NETWORK THEOREMS

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Superposition Theorem

Thevenin's Theorem

Norton's Theorem

Maximum Power Transfer

LINEAR CIRCUITS

A linear circuit is one whose output is directly proportional to its input.

Linear circuits obey both the properties of homogeneity (scaling) and additivity.

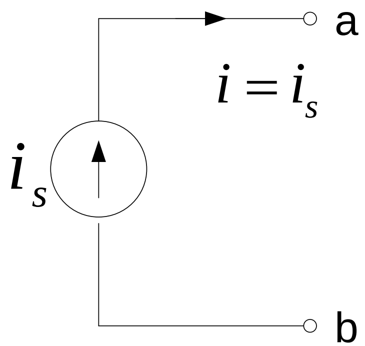
SUPERPOSITION PRINCIPLE

Because the circuit is linear we can find the response of the circuit to each source acting alone, and then add them up to find the response of the circuit to all sources acting together. This is known as the **superposition principle**.

*The **superposition principle** states that the voltage across (or the current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.*

TURNING SOURCES OFF

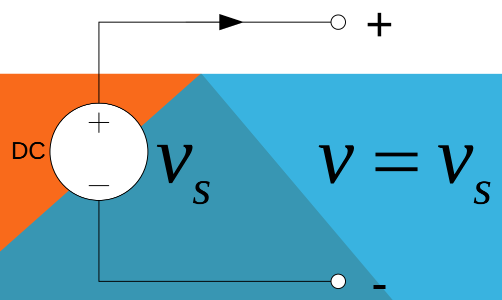
Current source:



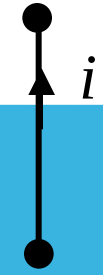
We replace it by a current source where $i_s \equiv 0$

An open-circuit

Voltage source:



We replace it by a voltage source where $v_s \equiv 0$



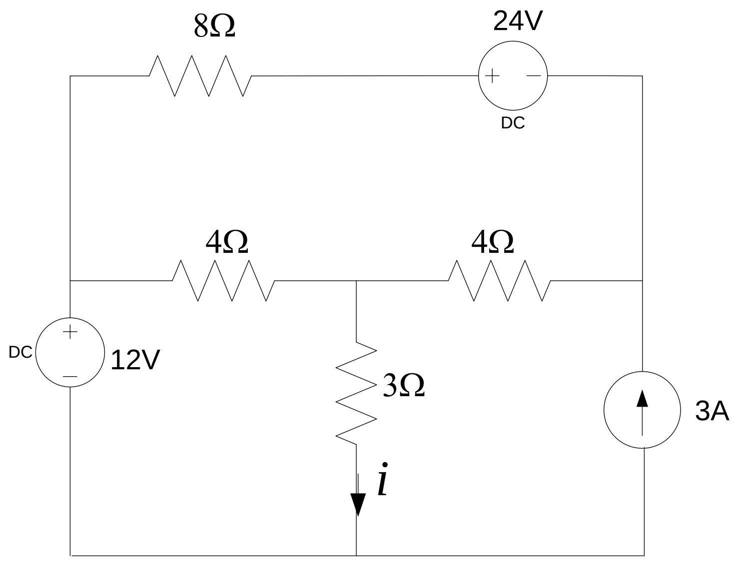
An short-circuit

STEPS IN APPLYING THE SUPERPOSITION PRINCIPLE

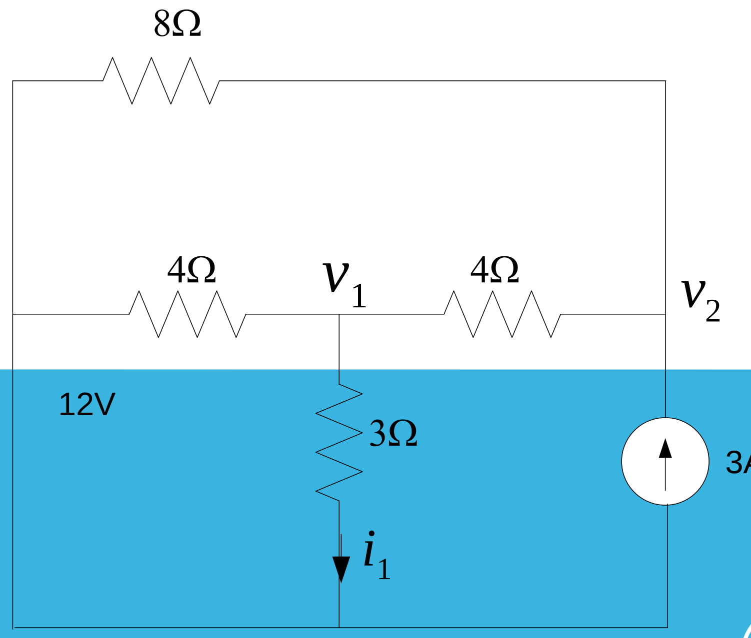
1. **Turn off all independent sources except one. Find the output (voltage or current) due to the active source.**
2. **Repeat step 1 for each of the other independent sources.**
3. **Find the total output by adding algebraically all of the results found in steps 1 & 2 above.**

In some cases, but certainly not all, superposition can simplify the analysis.

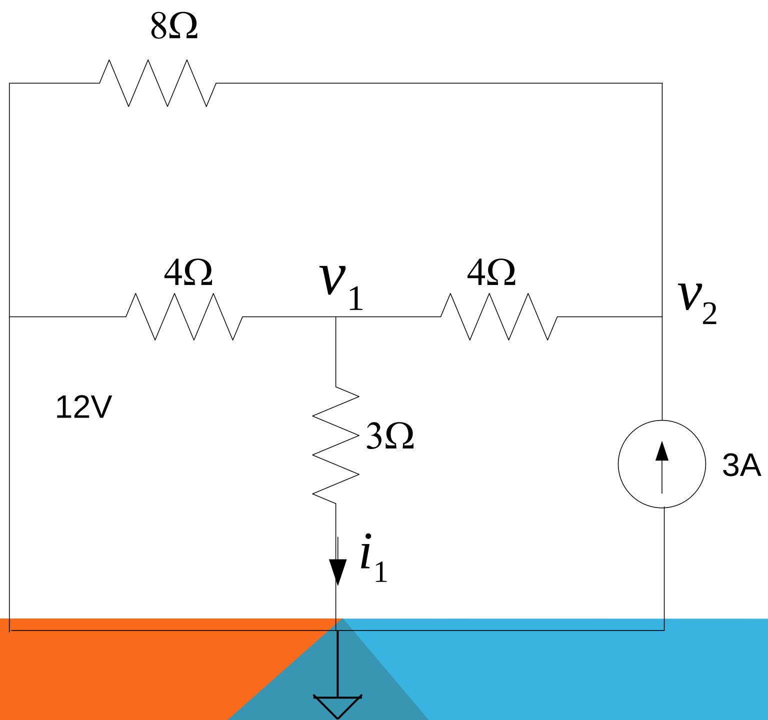
Example: In the circuit below, find the current *i* by superposition



Turn off the two voltage sources (replace by short circuits).



$$\begin{pmatrix} 1/4+1/3+1/4 & -1/4 \\ -1/4 & 1/4+1/8 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 3 \end{pmatrix}$$



$$\frac{5}{6}v_1-\frac{1}{4}v_2=0$$

$$-\frac{1}{4}v_1+\frac{3}{8}v_2=3$$

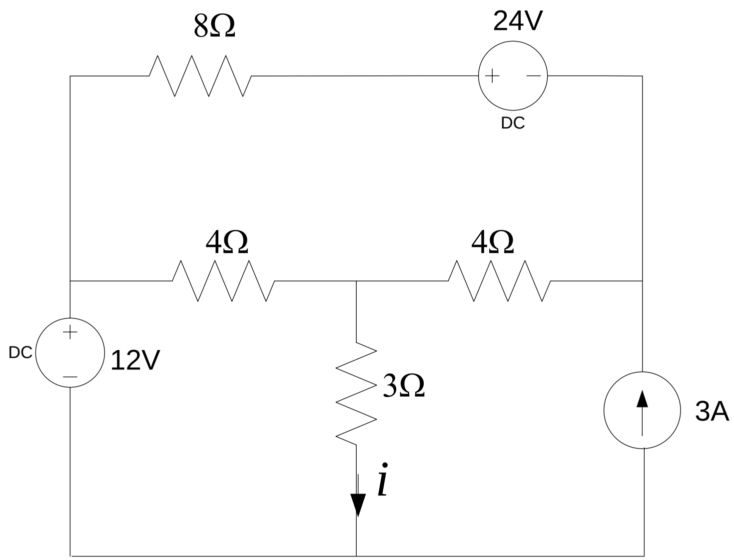
$$v_2=\frac{10}{3}v_1$$

$$v_1\left(\frac{10}{8}-\frac{2}{8}\right)=3$$

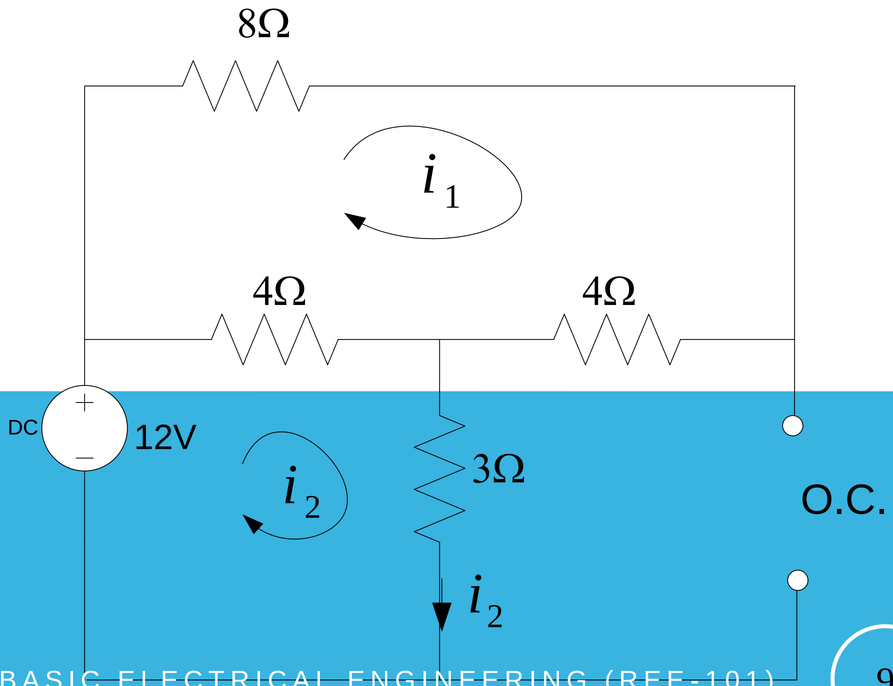
$$i_1=1$$

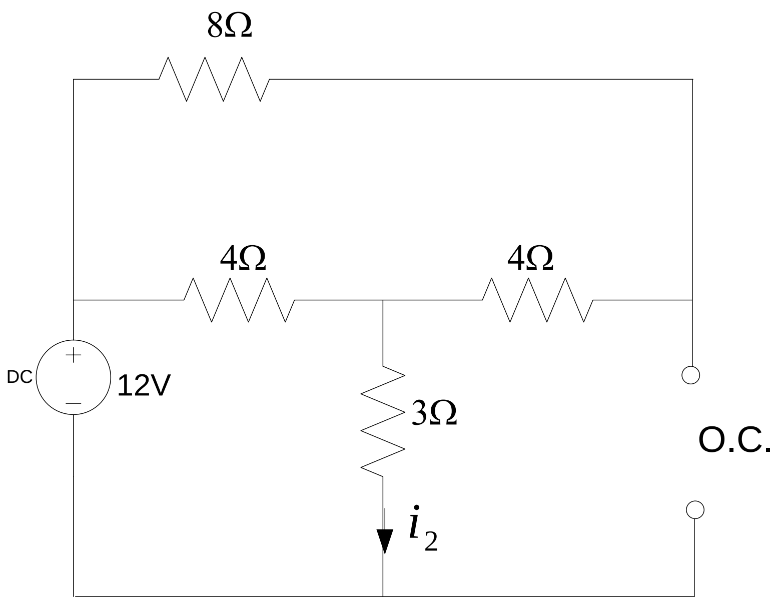
$$v_1=3$$

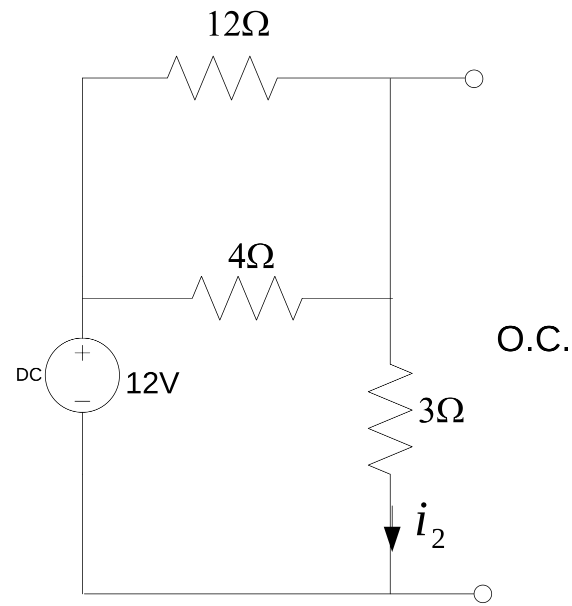
Example: In the circuit below, find the current *i* by superposition



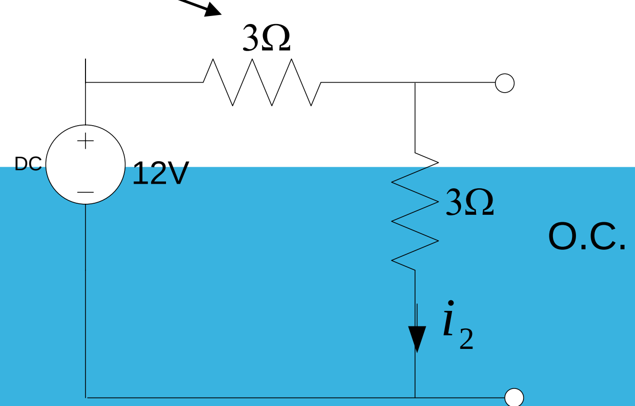
Turn off the 24V & 3A sources:





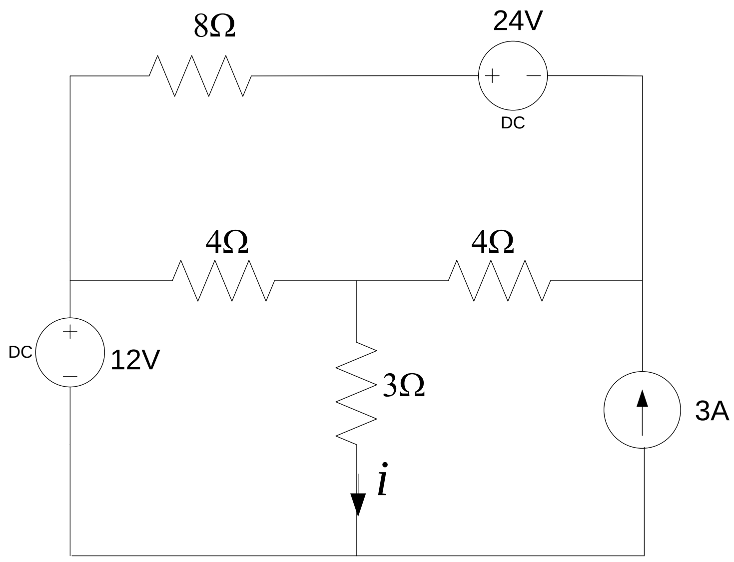


$$\frac{12 \times 4}{16} = 3$$

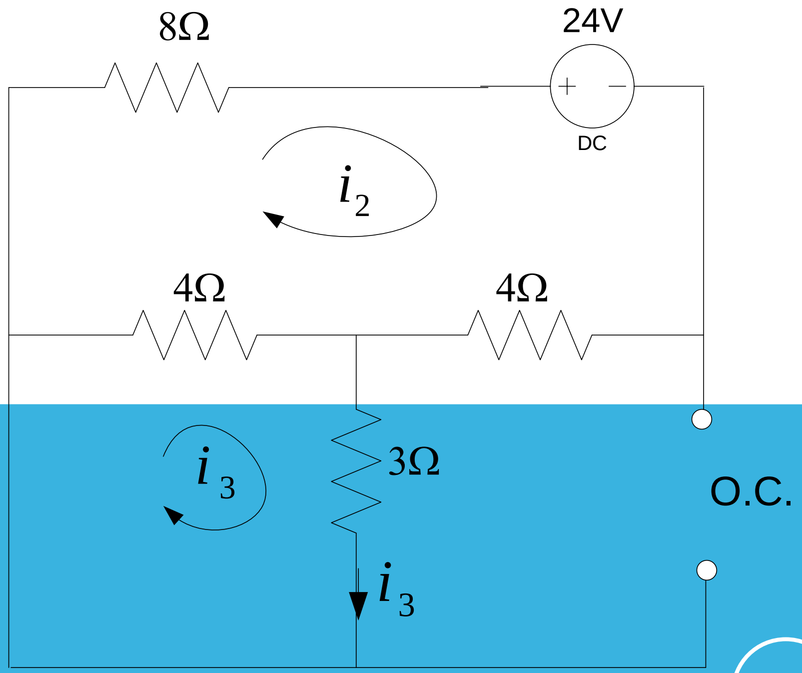


$$i_2 = \frac{12}{6} = 2$$

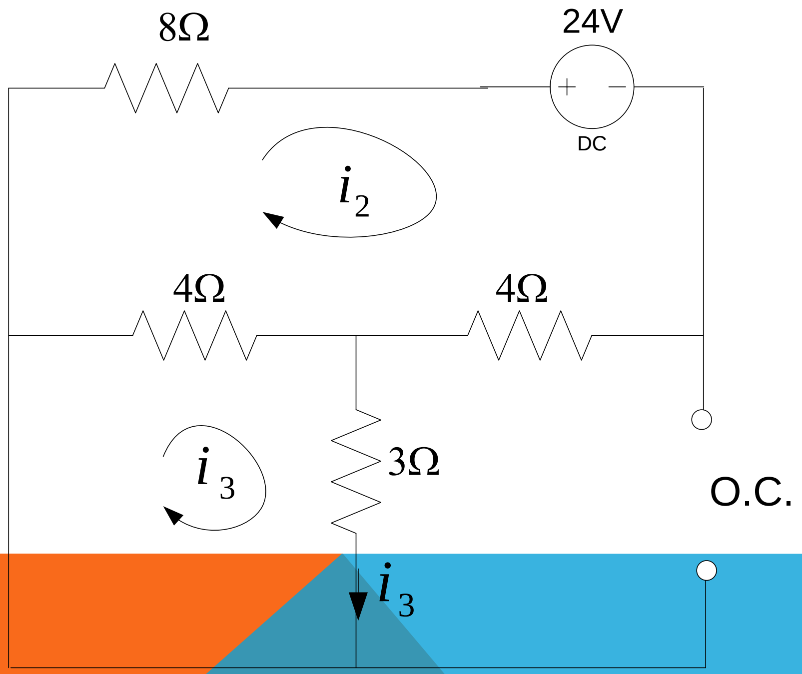
Example: In the circuit below, find the current *i* by superposition



Turn off the 3A & 12V sources:



$$\begin{pmatrix} 4+8+4 & -4 \\ -4 & 4+3 \end{pmatrix} \begin{pmatrix} i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} -24 \\ 0 \end{pmatrix}$$



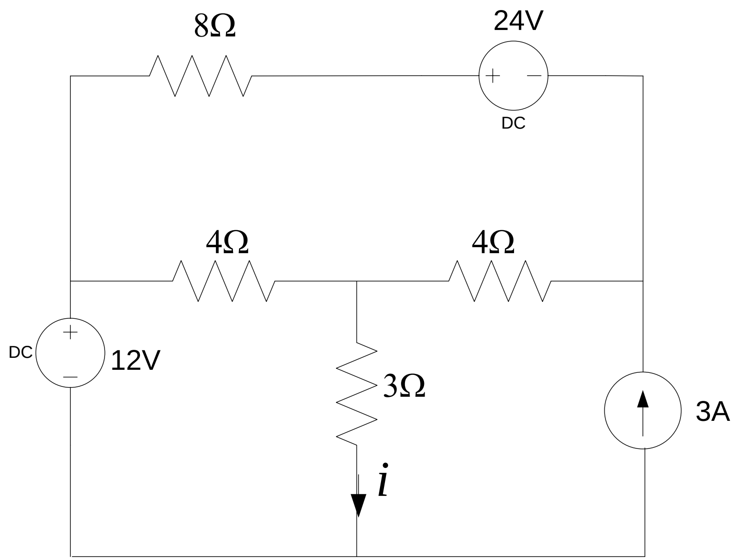
$$16i_2 - 4i_3 = -24$$

$$-4i_2 + 7i_3 = 0$$

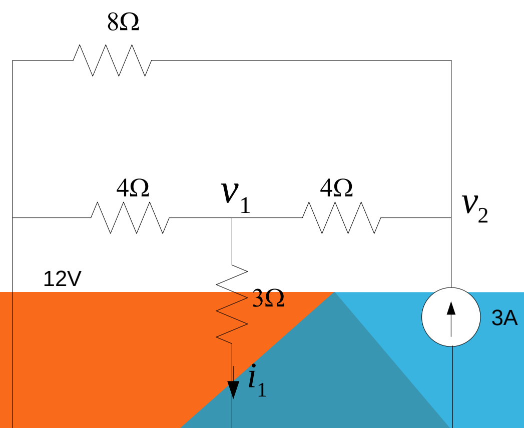
$$i_2 = \frac{7}{4} i_3$$

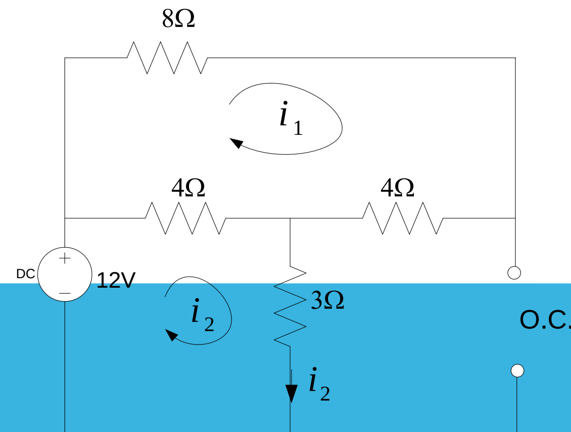
$$i_3(28-4) = -24$$

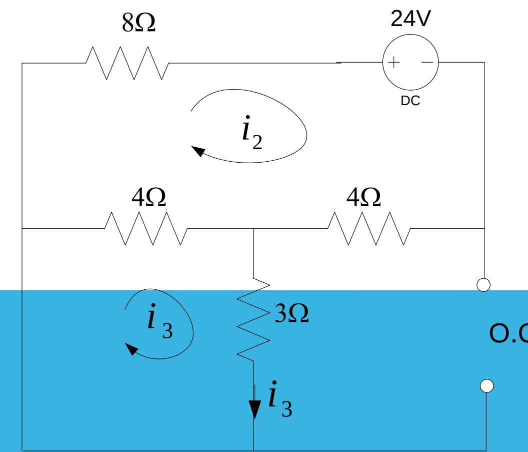
$$i_3 = -1$$



$$i = i_1 + i_2 + i_3 = 1A + 2A - 1A = 2A$$







$$i_1 = 1$$

$$i_2 = 2$$

$$i_3 = -1$$