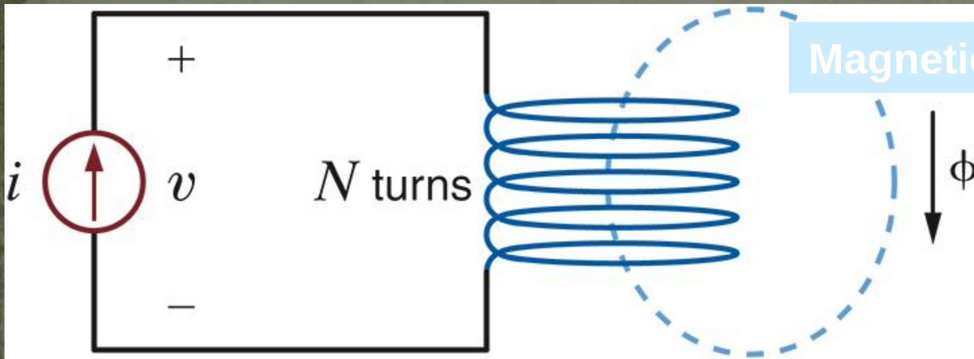
Mutual coupling with dot convention,

BASIC CONCEPTS – A REVIEW



Magnetic field

$$\lambda = N\phi$$

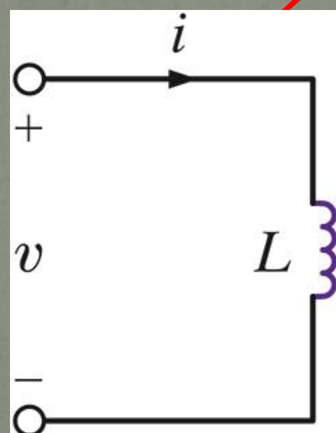
Total magnetic flux linked by N-turn coil

$$\lambda = Li$$

Ampere's Law (linear model)

$$v = \frac{d\lambda}{dt}$$

Faraday's Induction Law



Ideal Inductor

$$v = L\frac{di}{dt}$$

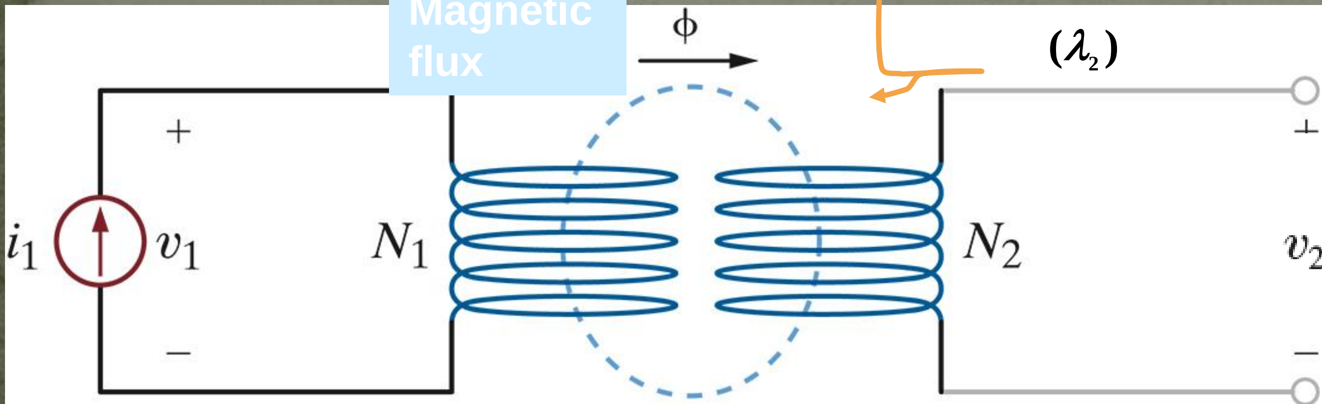
Assumes constant L and linear models!

MUTUAL INDUCTANCE

Overview of Induction Laws

Magnetic flux

Induced links on second



$$v_2 = \frac{d\lambda_2}{dt} = \frac{d}{dt}(N_2\phi)$$

$$= \frac{d}{dt}\left(N_2\left(\frac{L_1}{N_1}i_1\right)\right) = \frac{N_2}{N_1}L_1\frac{di_1}{dt} = L_{21}\frac{di_1}{dt}$$

Total flux linkage

$\lambda = N\phi$ (webers)

If linkage is created by a current flowing through the coils…

$\lambda = Li$ (Ampere's Law)

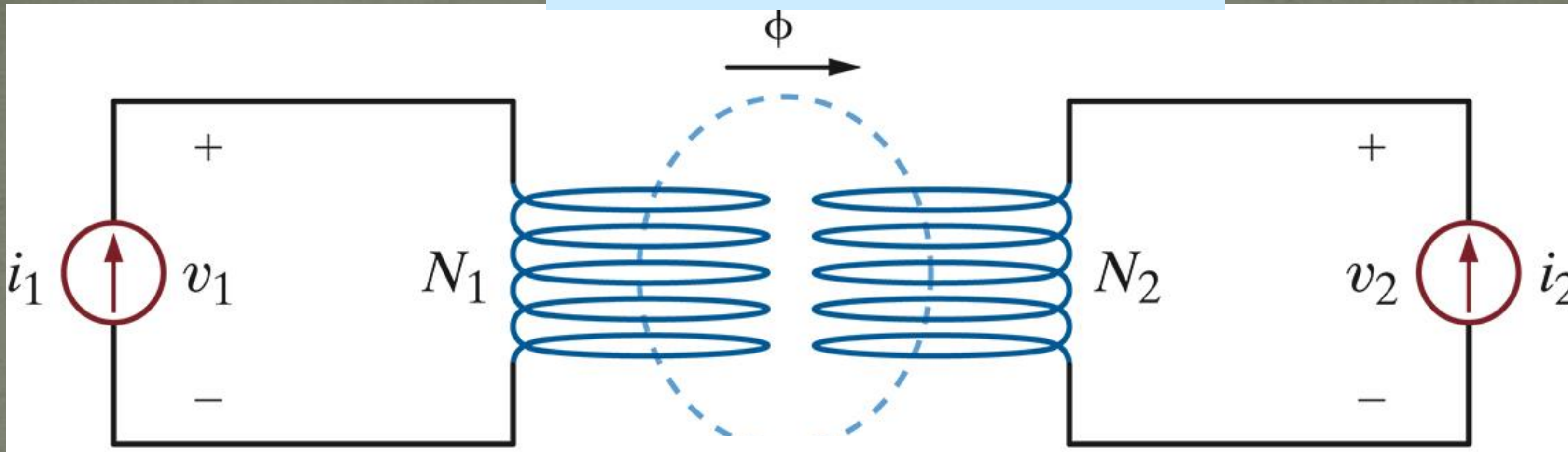
The voltage created at the terminals of the components is

$v = L\frac{di}{dt}$ (Faraday's Induction Law)

One has the effect of mutual inductance

TWO-COIL SYSTEM

(both currents contribute to flux)



$$\lambda_1 = L_1 i_1 + L_{12} i_2$$

$$\lambda_2 = L_{21} i_1 + L_2 i_2$$

$$v_1 = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$

$$v_2 = \frac{d\lambda_2}{dt} = L_{21} \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Self-induced

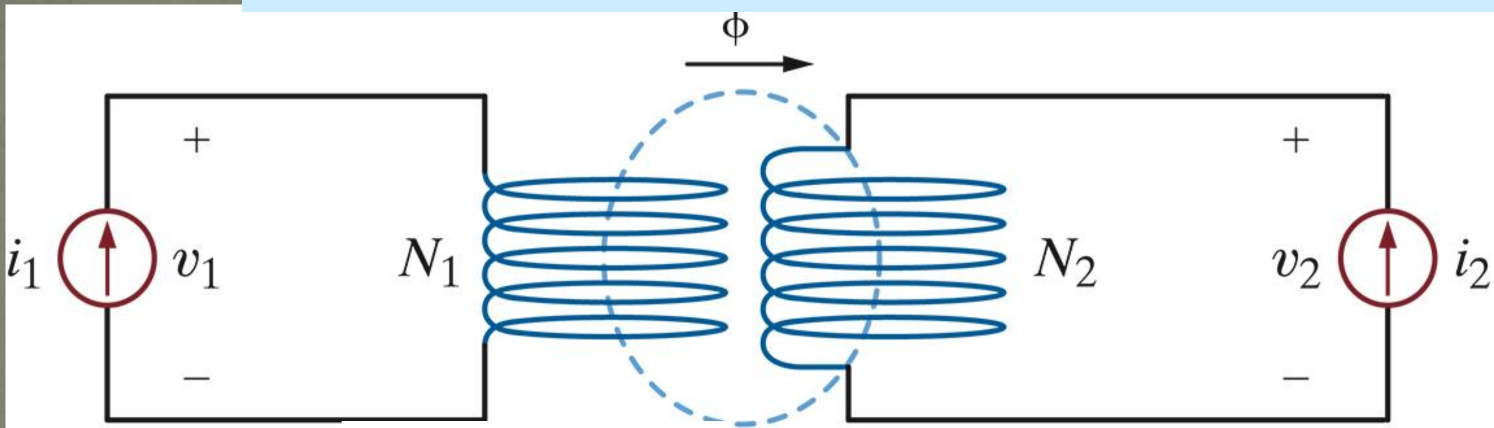
Mutual-induced

$$L_{12} = L_{21} = M$$

Linear model simplifying notation

THE 'DOT' CONVENTION

COUPLED COILS WITH DIFFERENT WINDING CONFIGURATION

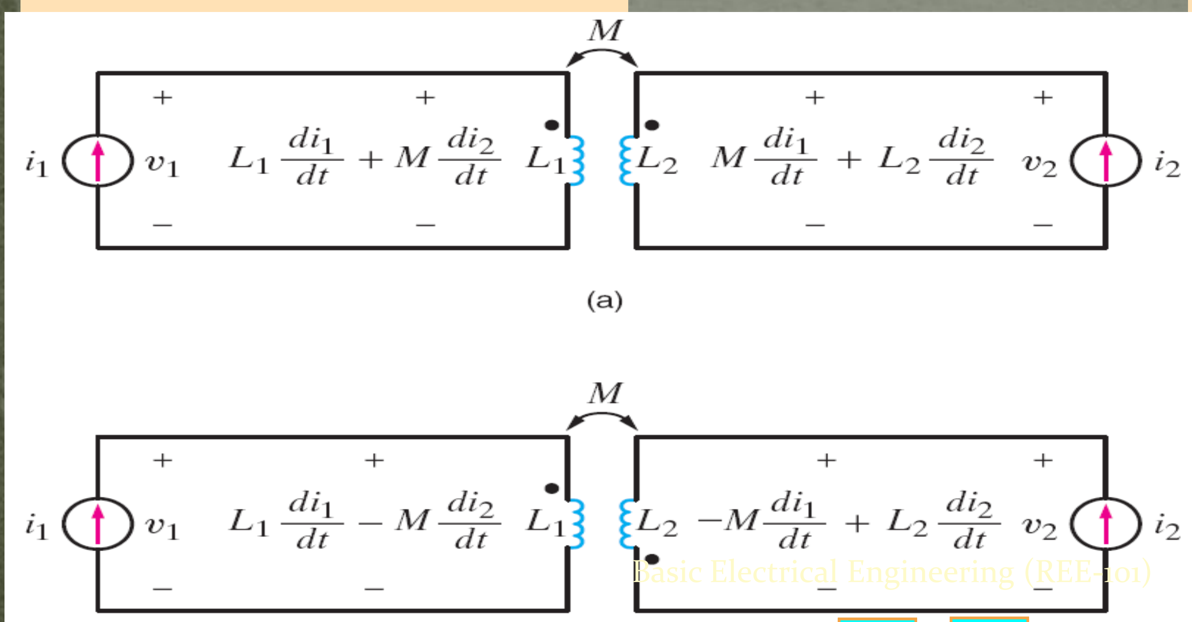


$$\lambda_1 = L_1 i_1 - M i_2$$

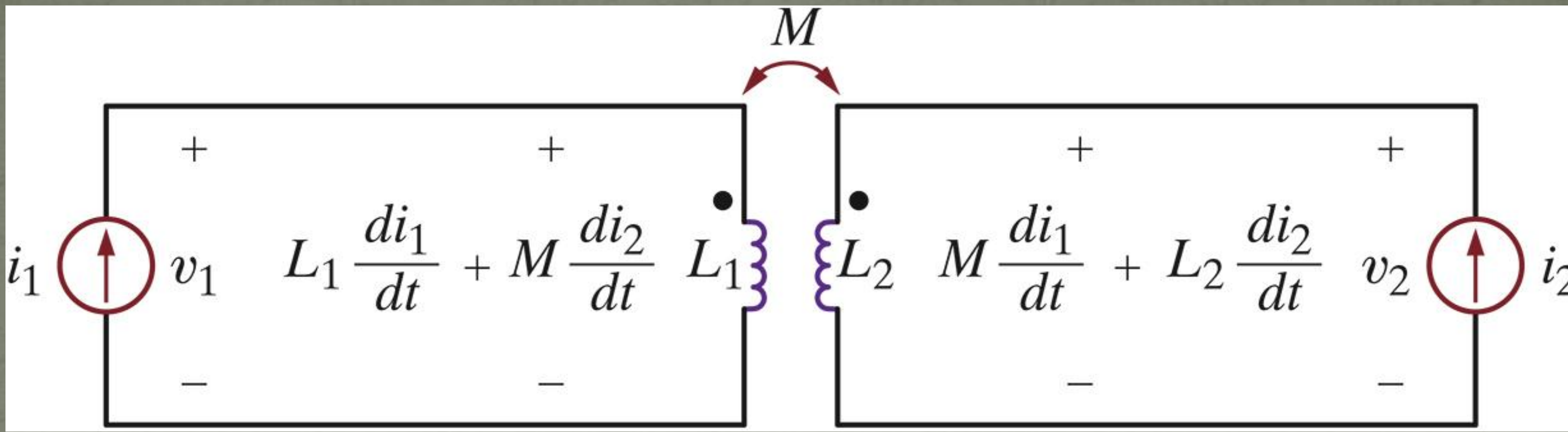
$$\lambda_2 = -M i_1 + L_2 i_2$$

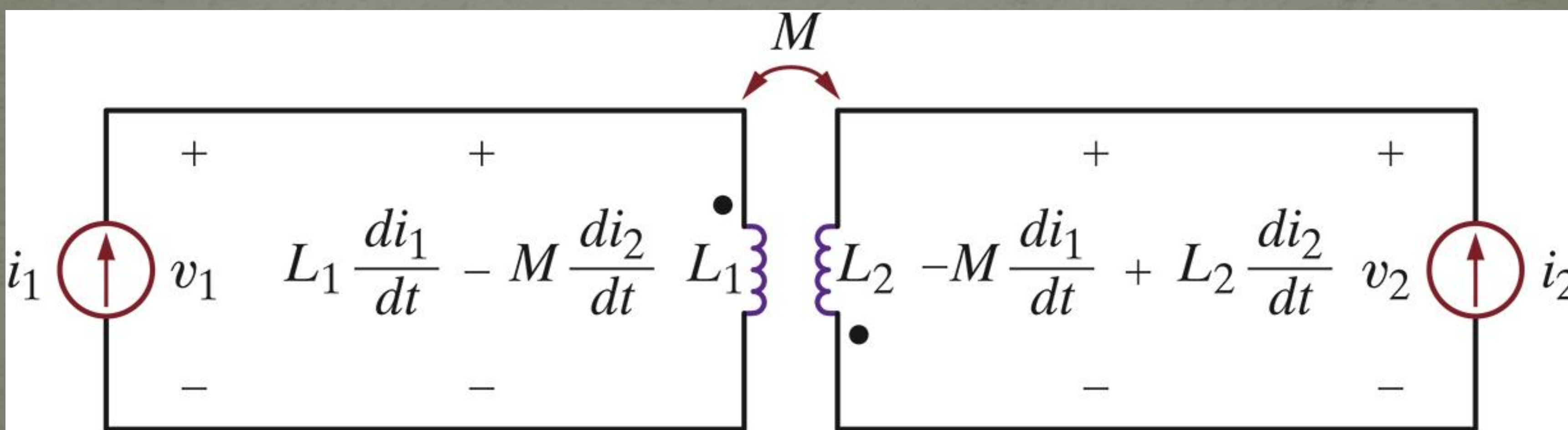
$$v_1 = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = \frac{d\lambda_2}{dt} = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$



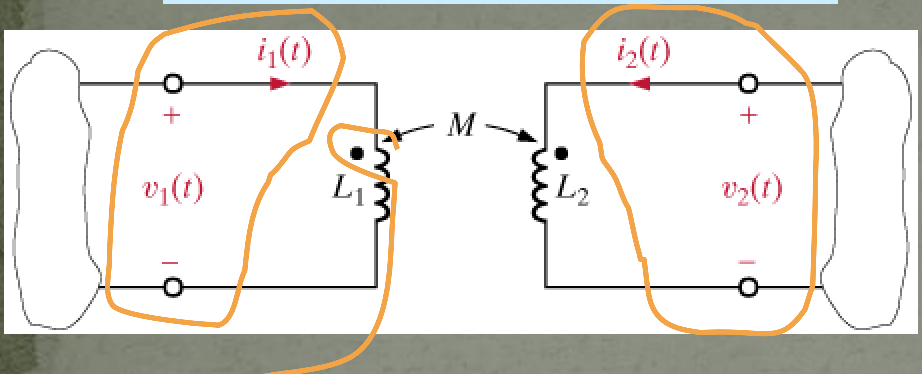
Dots mark reference polarity for voltages induced by each flux


M
i_1
v_1
$L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$
L_1
L_2
$M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$
v_2
i_2


M
i_1
v_1
$L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$
L_1
L_2
$-M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$
v_2
i_2

THE DOT CONVENTION REVIEW

Currents and voltages follow passive sign convention



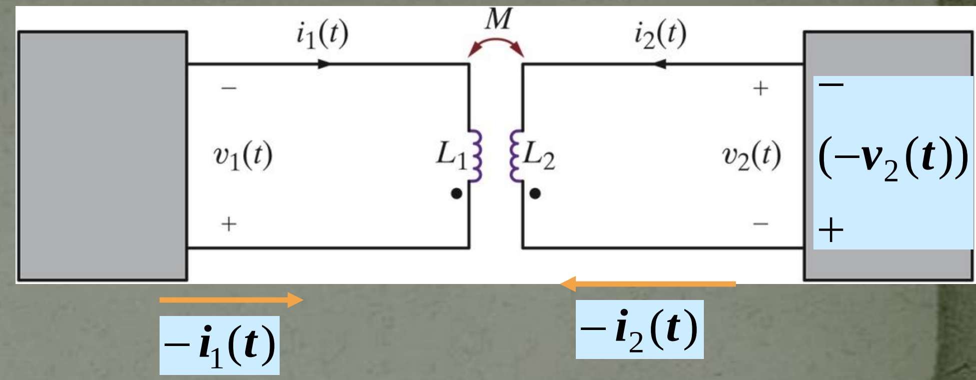
Flux 2 induced voltage has + at dot

$$v_1(t) = L_1 \frac{di_1}{dt}(t) + M \frac{di_2}{dt}(t)$$

$$v_2(t) = M \frac{di_1}{dt}(t) + L_2 \frac{di_2}{dt}(t)$$

For other cases change polarities or current directions to convert to this basic case

LEARNING EXAMPLE



$$v_1(t) = L_1\left(-\frac{di_1}{dt}\right) + M\left(-\frac{di_2}{dt}\right)$$

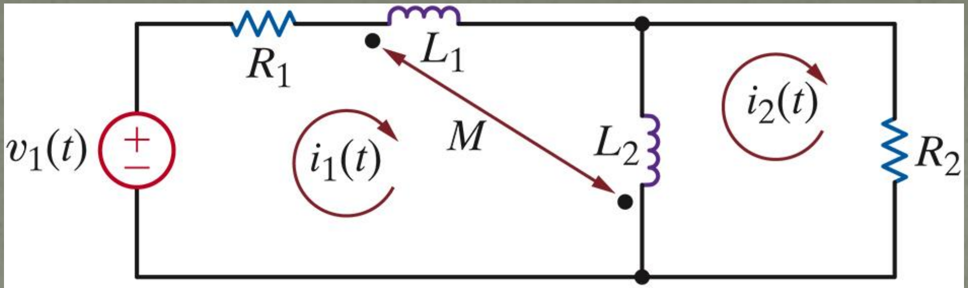
$$-v_2(t) = M\left(-\frac{di_1}{dt}\right) + L_2\left(-\frac{di_2}{dt}\right)$$

$$v_1 = -L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

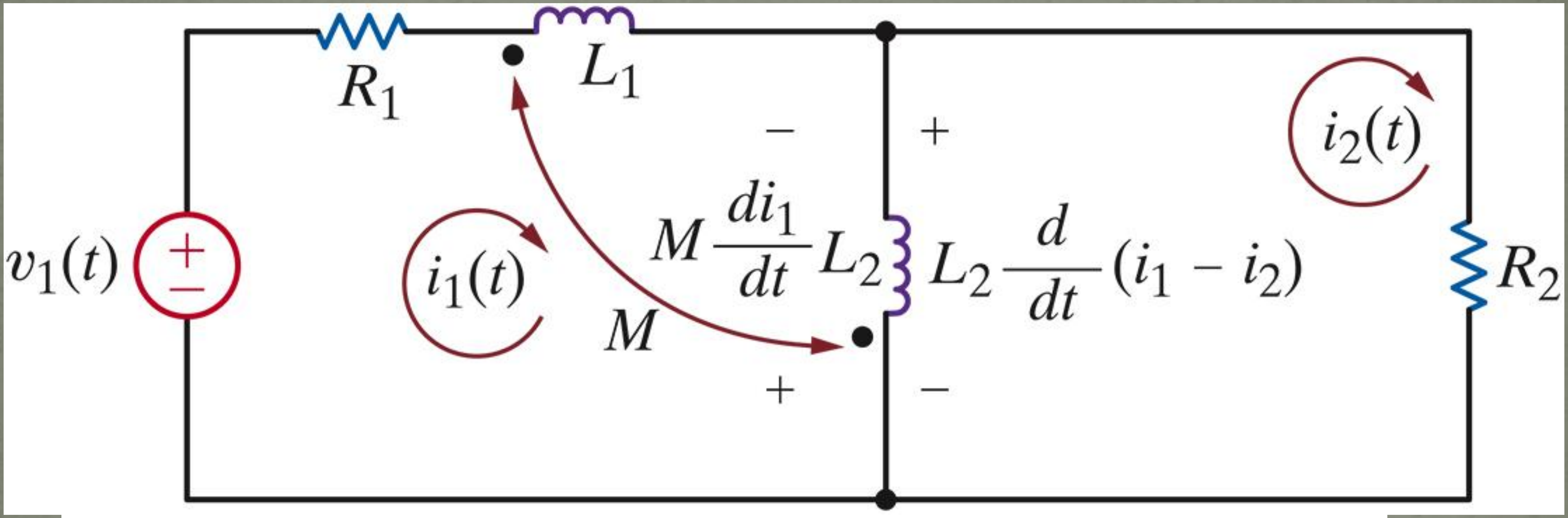
$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

LEARNING EXAMPLE

Write mesh equations for the circuit using the assigned mesh currents.

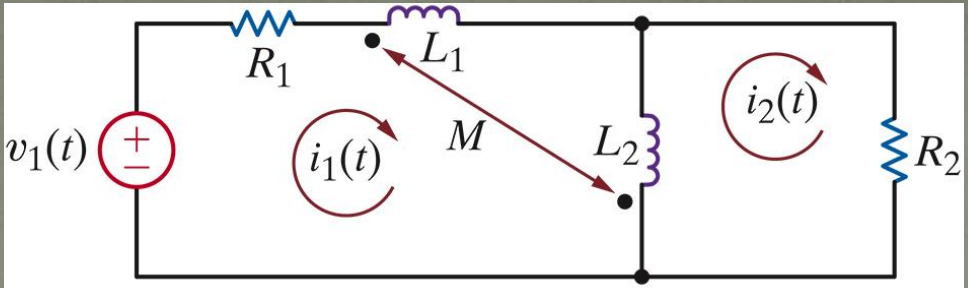


Mesh 1



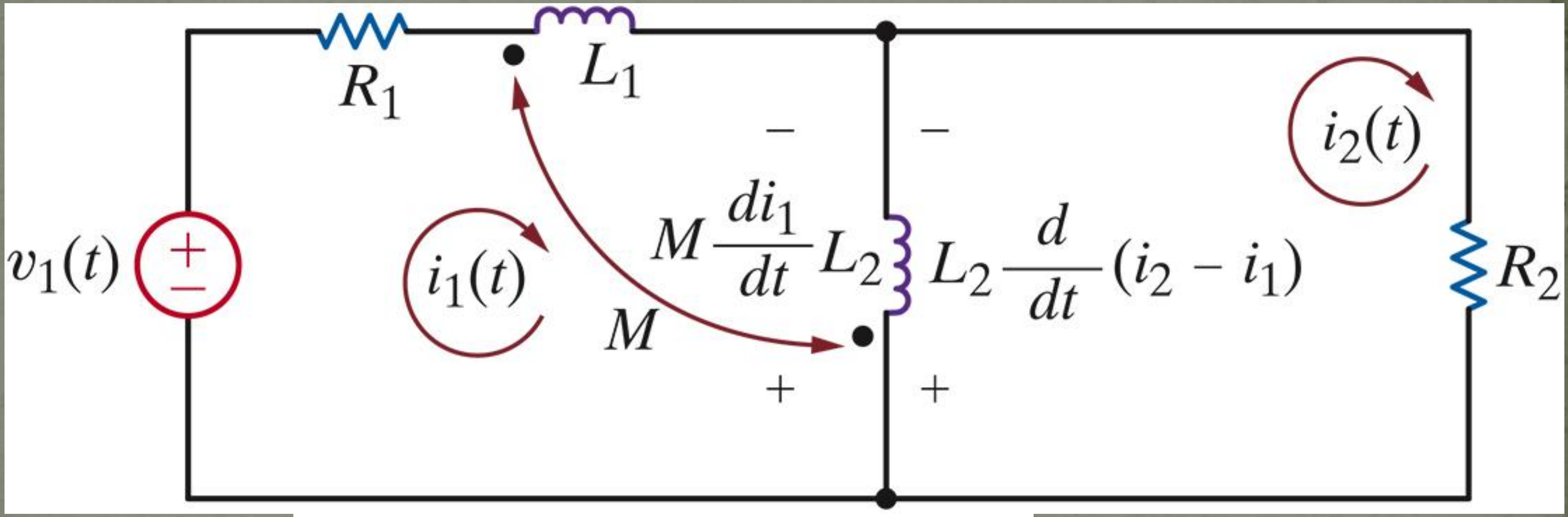
$$v_1(t) = R_1 i_1(t) + L_1\frac{di_1}{dt} + M\frac{d}{dt}(i_2 - i_1) + L_2\frac{d}{dt}(i_1 - i_2) - M\frac{di_1}{dt}$$

LEARNING EXAMPLE - CONTINUED



Mesh 2

Voltage Terms



$$R_2 i_2(t) + L_2 \frac{d}{dt}(i_2 - i_1) + M\frac{di_1}{dt} = 0$$