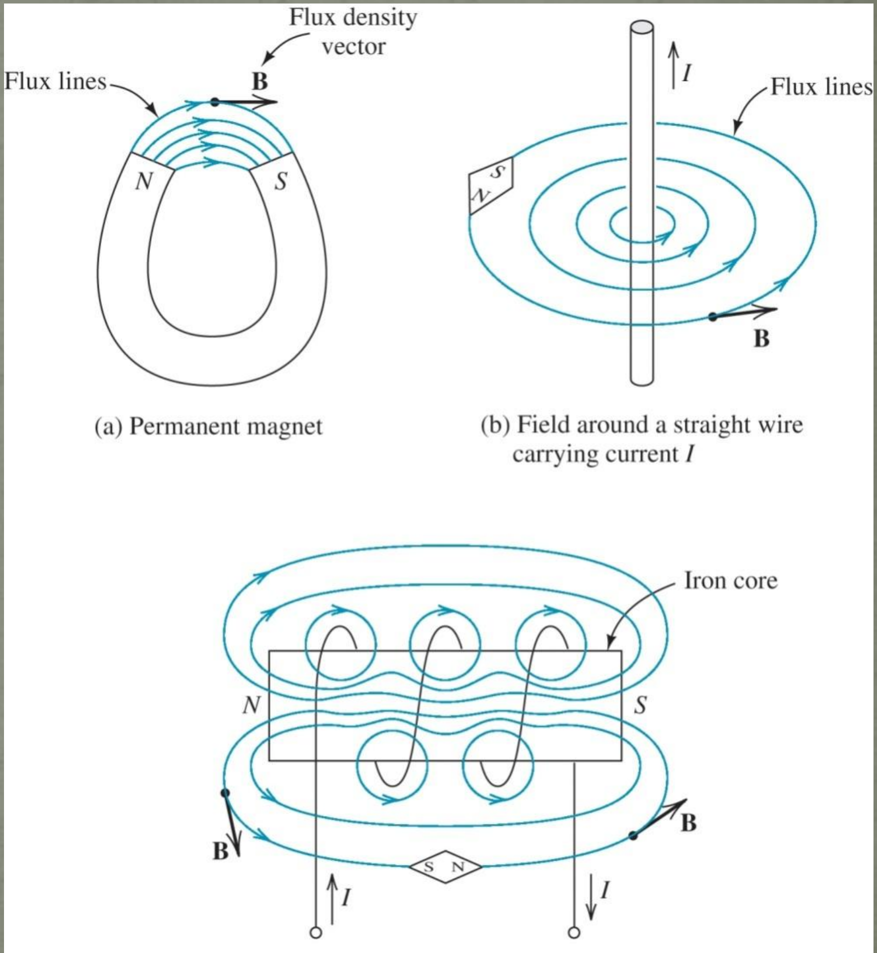
Magnetic Fields

Magnetic fields can be visualized as lines of flux that form closed paths.

Using a compass, we can determine the direction of the flux lines at any point.

Note that the **flux density vector B** is tangent to the lines of flux.

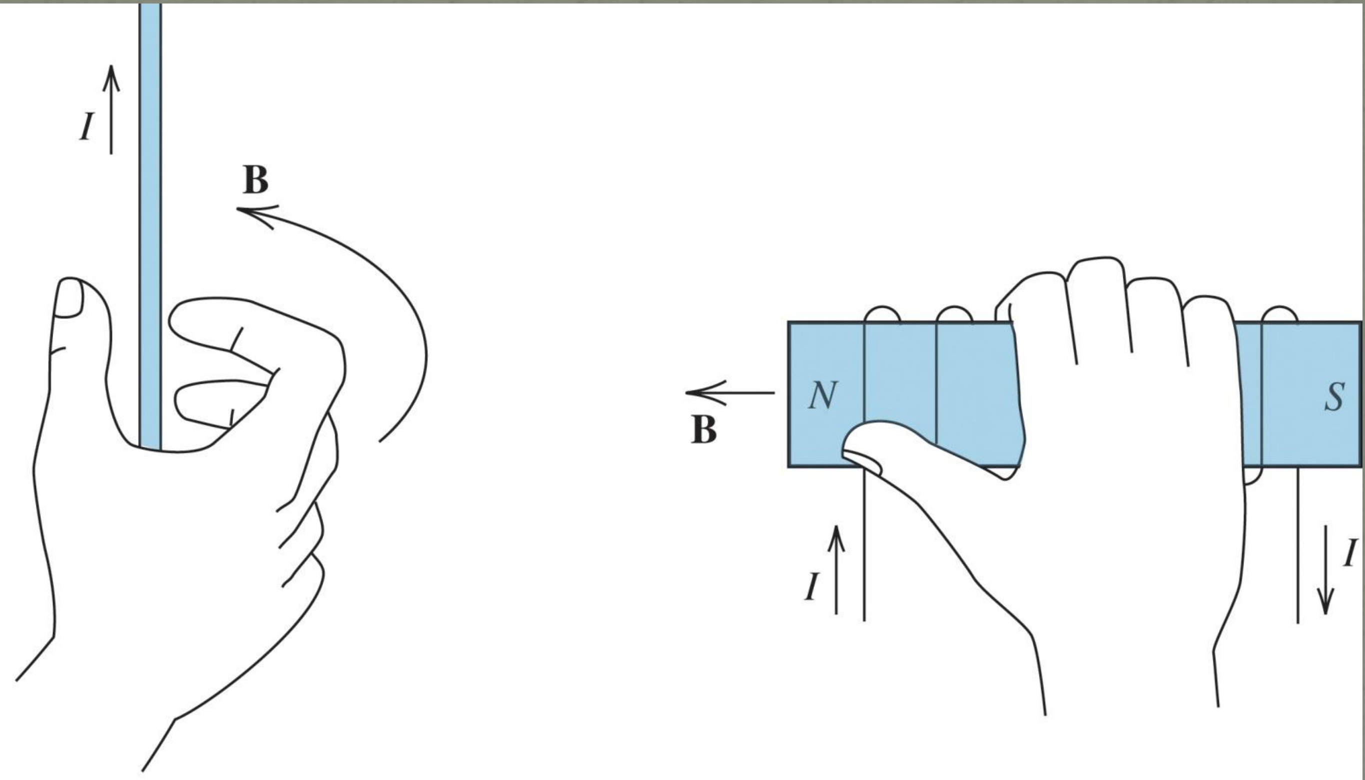


(a) Permanent magnet

(b) Field around a straight wire carrying current I

(c) Field for a coil of wire

Illustrations of the right-hand rule

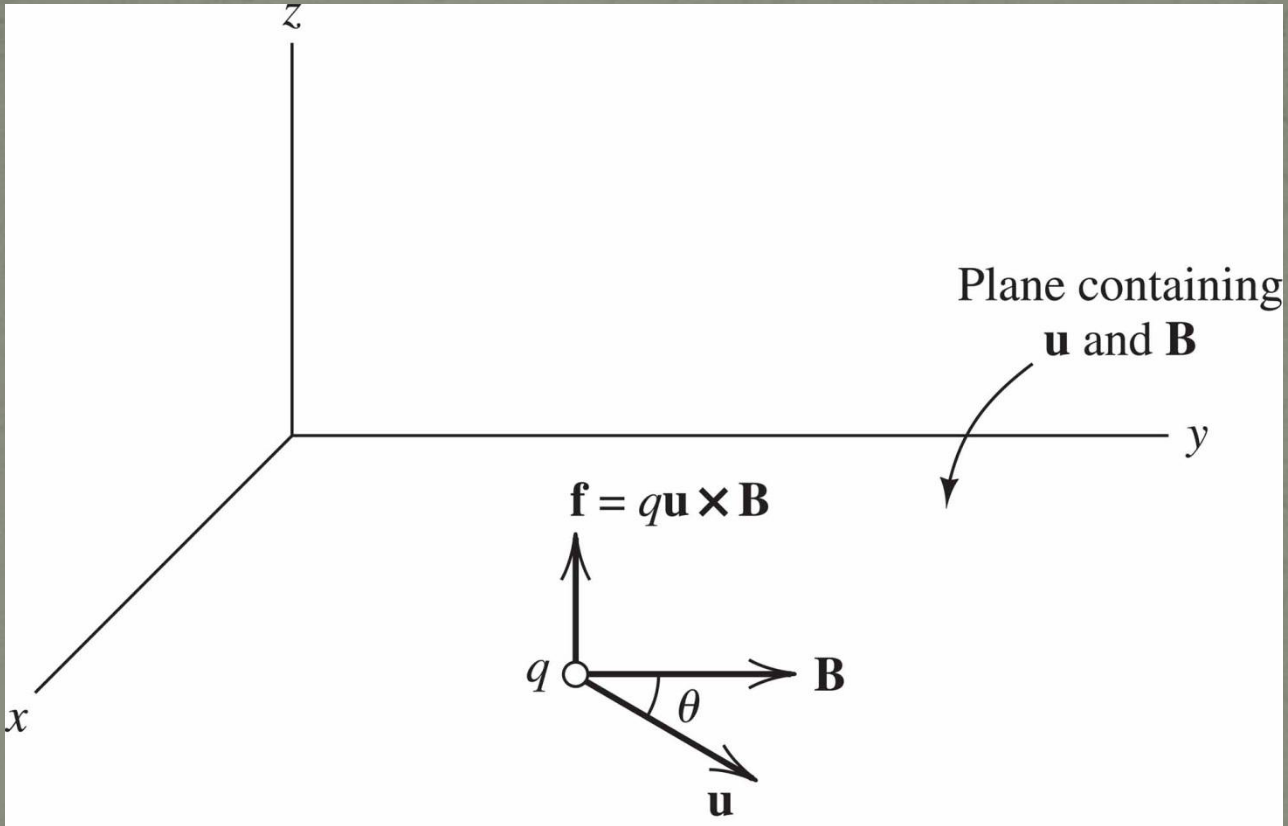


(a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field

(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

Force on Moving Electric Charge

A charge moving through a magnetic field experiences a force **f** perpendicular to both the velocity **u** and flux density **B**.



Force on Moving Electric Charge

A charge q moving through a magnetic field experiences a force **f** perpendicular to both the velocity **u** and flux density **B**.

$$f = qu \times B$$

where u is the velocity vector and B is a magnetic field.

The magnitude of this force is

$$|f| = quB\sin(\theta)$$

Current that flows through a conductor are electron charges in motion so the force acting on the wire with current in the magnetic field is

and in the straight wire of the length l crossing the field under angle

$$df = i\ dl \times B$$

$$f = i\ l\ B \sin(\theta)$$

Flux Linkage and Induced Voltage

When the flux linking a coil changes, a voltage is induced in the coil.

The polarity of the voltage is such that if a circuit is formed by placing a resistance across the coil terminals, the resulting current produces a field that tends to oppose the original change in the field.

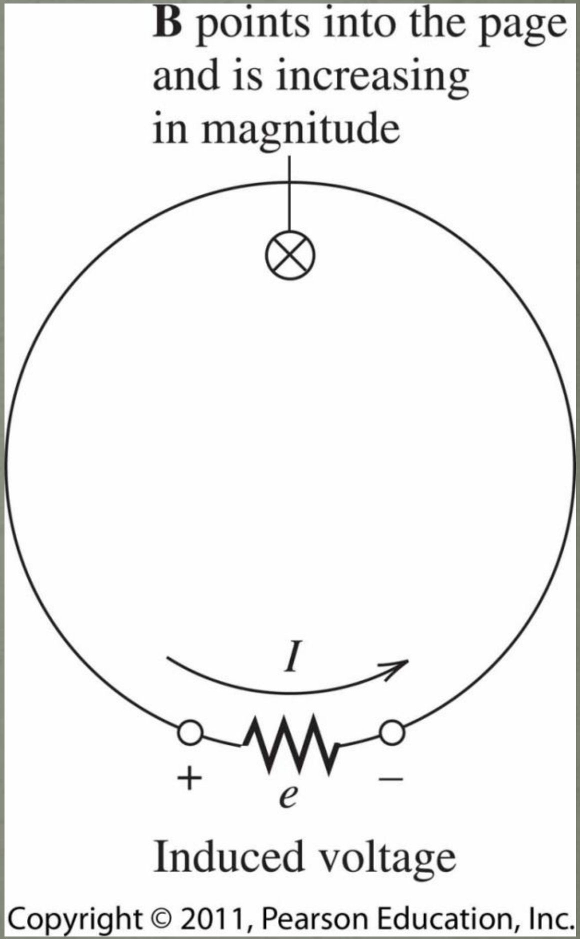
Faraday Law of magnetic induction: voltage **e** induced by the flux changes is

$$e = \frac{d\lambda}{dt}$$

where **total flux linkage** λ is

$$\lambda = N\phi = N\int_A B\,dA$$

N-number of turns, ϕ magnetic flux passing through the surface area **A**, and **B** is the magnetic field



Induced Voltage in a Moving Conductor

A voltage is also induced in a conductor moving through a magnetic field in the direction such that the conductor cuts through magnetic flux lines.
The flux linkage of the coil is (with uniform magnetic field B)

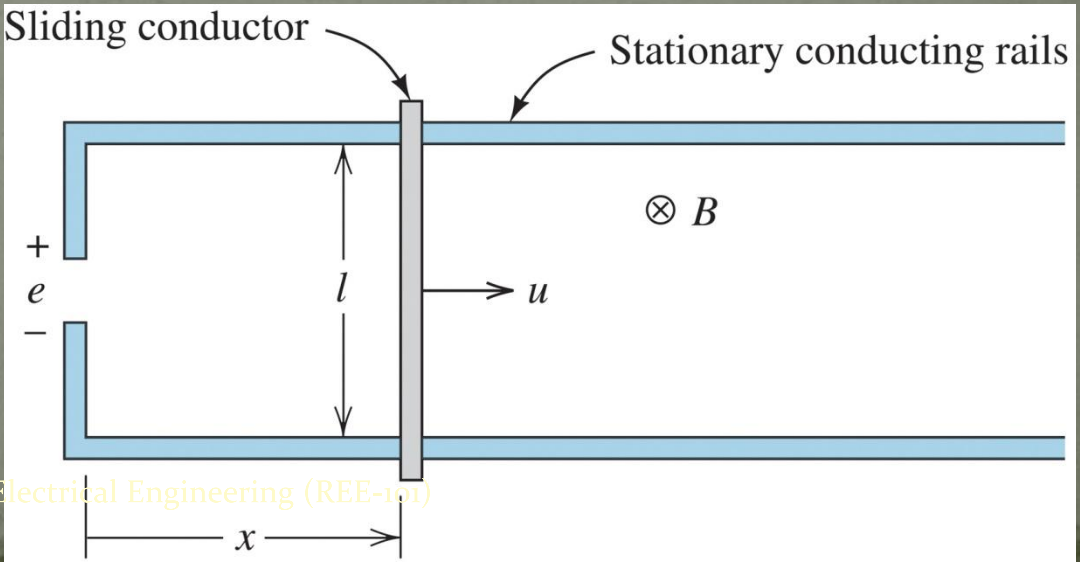
$$\lambda = \phi = BA$$

so according to Faraday's law the voltage induced in the coil is

$$e = \frac{d\lambda}{dt} = Bl\frac{dx}{dt} = Blu$$

where

$$u = \frac{dx}{dt}$$



Ampère's Law

Ampère's law (generalization of Kirchhoff's law) states that the line integral of **magnetic field intensity H** around a closed path is equal to the sum of the currents flowing through the surface bounded by the path.

$$\oint H \cdot dl = \sum i$$

where magnetic field intensity H is related to flux density B and **magnetic permeability**

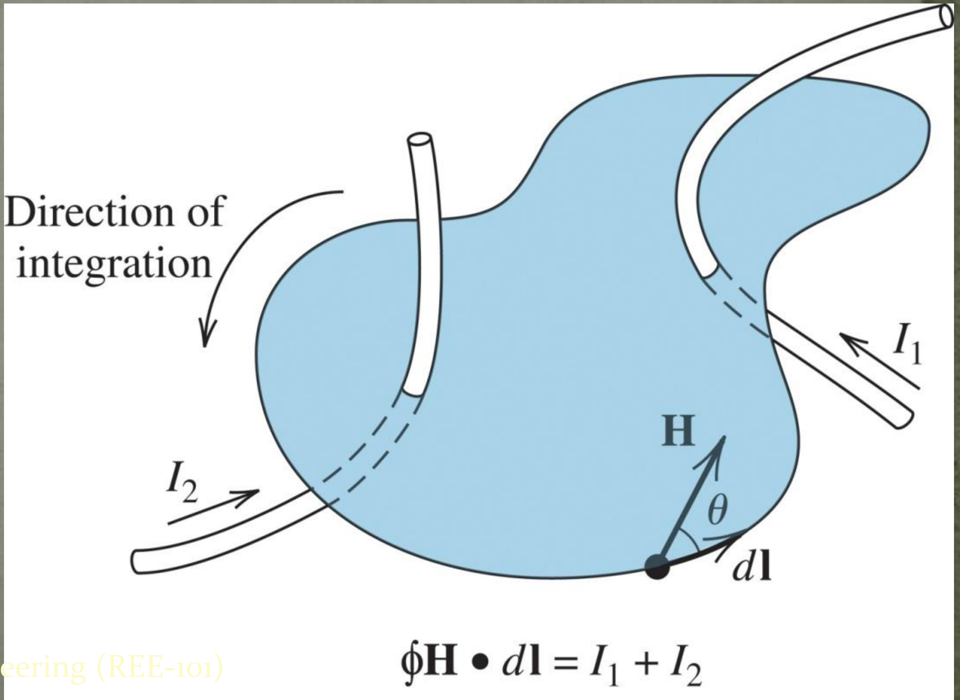
$$\mu$$

since $$H = \frac{B}{\mu}\left[\frac{A}{m}\right]$$

so if H and dl point in the same direction

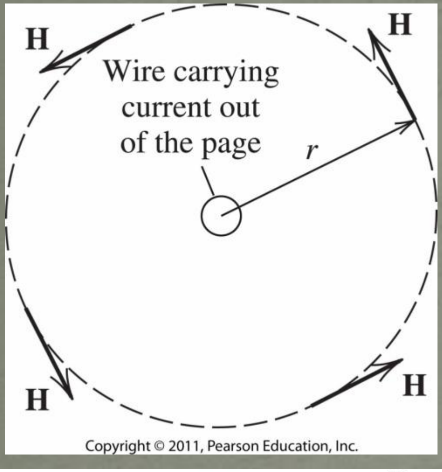
$$H \cdot dl = H\ dl \cos(\theta)$$

$$H\ l = \sum i$$



Ampère's Law

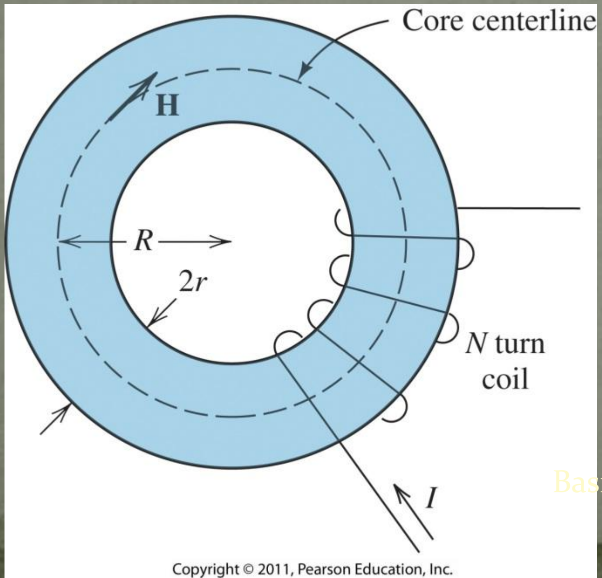
The magnetic field around a long straight wire carrying a current can be determined with Ampère's law aided by considerations of symmetry.



$$H\,l = H\,2\pi r = I$$

So the magnetic flux density

$$B = \mu H = \frac{\mu I}{2\pi r} \qquad (*)$$



Using Ampère's law in the toroidal coil, filed intensity is

$$H\,l = H\,2\pi R = NI$$

Using (*) we get inside the toroidal coil:

$$B = \frac{\mu NI}{2\pi R}$$

Reluctance of a Magnetic Path

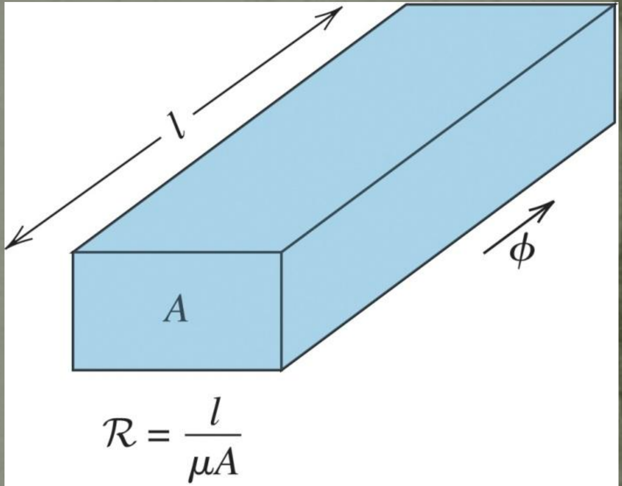
Magnetic circuits are analogue of electrical circuits.

The **magnetomotive force** of N-turn current carrying coil is

$$F = Ni$$

The **reluctance R** of a magnetic path depends on the mean length *l*, the area *A*, and the **permeability** *μ* of the material.

$$\mathrm{R} = \frac{l}{\mu A}$$



Magnetic flux is analogous to current in electrical circuit and is related to F and R in a similar way as Ohm's law

$$F = \mathrm{R}\,\phi$$

Magnetic Circuits

The magnetic circuit for the toroidal coil can be analyzed to obtain an expression for flux.

Magnetomotive force is

$$F = NI = \mathcal{R}\phi$$

Where the reluctance is

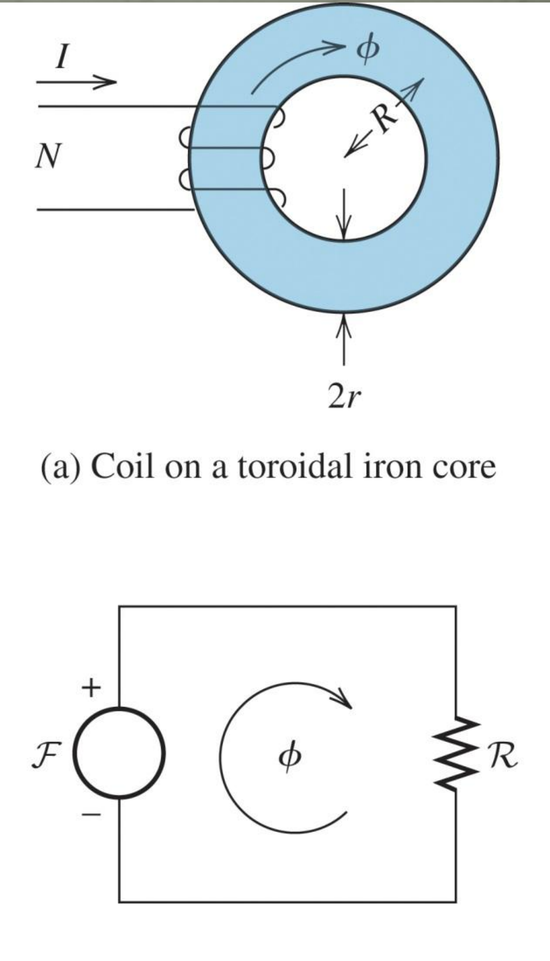
$$\mathcal{R} = \frac{l}{\mu A} = \frac{2\pi R}{\mu \pi r^2} = \frac{2R}{\mu r^2}$$

so

$$NI = \frac{2R}{\mu r^2}\phi$$

and the magnetic flux is

$$NI = \frac{2R}{\mu r^2}\phi \quad \text{so} \quad \phi = \frac{NI\mu r^2}{2R}$$



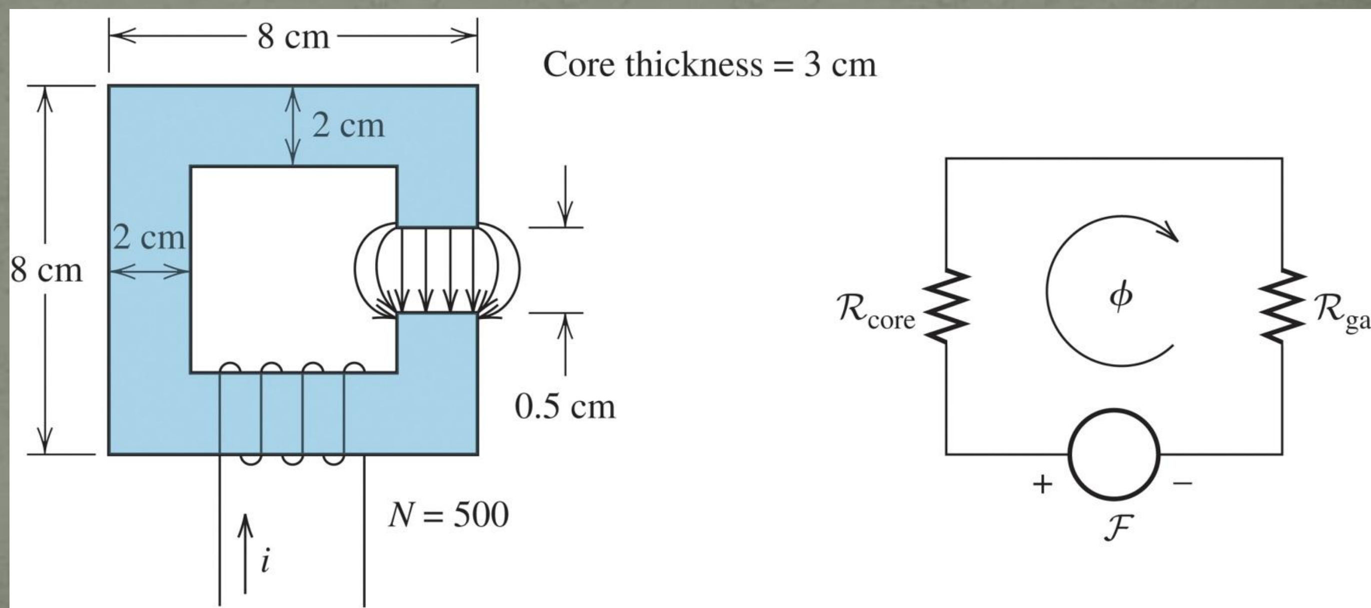
(a) Coil on a toroidal iron core

(b) Magnetic circuit

Magnetic Circuits

Example

Magnetic circuit below relative permeability of the core material is 6000 its rectangular cross section is 2 cm by 3 cm. The coil has 500 turns. Find the current needed to establish a flux density in the gap of B_{gap}=0.25 T.



(a) Iron core with an air gap (b) Magnetic circuit

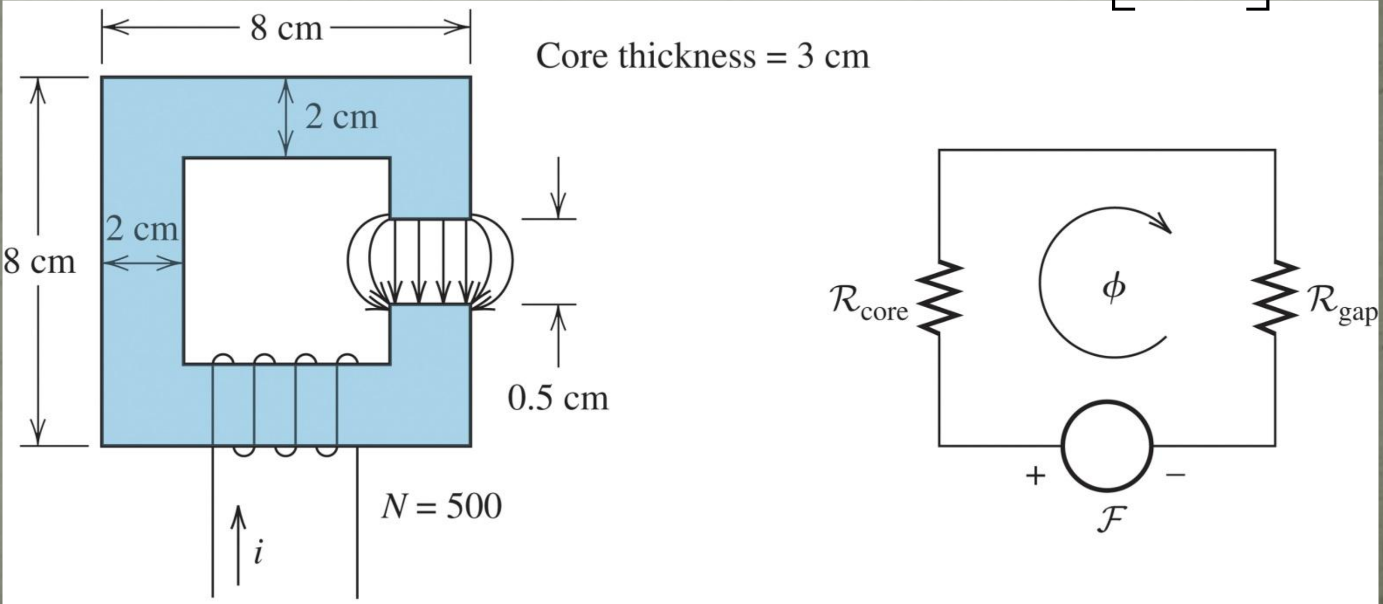
Magnetic Circuits

Example

Magnetic circuit below relative permeability of the core material is 6000 its rectangular cross section is 2 cm by 3 cm. The coil has 500 turns. Find the current needed to establish a flux density in the gap of B_{gap}=0.25 T.

Medium length of the magnetic path in the core is l_{core}=4*6-0.5=23.5cm, and
the cross section area is A_{core}= 2cm*3cm = $6*10^{-4}$ m^2
the core permeability is

$$\mu_{core} = \mu_r \mu_0 = 6000 \times 4\pi \times 10^{-7} = 7.54 \times 10^{-3} \left[\frac{Wb}{Am} \right]$$



(a) Iron core with an air gap (b) Magnetic circuit

Magnetic Circuits

Example.

The core reluctance is

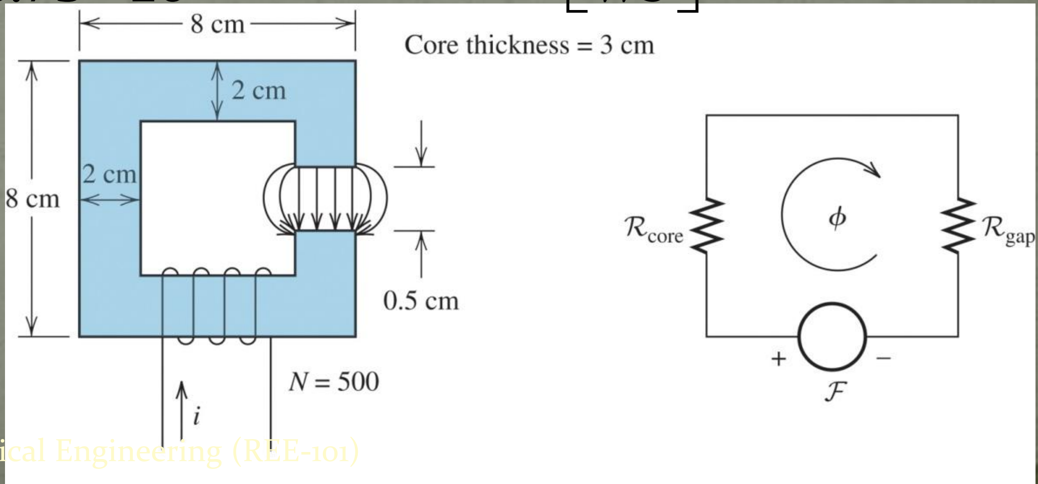
$$\mathrm{R}_{core} = \frac{l_{core}}{\mu_{core} A_{core}} = \frac{23.5\times10^{-2}}{7.54\times10^{-3}\times6\times10^{-4}} = 5.195\times10^{4}\left[\frac{A}{Wb}\right]$$

the gap area is computed by adding the gap length to each dimension of cross-section:

$$A_{gap} = (2cm + 0.5cm)\times(3cm + 0.5cm) = 8.75\times10^{4}\left[m^{2}\right]$$

thus the gap reluctance is:

$$\mathrm{R}_{gap} = \frac{l_{gap}}{\mu_0 A_{gap}} = \frac{0.5\times10^{-2}}{4\pi\times10^{-7}\times8.75\times10^{-4}} = 4.547\times10^{6}\left[\frac{A}{Wb}\right]$$



(a) Iron core with an air gap (b) Magnetic circuit