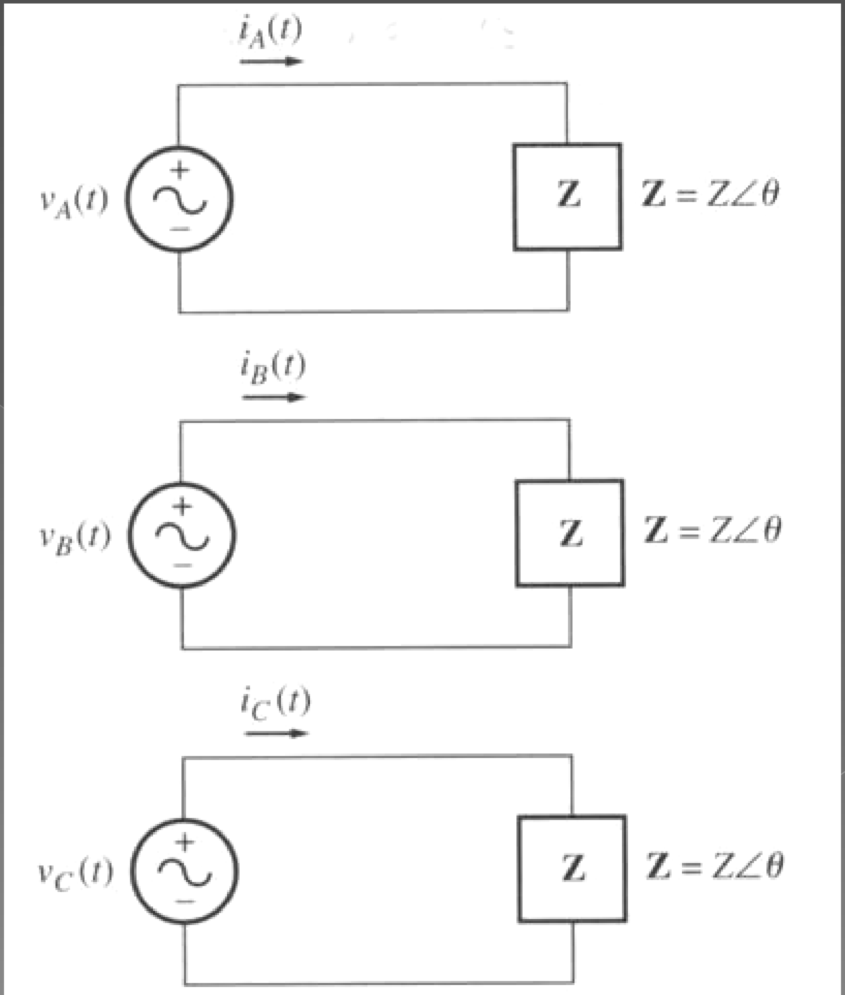
Generation of three-phase voltages and currents

Each of three-phase generators can be connected to one of three identical loads.

This way the system would consist of three single-phase circuits differing in phase angle by 120^0.

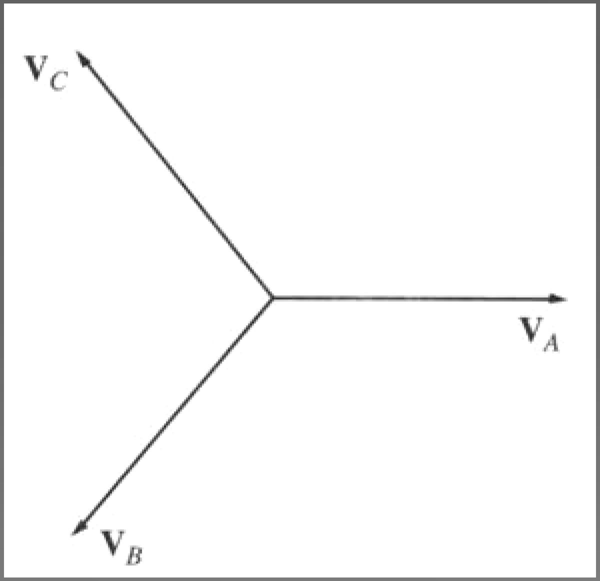
The current flowing to each load can be found as

$$I = \frac{V}{Z}$$



Generation of three-phase voltages and currents

Therefore, the currents flowing in each phase are



$$I_A = \frac{V\angle 0^0}{Z\angle\theta} = I\angle -\theta$$

$$I_B = \frac{V\angle -120^0}{Z\angle\theta} = I\angle -120-\theta$$

$$I_A = \frac{V\angle -240^0}{Z\angle\theta} = I\angle -240-\theta$$

Generation of three-phase voltages and currents

The current flowing through a neutral can be found as

$$I_N = I_A + I_B + I_C = I\angle -\theta + I\angle -\theta - 120^0 + I\angle -\theta - 240^0$$

$$= I\cos(-\theta) + jI\sin(-\theta) + I\cos(-\theta - 120^0) + jI\sin(-\theta - 120^0) + I\cos(-\theta - 240^0) + jI\sin(-\theta - 240^0)$$

$$= I\left[\cos(-\theta) + \cos(-\theta - 120^0) + \cos(-\theta - 240^0)\right] + jI\left[\sin(-\theta) + \sin(-\theta - 120^0) + \sin(-\theta - 240^0)\right]$$

$$= I\left[\cos(-\theta) + \cos(-\theta)\cos(120^0) + \sin(-\theta)\sin(120^0) + \cos(-\theta)\cos(240^0) + \sin(-\theta)\sin(240^0)\right]$$

$$+ jI\left[\sin(-\theta) + \sin(-\theta)\cos(120^0) - \cos(-\theta)\sin(120^0) + \sin(-\theta)\cos(240^0) - \cos(-\theta)\sin(240^0)\right]$$

Which is:

$$I_N = I\left[\cos(-\theta) - \frac{1}{2}\cos(-\theta) + \frac{\sqrt{3}}{2}\sin(-\theta) - \frac{1}{2}\cos(-\theta) - \frac{\sqrt{3}}{2}\sin(-\theta)\right]$$

$$+ jI\left[\sin(-\theta) - \frac{1}{2}\sin(-\theta) + \frac{\sqrt{3}}{2}\cos(-\theta) - \frac{1}{2}\sin(-\theta) - \frac{\sqrt{3}}{2}\cos(-\theta)\right]$$

$$= 0 \qquad (3.7.2)$$

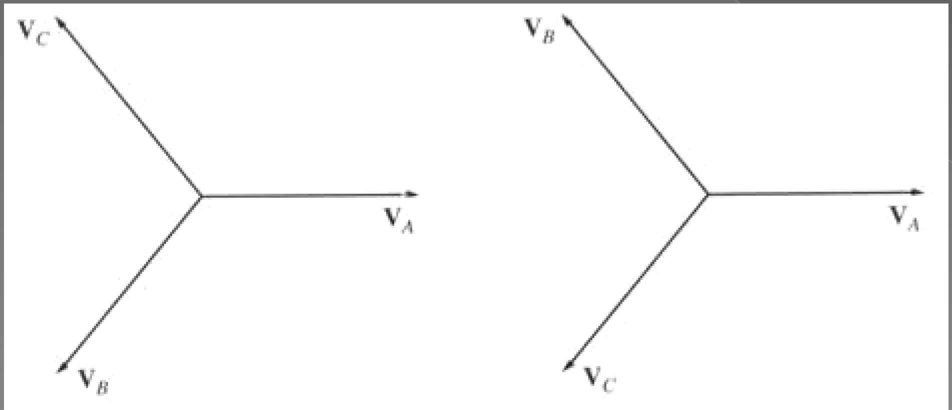
Generation of three-phase voltages and currents

As long as the three loads are equal, the return current in the neutral is zero!

Such three-phase power systems (equal magnitude, phase differences of 120^0, identical loads) are called balanced.

In a balanced system, the neutral is unnecessary!

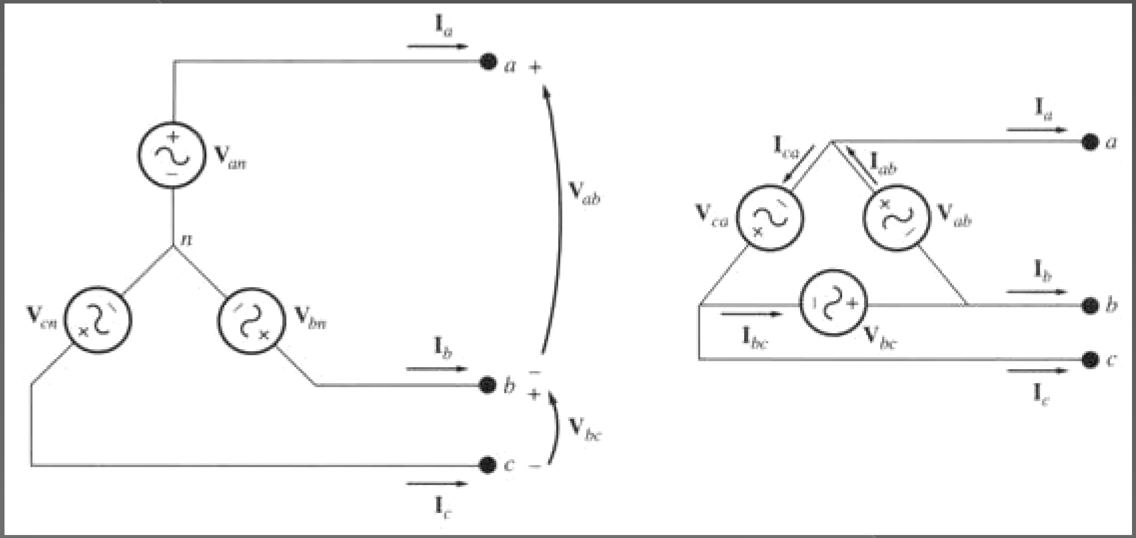
Phase Sequence is the order in which the voltages in the individual phases peak.



abc *acb*

Voltages and currents

There are two types of connections in three-phase circuits: Y and Δ.



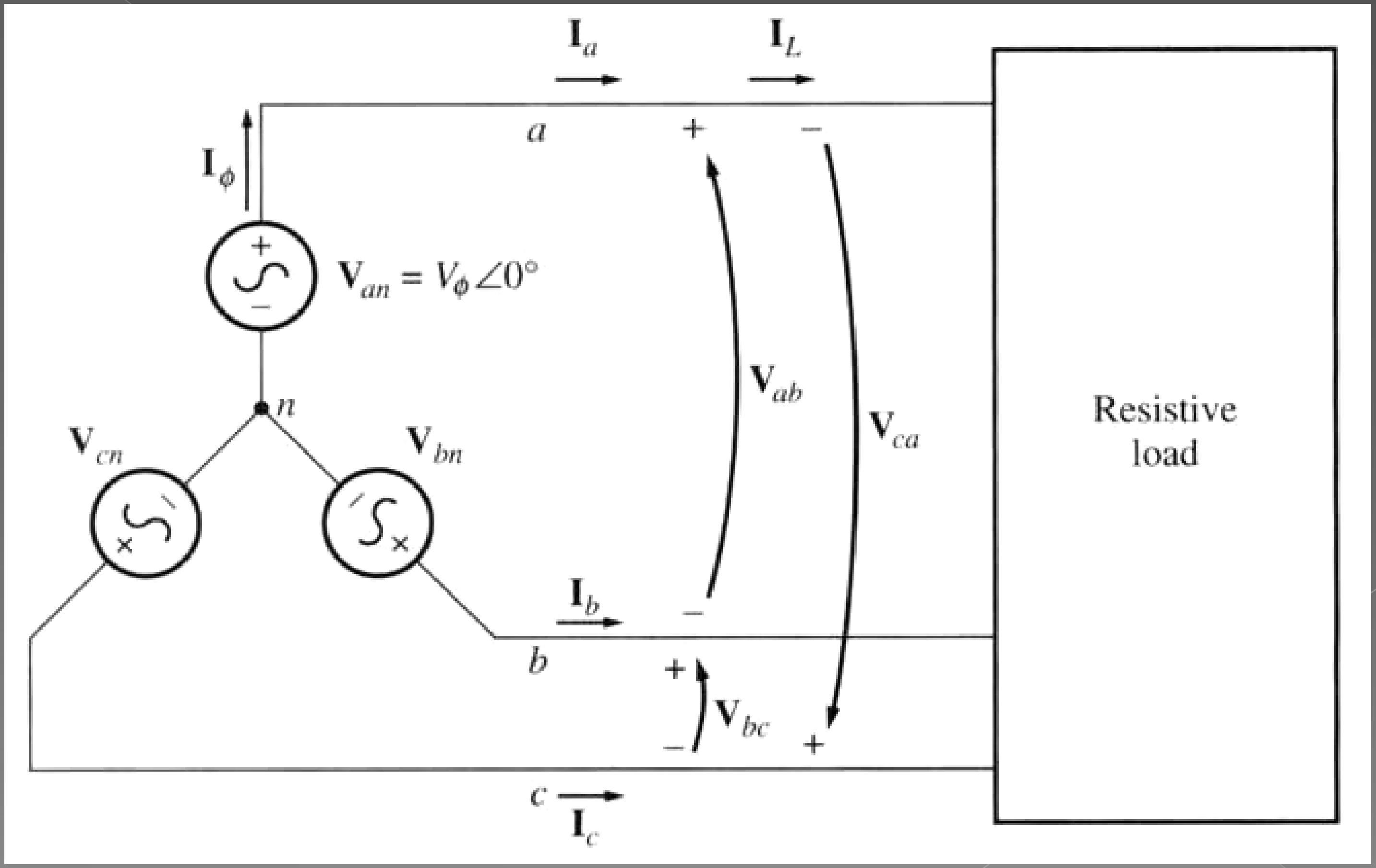
Each generator and each load can be either Y- or Δ-connected. Any number of Y- and Δ-connected elements may be mixed in a power system.

Phase quantities - voltages and currents in a given phase.
Line quantities – voltages between the lines and currents in the lines connected to the generators.

Voltages and currents

Y-connection

Assuming a resistive load…



Voltages and currents

Y-connection (cont)

$$V_{an} = V_{\phi} \angle 0^0$$

$$V_{bn} = V_{\phi} \angle -120^0$$

$$V_{cn} = V_{\phi} \angle -240^0$$

Since we assume a resistive load:

$$I_a = I_{\phi} \angle 0^0$$

$$I_b = I_{\phi} \angle -120^0$$

$$I_c = I_{\phi} \angle -240^0$$

Voltages and currents

Y-connection (cont. 2)

The current in any line is the same as the current in the corresponding phase.

$$I_L = I_\phi$$

Voltages are:

$$V_{ab} = V_a - V_b = V_\phi \angle 0^0 - V_\phi \angle -120^0 = V_\phi - \left(-\frac{1}{2} V_\phi - j \frac{\sqrt{3}}{2} V_\phi \right) = \frac{3}{2} V_\phi + j \frac{\sqrt{3}}{2} V_\phi$$

$$= \sqrt{3} V_\phi \left(\frac{\sqrt{3}}{2} + j \frac{1}{2} \right) = \sqrt{3} V_\phi \angle 30^0$$

Voltages and currents

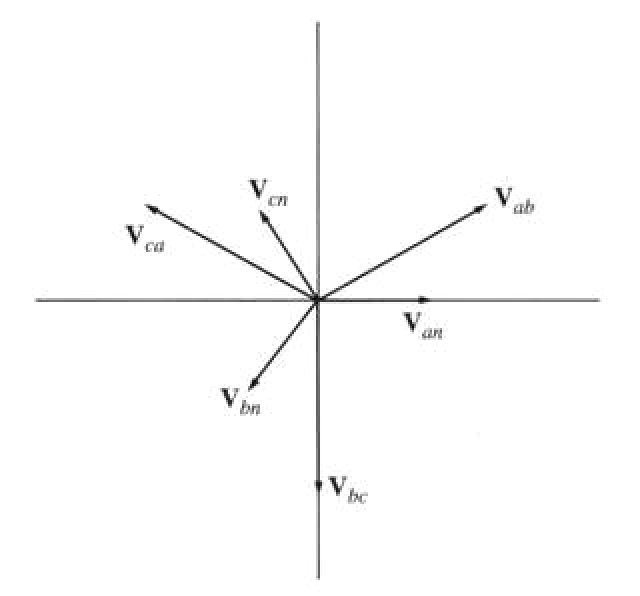
Y-connection (cont. 3)

Magnitudes of the line-to-line voltages and the line-to-neutral voltages are related as:

$$V_{LL} = \sqrt{3} V_{\phi}$$

In addition, the line voltages are shifted by 30^0 with respect to the phase voltages.

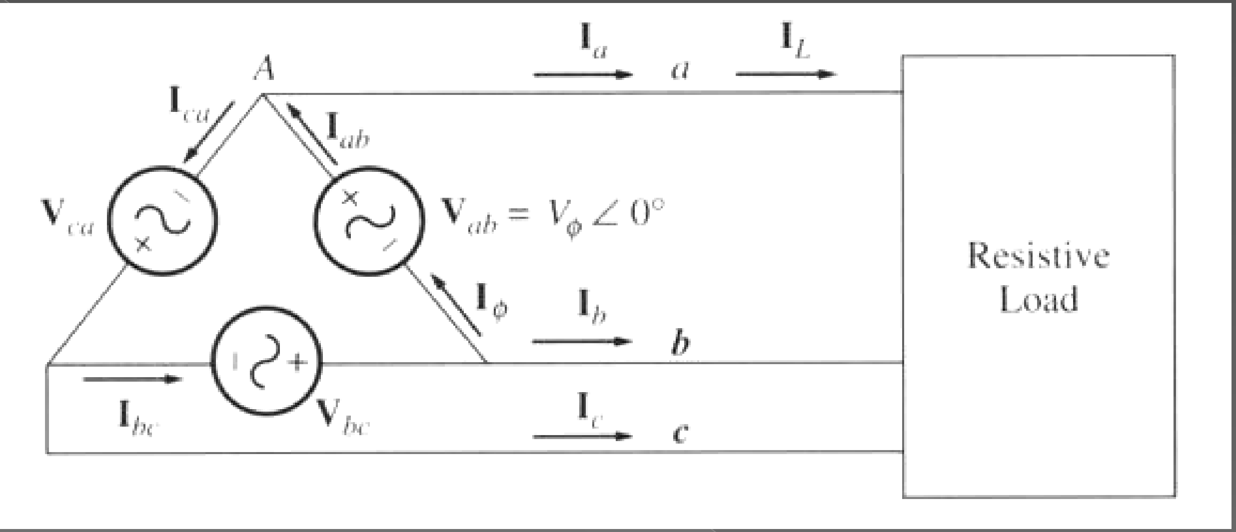
In a connection with *abc* sequence, the voltage of a line **leads** the phase voltage.



Voltages and currents

Δ-connection

assuming a resistive load:



$$V_{ab} = V_\phi \angle 0^0$$
$$V_{bc} = V_\phi \angle -120^0 \qquad (3.14.1)$$
$$V_{ca} = V_\phi \angle -240^0$$

$$I_{ab} = I_\phi \angle 0^0$$
$$I_{bc} = I_\phi \angle -120^0$$
$$I_{ca} = I_\phi \angle -240^0$$

Voltages and currents

Δ-connection (cont.)

$$V_{LL} = V_{\phi}$$

The currents are:

$$I_a = I_{ab} - I_{ca} = I_{\phi}\angle 0^0 - I_{\phi}\angle 240^0 = I_{\phi} - \left(-\frac{1}{2}I_{\phi} + j\frac{\sqrt{3}}{2}I_{\phi}\right)$$

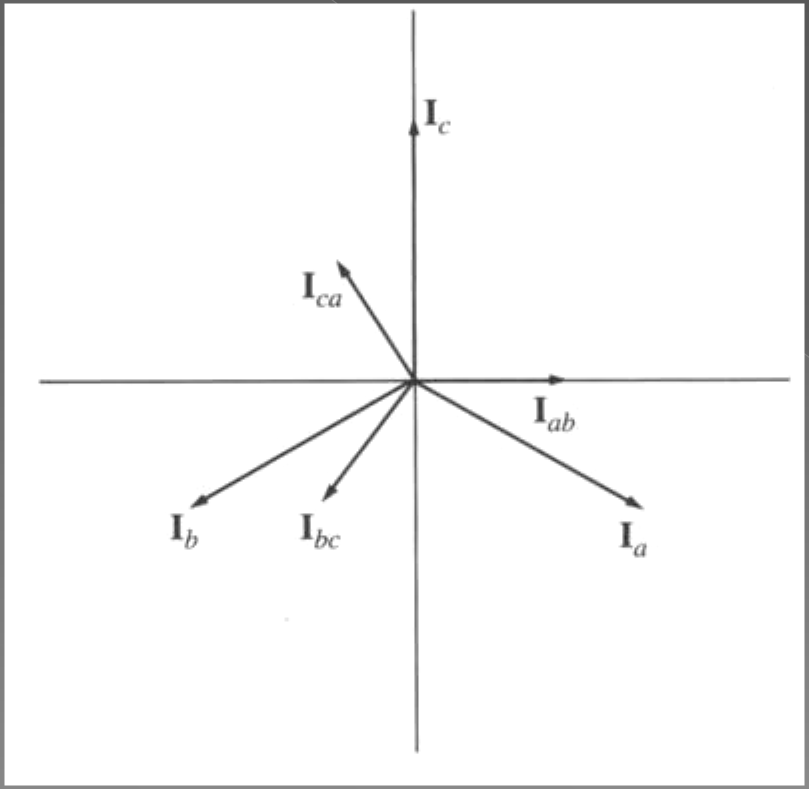
$$= \frac{3}{2}I_{\phi} - j\frac{\sqrt{3}}{2}I_{\phi} = \sqrt{3}I_{\phi}\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) = \sqrt{3}I_{\phi}\angle -30^0$$

The magnitudes:

$$I_L = \sqrt{3}I_{\phi}$$

Voltages and currents

For the connections with the *abc* phase sequences, the current of a line **lags** the corresponding phase current by 30^0 (see Figure below).



For the connections with the *acb* phase sequences, the line current **leads** the corresponding phase current by 30^0.

Power relationships

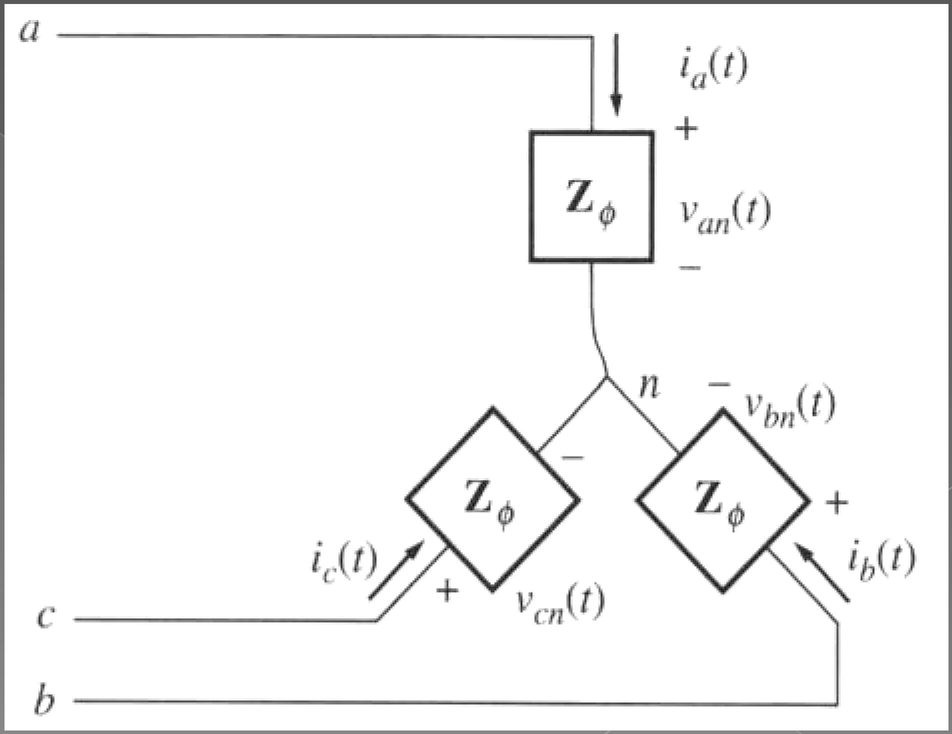
For a balanced Y-connected load with the impedance $Z_\phi = Z\angle\theta^0$:

and voltages:

$$v_{an}(t) = \sqrt{2}V \sin \omega t$$
$$v_{bn}(t) = \sqrt{2}V \sin(\omega t - 120^0)$$
$$v_{cn}(t) = \sqrt{2}V \sin(\omega t - 240^0) \quad (3.17.1)$$

The currents can be found:

$$i_a(t) = \sqrt{2}I \sin(\omega t - \theta)$$
$$i_b(t) = \sqrt{2}I \sin(\omega t - 120^0 - \theta)$$
$$i_c(t) = \sqrt{2}I \sin(\omega t - 240^0 - \theta) \quad (3.17.2)$$



Power relationships

The instantaneous power is:

$$p(t) = v(t)i(t)$$

Therefore, the instantaneous power supplied to each phase is:

$$p_a(t) = v_{an}(t)i_a(t) = 2VI\sin(\omega t)\sin(\omega t - \theta)$$
$$p_b(t) = v_{bn}(t)i_b(t) = 2VI\sin(\omega t - 120^0)\sin(\omega t - 120^0 - \theta)$$
$$p_c(t) = v_{cn}(t)i_c(t) = 2VI\sin(\omega t - 240^0)\sin(\omega t - 240^0 - \theta)$$

Since

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$$

Power relationships

Therefore

$$p_a(t) = VI\left[\cos\theta - \cos(2\omega t - \theta)\right]$$

$$p_b(t) = VI\left[\cos\theta - \cos(2\omega t - 240^0 - \theta)\right]$$

$$p_c(t) = VI\left[\cos\theta - \cos(2\omega t - 480^0 - \theta)\right]$$

The total power on the load

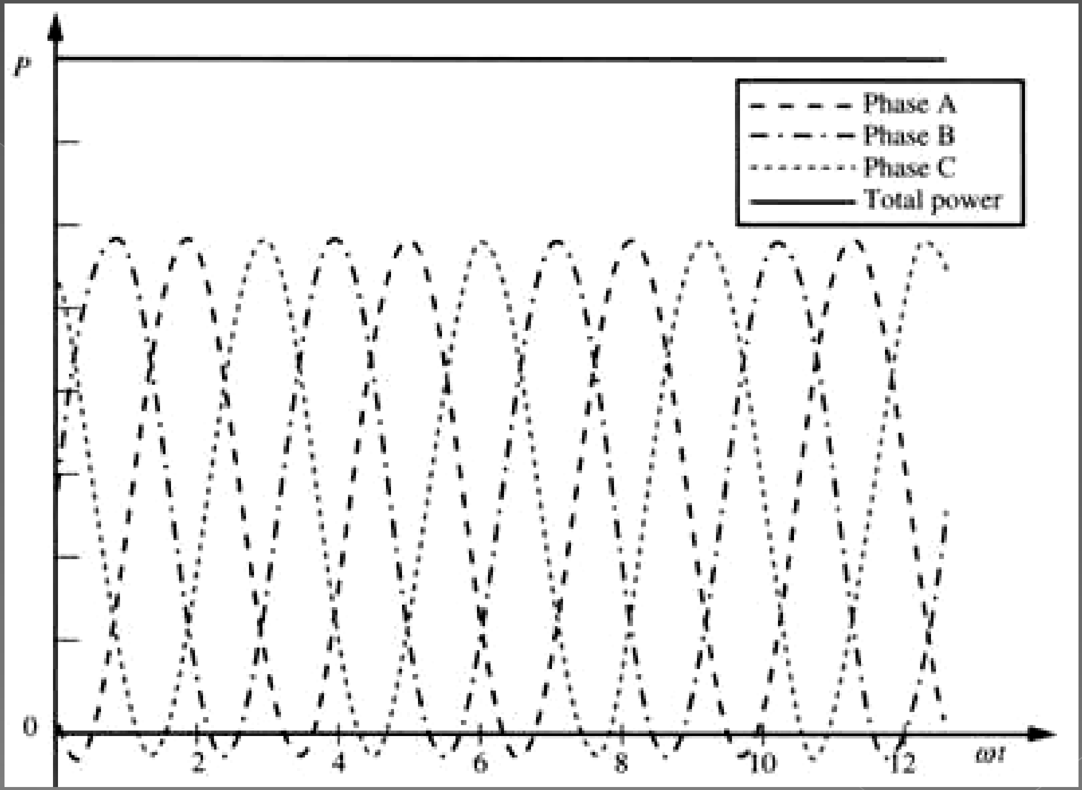
$$p_{tot}(t) = p_a(t) + p_b(t) + p_c(t) = 3VI\cos\theta$$

The pulsing components cancel each other because of 120^0 phase shifts.

Power relationships

The instantaneous power in phases.

The total power supplied to the load is constant.



Power relationships

Phase quantities in each phase of a Y- or Δ-connection.

Real $$P = 3V_\phi I_\phi \cos\theta = 3I_\phi^{\ 2} Z \cos\theta$$

Reactive $$Q = 3V_\phi I_\phi \sin\theta = 3I_\phi^{\ 2} Z \sin\theta$$

Apparent $$S = 3V_\phi I_\phi = 3I_\phi^{\ 2} Z$$

Note: these equations are valid for balanced loads only.

Power relationships

Line quantities: Y-connection.

Power consumed by a load: $P = 3V_{\phi} I_{\phi} \cos\theta$

Since for this load $I_L = I_{\phi} \;\; and \;\; V_{LL} = \sqrt{3} V_{\phi}$

Therefore: $P = 3\frac{V_{LL}}{\sqrt{3}} I_L \cos\theta$

Finally: $P = \sqrt{3} V_{LL} I_L \cos\theta$

Note: these equations are valid for balanced loads only.

Power relationships

Line quantities: Δ-connection.

Power consumed by a load: $P = 3V_\phi I_\phi \cos\theta$

Since for this load $I_L = \sqrt{3} I_\phi \;\; and \;\; V_{LL} = V_\phi$

Therefore: $P = 3\dfrac{I_L}{\sqrt{3}} V_{LL} \cos\theta$

Finally: $P = \sqrt{3} V_{LL} I_L \cos\theta$

Same as for a Y-connected load!

Note: these equations were derived for a balanced load.

Power relationships

Line quantities: Y- and Δ-connection.

Reactive power

$$Q = \sqrt{3} V_{LL} I_L \sin\theta$$

Apparent power

$$S = \sqrt{3} V_{LL} I_L$$

Note: θ is the angle between the **phase voltage** and the **phase current – the impedance angle.**