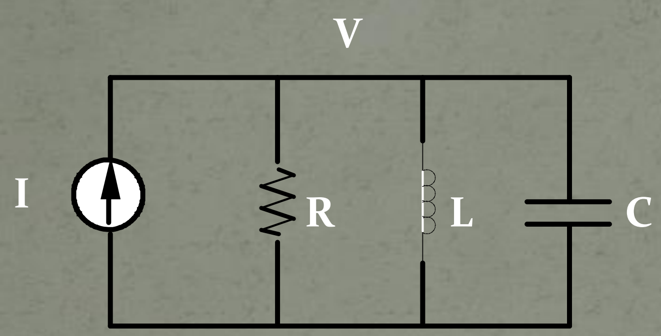
Program Output



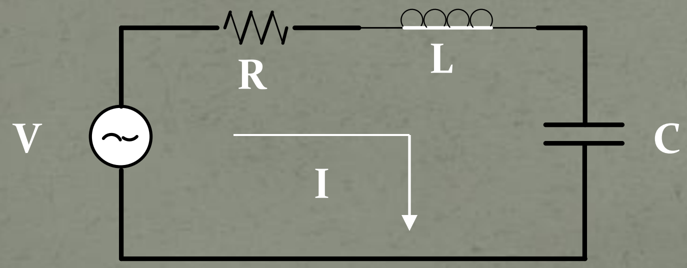
Parallel Resonance

Background

Consider the circuits shown below:



$$I = V\left[\frac{1}{R} + jwC + \frac{1}{jwL}\right]$$



$$V = I\left[R + jwL + \frac{1}{jwC}\right]$$

Series Resonance

Duality

$$I = V\left[\frac{1}{R} + jwC + \frac{1}{jwL}\right] \qquad V = I\left[R + jwL + \frac{1}{jwC}\right]$$

We notice the above equations are the same provided:

$I \longleftrightarrow V$

$R \longleftrightarrow \frac{1}{R}$

$L \longleftrightarrow C$

If we make the inner-change, then one equation becomes the same as the other.

For such case, we say the one circuit is the dual of the other.

Parallel Resonance

Background

What this means is that for all the equations we have derived for the parallel resonant circuit, we can use for the series resonant circuit provided we make the substitutions:

$$R \quad \text{replaced be} \quad \frac{1}{R}$$

$$L \quad \text{replaced by} \quad C$$

$$C \quad \text{replaced by} \quad L$$

Parallel Resonance

Parallel Resonance

$$w_o = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{w_o L}{R}$$

$$BW = (w_2 - w_1) = w_{BW} = \frac{R}{L}$$

$$w_1, w_2 = \left[\frac{\mp R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]$$

$$w_1, w_2 = w_o \left[\frac{\mp 1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$$

$w_1, w w_2$

Series Resonance

$$w_o = \frac{1}{\sqrt{LC}}$$

$$Q = w_o RC$$

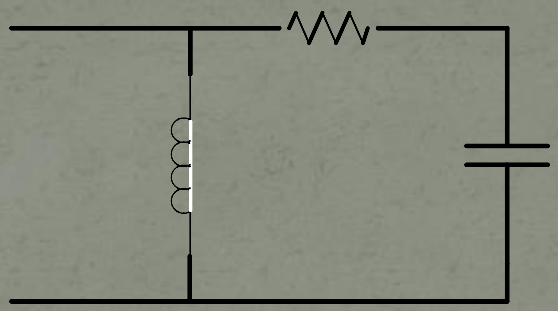
$$BW = w_{BW} = \frac{1}{RC}$$

$$w_1, w_2 = \left[\frac{\mp 1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \right]$$

$$w_1, w_2 = w_o \left[\frac{\mp 1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$$

Resonance

Example 1: **Determine the resonant frequency for the circuit below.**



$$Z_{IN} = \frac{jwL(R + \frac{1}{jwC})}{R + jwL + \frac{1}{jwC}} = \frac{(-w^2LRC + jwL)}{(1 - w^2LC) + jwRC}$$

At resonance, the phase angle of Z must be equal to zero.

Resonance

Analysis

$$\frac{(-w^2LRC + jwL)}{(1 - w^2LC) + jwRC}$$

For zero phase;

$$\frac{wL}{(-w^2LCR)} = \frac{wRC}{(1 - w^2LC}$$

This gives;

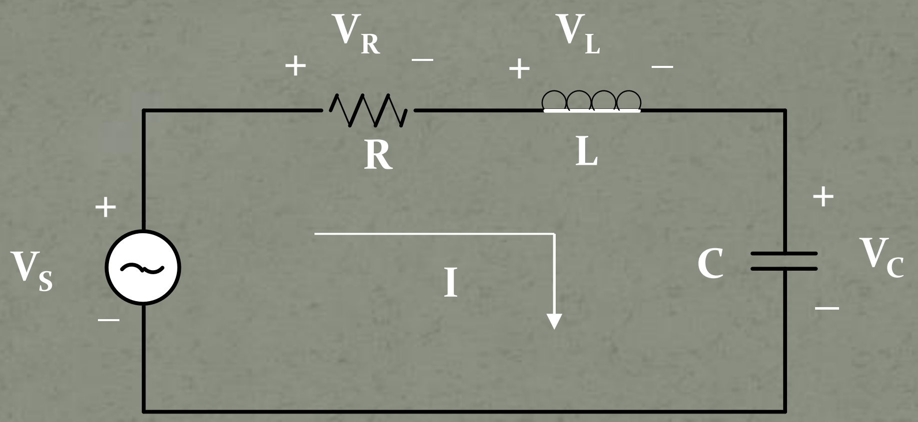
$$w^2LC - w^2R^2C^2 = 1$$

or

$$w_o = \frac{1}{\sqrt{(LC - R^2C^2)}}$$

Extension of Series Resonance

Peak Voltages and Resonance:



We know the following:

- When $w = w_o = \dfrac{1}{\sqrt{LC}}$, V_S and I are in phase, the driving point impedance is purely real and equal to R.
- A plot of |I| shows that it is maximum at $w = w_o$. We know the standard equations for series resonance applies: Q, w_{BW}, etc.

Extension of Series Resonance

Reflection:

- A question that arises is what is the nature of V_R, V_L, and V_C? A little reflection shows that V_R is a peak value at w_o. But we are not sure about the other two voltages. We know that at resonance they are equal and they have a magnitude of $Q x V_S$.

- Irwin shows that the frequency at which the voltage across the capacitor is a maximum is given by;

$$w_{max} = w_o \sqrt{1 - \frac{1}{2Q^2}}$$

- The above being true, we might ask, what is the frequency at which the voltage across the inductor is a maximum?

We answer this question by simulation

Extension of Resonant Circuits

Resonant Condition Analysis:

What is the significance of w_r and w_o in the previous two equations? Clearly w_r is a lower frequency of the two. To answer this question, consider the following example.

Given the following circuit with the indicated parameters. Write a Matlab program that will determine the frequency response of the transfer function of the voltage to the current as indicated.