The Quality Factor (Q)

- The **quality factor** (Q) of a series resonant circuit is defined as the ratio of the reactive power of either the inductor or the capacitor to the average power of the resistor at resonance.

$$Q_s = \frac{\text{reactive power}}{\text{average power}}$$

- The quality factor is also an indication of how much energy is placed in storage compared to that dissipated. The lower the level of dissipation for the same reactive power, the larger the Q_S factor and the more concentrated and intense the region of resonance.

The Quality Factor (Q)

- Substituting for an inductive reactance at resonance gives us

$$Q_s = \frac{X_L}{R} = \frac{\omega_s L}{R}$$

- If the resistance R is just the resistance of the coil (R_l), we can speak of the Q of the coil, where

$$Q_{coil} = Q_l = \frac{X_L}{R_l} \qquad R = R_l$$

The Quality Factor (Q)

- The Q of a coil is usually provided by the manufacturer.
- As the frequency increases the effective resistance of the coil also increases, due primarily to the skin effect phenomena, causing the Q_l to decrease. The capacitive effect between windings will increase, further reducing the Q_l of the coil.
- Q_l must be specified for a particular frequency or frequency range.
- Q_S in terms of circuit parameters:

$$Q_s = \frac{1}{R}\sqrt{\frac{L}{C}}$$

- For series circuits u unication systems Q_S is usually greater tha

Z_T Versus Frequency

- From previous discussions we found that total impedance of a series R-L-C circuit at any frequency is determined by

$$Z_T = R + jX_L - jX_C \quad \text{or} \quad Z_T = R + j(X_L - X_C)$$

- The total-impedance-versus-frequency curve for the series resonant circuit can be found by applying the impedance-versus-frequency curve for each element of the equation:

$$Z_T(f) = \sqrt{[R(f)]^2 + [X_L(f) - X_C(f)]^2}$$

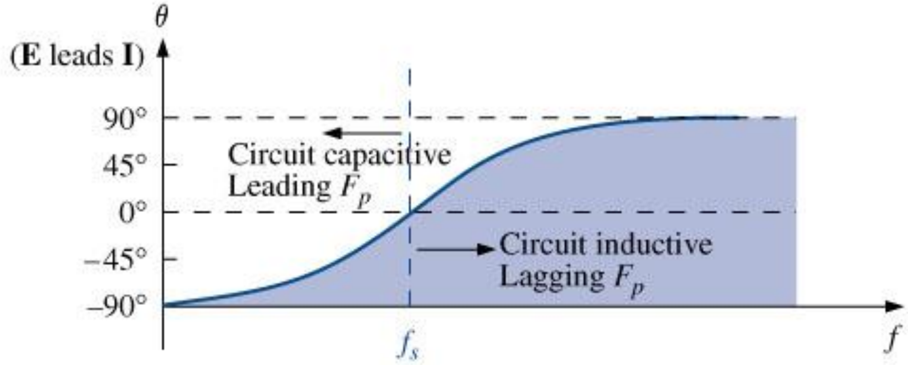
Z_T Versus Frequency

- The phase angle associated with the total impedance:

$$\theta = \tan^{-1}\frac{(X_L - X_C)}{R}$$

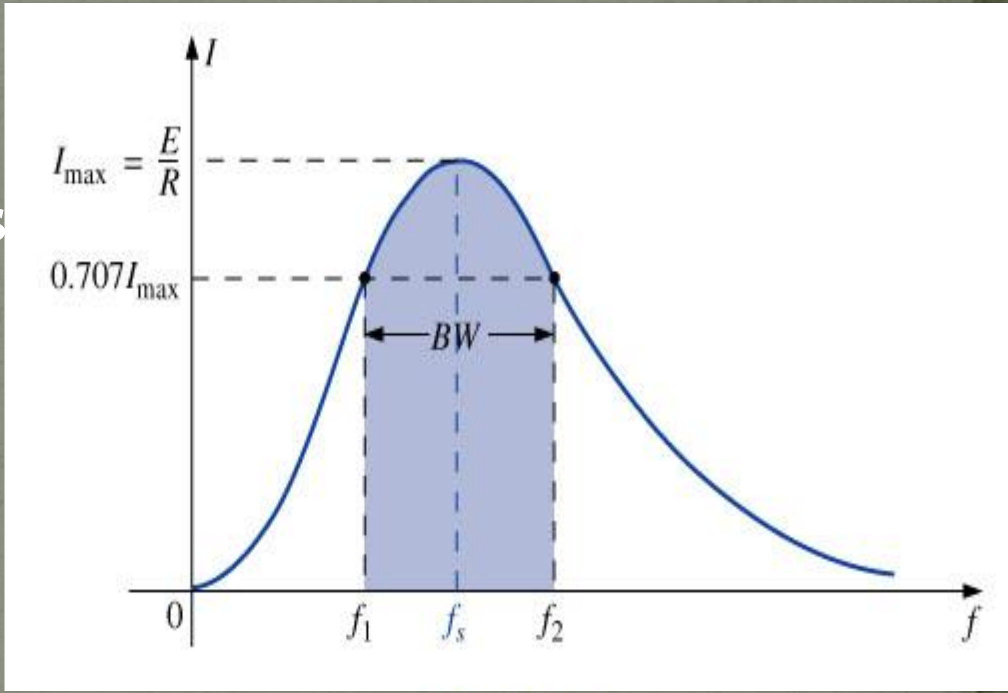
- For a series resonant circuit

$f < f_s$:	network capacitive; **I** leads **E**
$f > f_s$:	network inductive; **E** leads **I**
$f = f_s$:	network resistive; **E** and **I** are in phase



Selectivity

- If we now plot the magnitude of the current $I=E/Z_T$ versus frequency for a fixed applied voltage E, we obtain a curve that is actually the inverse of the impedance-versus-frequency curve.
- Z_T is not absolutely symmetrical about the resonant frequency, and the curve of *I* vs *f* is also non-symmetrical about resonance.



Selectivity

- The frequencies corresponding to 0.707 of the maximum current are called the **band frequencies, cutoff frequencies**, or **half-power frequencies** (f_1, f_2).
  - Half-power frequencies are those frequencies at which the power delivered is one-half that delivered at resonant frequency.
- The range of frequencies between the two are referred to as **bandwidth** (abbreviated BW) of the resonant circuit.
- Since the resonant circuit is adjusted to select a band of frequencies it is called a **selectivity curve.**

$$BW = \frac{f_s}{Q_s}$$

Selectivity

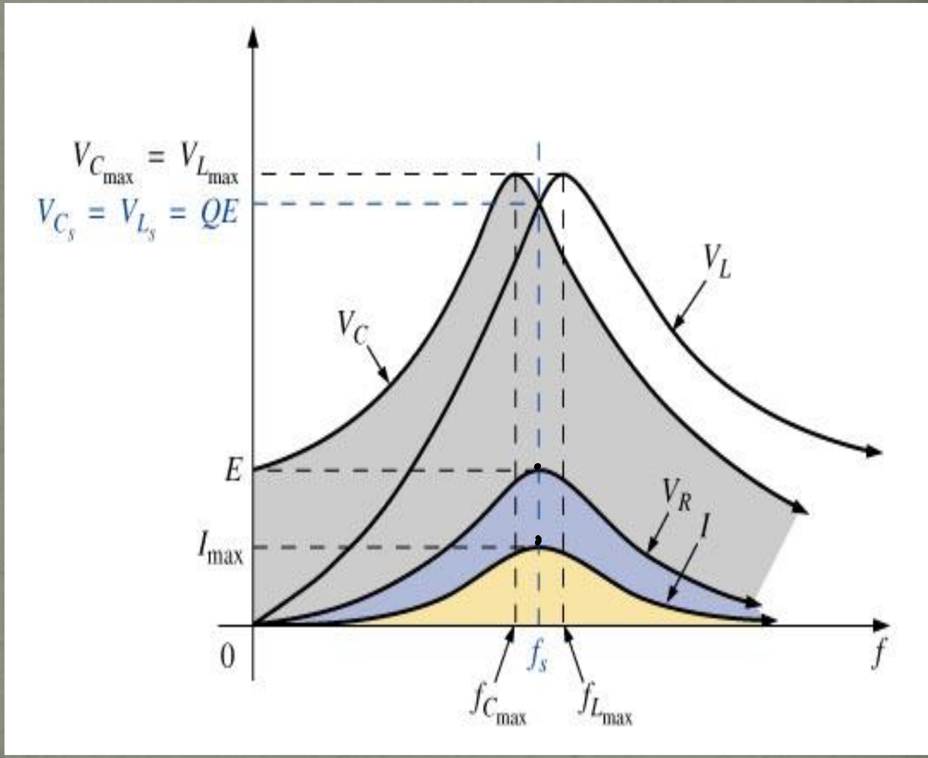
- The shape of the curve depends on each element of the series R-L-C circuit.
  - If resistance is made smaller with a fixed inductance and capacitance, the bandwidth decreases and the selectivity increases.
  - If the ratio L/C increases with fixed resistance, the bandwidth again decreases with an increase in selectivity.

Selectivity

- In terms of Q_S, if R is larger for the same X_L, then Q_S is less, as determined by the equation $Q_S = \omega_S L/R$.
  - A small Q_S, therefore, is associated with a resonant curve having a large bandwidth and a small selectivity, while a large Q_S indicates the opposite.
  - For circuits where $Q_S \geq 10$, a widely accepted approximation is that the resonant frequency bisects the bandwidth and that the resonant curve **is** symmetrical about the resonant frequency.

V_R , V_L , and V_C

- Plotting the magnitude (effective value) of the voltages V_R , V_L , and V_C and the current I versus frequency for the series resonant circuit on the same set of axes
  - The V_R curve has the same shape as the I curve and the peak value equal to the magnitude of the input voltage E.



V_R , V_L , and V_C

- The V_C curve builds up slowly at first from a value equal to the input voltage since the reactance of the capacitor is infinite (open circuit) at zero frequency and the reactance of the inductor is zero (short circuit) at this frequency.
- As the Frequency increases, $1/\omega C$ of the equation becomes smaller, but *I* increases at a rate faster than that at which $1/\omega C$ drops.

V_R , V_L , and V_C

- The curve for V_L increases steadily from zero to the resonant frequency since both quantities ωL and *I* of the equation $V_L = IX_L = (I)(\omega L)$ increases over this frequency range.
- At resonance, *I* has reached its maximum value, but ωL is still rising.
- V_L will reach its maximum value after resonance.

V_R, V_L, and V_C

- Review
  - V_C and V_L are at their maximum values at or near resonance (depending on Q_S).
  - At very low frequencies, V_C is very close to the source voltage and V_L is very close to zero volts, whereas at very high frequencies, V_L approaches the source voltage and V_C approaches zero volts.
  - Both V_R and *I* peak at the resonance frequency and have the same shape.