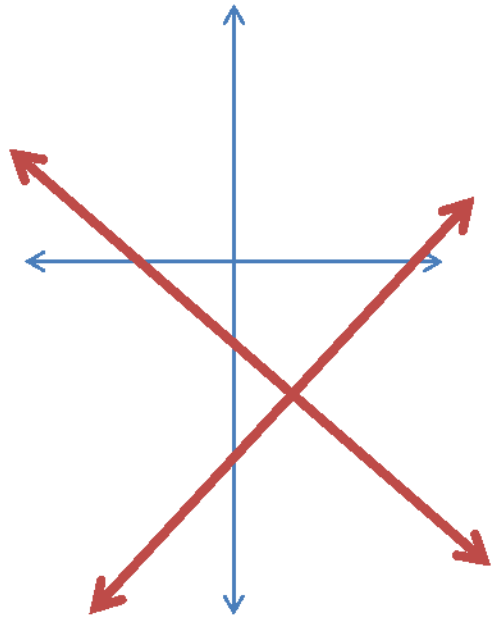


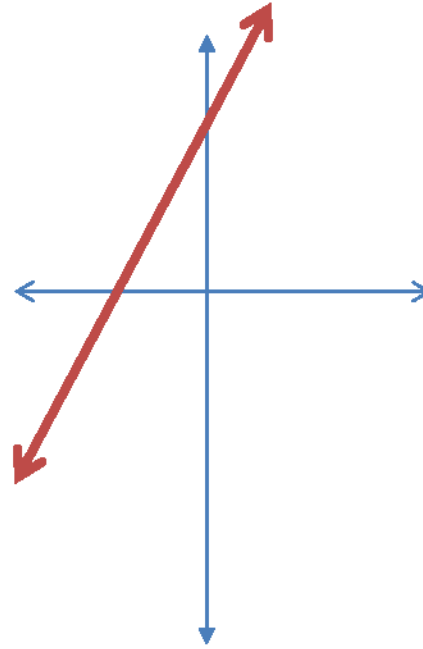
CONSISTENT AND INCONSISTENT SYSTEM



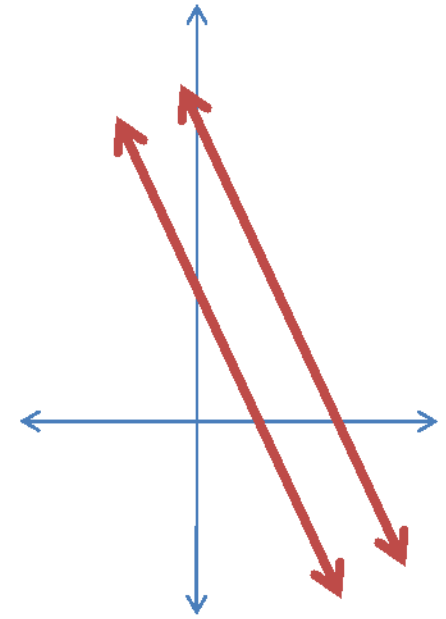
When the graph of two linear equations are drawn in the coordinate plane they may be related to each other as shown below.



- **FIGURE 1.**
- $x + y = -2$
- $x - 2y = 7$
- The graphs intersect in **exactly** one point.

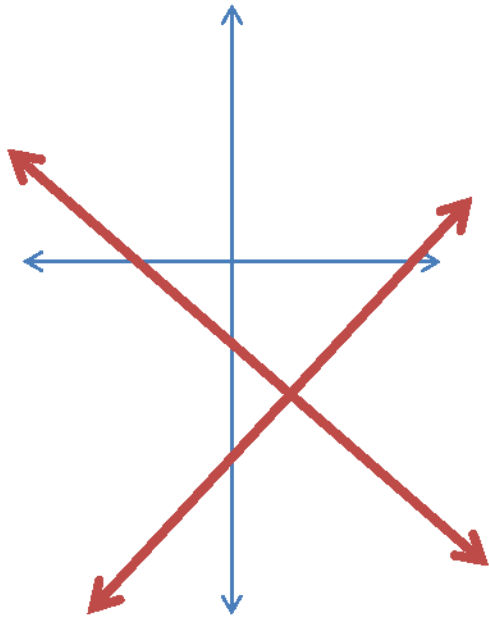


- **FIGURE 2.**
- $y = 2x + 3$
- $2y = 4x + 6$
- The graphs **coincide**; that is, they have an **infinite number** of points in common.

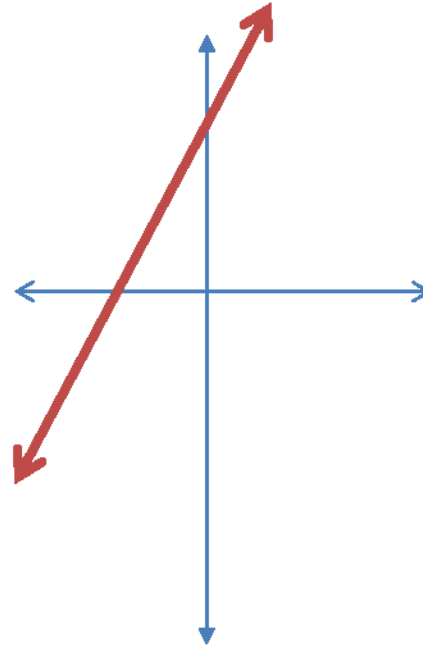


- **FIGURE 3.**
- $3x + y = 2$
- $3x + y = 7$
- The graphs are parallel; that is, they have **no points** in common.

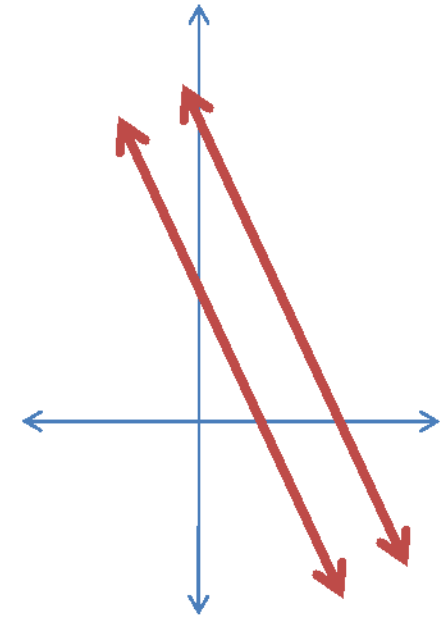
The graphs in figures 1 and 2 have at least one point in common. The system are said to be **CONSISTENT**.



- **FIGURE 1.**
- $x + y = -2$
- $x - 2y = 7$
- The graphs intersect in **exactly** one point.



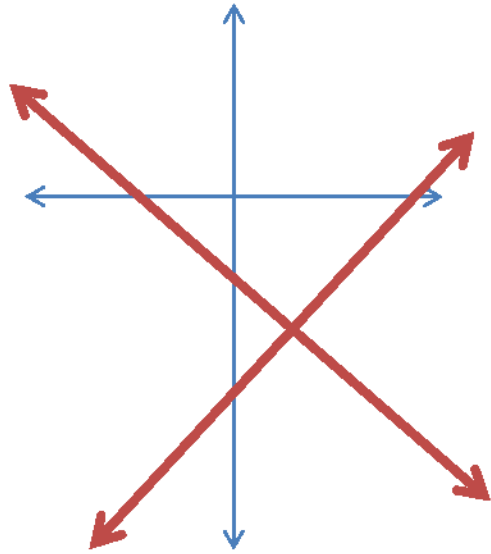
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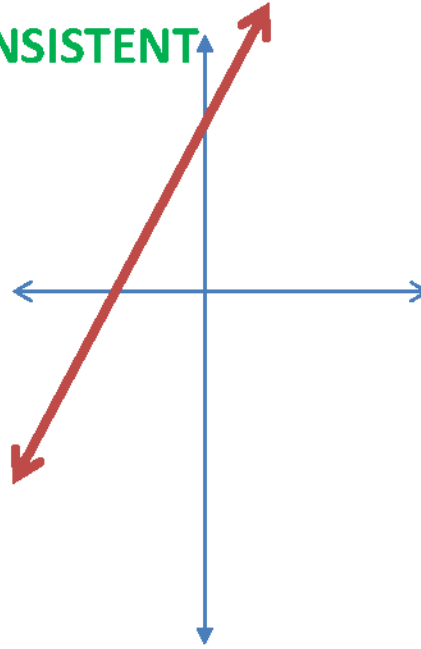
- **FIGURE 3.**
- $3x + y = 2$
- $3x + y = 7$
- The graphs are parallel; that is, they have **no points** in common.

The graphs in figures 3 have NO point in common.
This system is said to be INCONSISTENT.

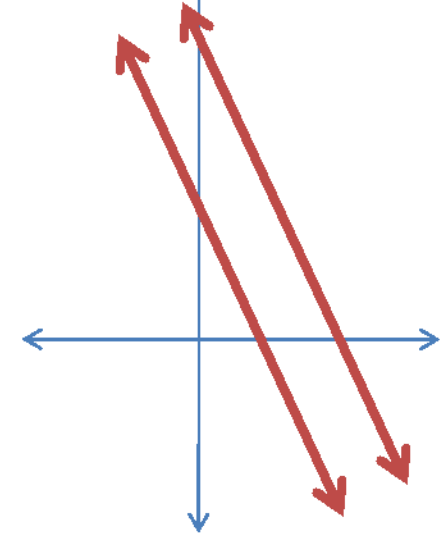
CONSISTENT



CONSISTENT



INCONSISTENT



- **FIGURE 1.**
- $x + y = -2$
- $x - 2y = 7$
- The graphs intersect in **exactly** one point.

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- $y = 2x + 3$
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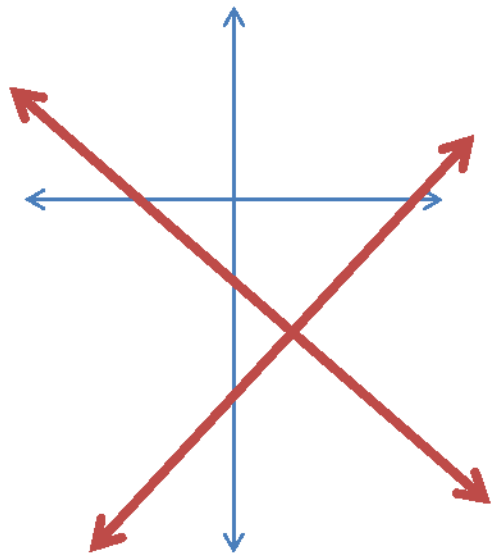
- **FIGURE 3.**
- $3x + y = 2$
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DEFINITIONS

- A **CONSISTENT SYSTEM** of equations or inequalities is one whose solution set contains *at least one ordered pair*.
- An **INCONSISTENT SYSTEM** of equations or inequalities is one whose solution set is *the empty set*.

Write the equations of the system below in slope-intercept form.

CONSISTENT



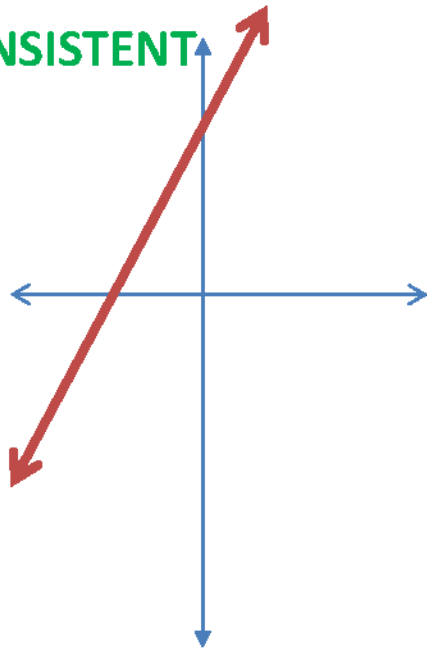
• **FIGURE 1.**

• $x + y = -2$

• $x - 2y = 7$

- The graphs intersect in **exactly** one point.

CONSISTENT



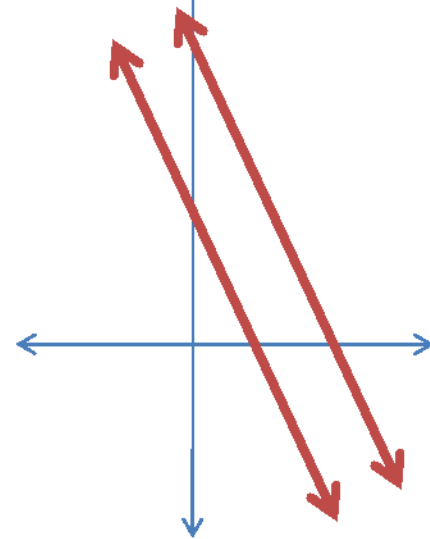
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• $y = 2x + 3$

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INCONSISTENT



• **FIGURE 3.**

• $3x + y = 2$

• $3x + y = 7$

- The graphs are parallel; that is, they have **no points** in common.

- **System 1**

- $x + y = -2$ (1)

- $x - 2y = 7$ (2)

- Slope-intercept form

- $y = -x - 2$ (1)

- $y = \frac{1}{2}x - \frac{7}{2}$ (2)

- **System 2**

- $y = 2x + 3$ (1)

- $2y = 4x + 6$ (2)

- Slope-intercept form

- $y = 2x + 3$ (1)

- $y = \frac{4}{2}x - \frac{6}{2}$ (2)

- **System 3**

- $3x + y = 2$ (1)

- $3x + y = 7$ (2)

- Slope-intercept form

- $y = -3x + 2$ (1)

- $y = -3x + 7$ (2)

- **System 1**

- $x + y = -2$ (1)

- $x - 2y = 7$ (2)

- Slope-intercept form

- $y = -x - 2$ (1)

- $y = \frac{1}{2}x - \frac{7}{2}$ (2)

- For system 1, exactly one ordered pair satisfies both equations. For this system,

- $m_1 = -1$ & $m_2 = \frac{1}{2}$

- Thus, $m_1 \neq m_2$

- **System 2**

- $y = 2x + 3$ (1)

- $2y = 4x + 6$ (2)

- Slope-intercept form

- $y = 2x + 3$ (1)

- $y = \frac{4}{2}x - \frac{6}{2}$ (2)

- For system 2, every ordered pair that satisfies equation 1 also satisfies Equation 2. The system is **DEPENDENT**. For this system

- $m_1 = 2$ & $m_2 = 2$

- Also, $b_1 = 3$ & $b_2 = 3$

- **System 3**

- $3x + y = 2$ (1)

- $3x + y = 7$ (2)

- **Slope-intercept form**

- $y = -3x + 2$ (1)

- $y = -3x + 7$ (2)

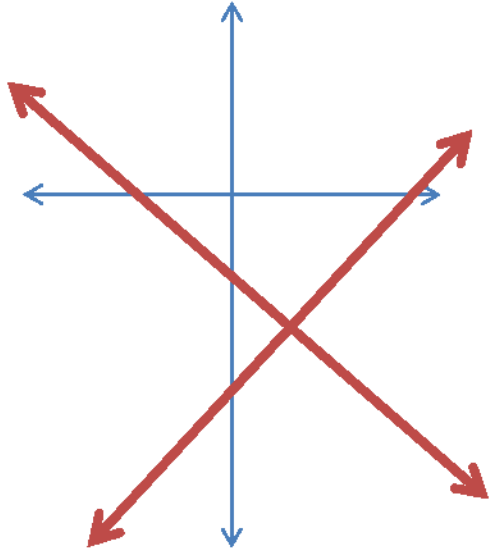
- For system 3, no ordered pair satisfies both equations. For this system,

- $m_1 = -3$ & $m_2 = -3$ *Also, $b_1 = 2$ & $b_2 = 7$*

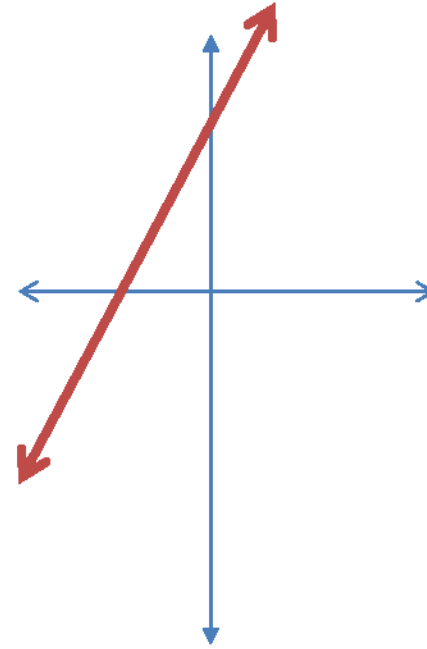
- **Thus, $m_1 = m_2$ and $b_1 \neq b_2$.**

CONSISTENT & DEPENDENT

CONSISTENT & INDEPENDENT

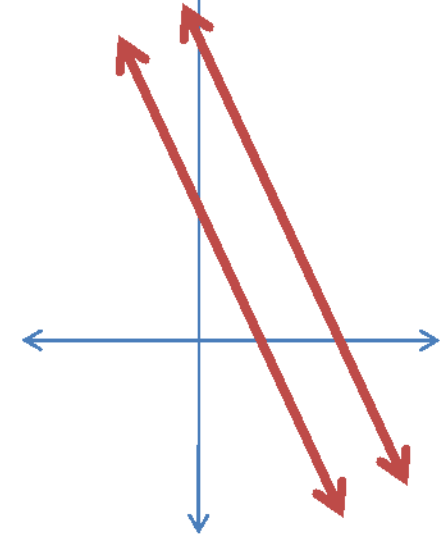


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INCONSISTENT



- **FIGURE 3.**
- $3x + y = 2$
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- The graphs are parallel; that is, they have **no points** in common.

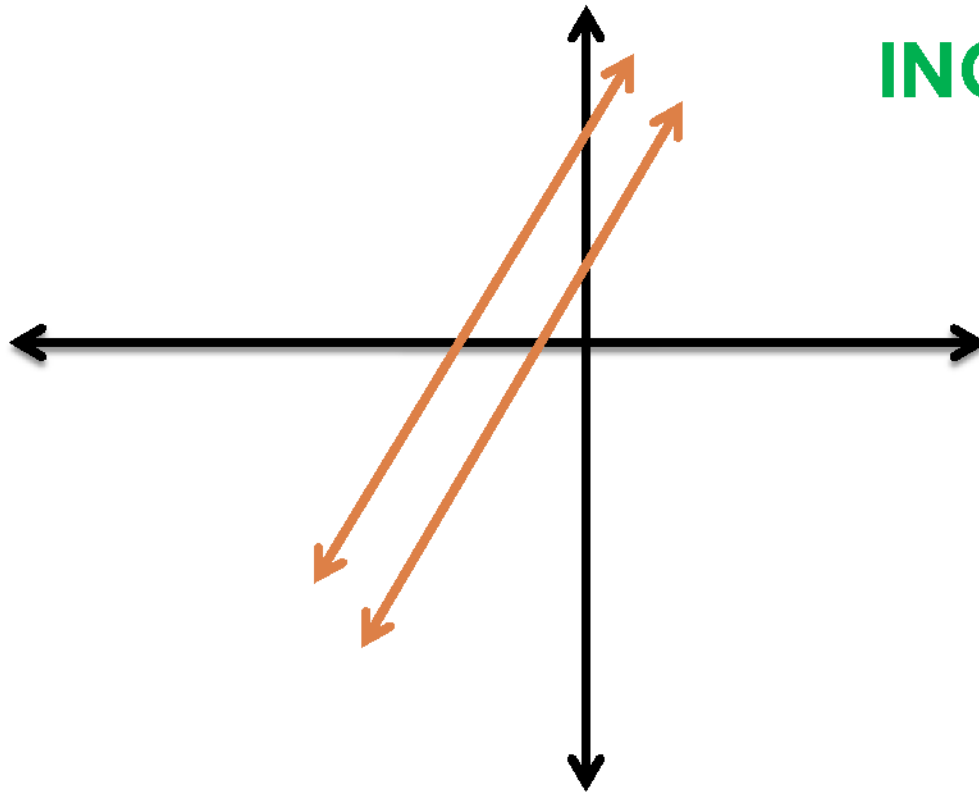
SUMMARY

Properties of a Linear System of Two Equations

$$y = m_1x + b_1 \text{ and } y = m_2x + b_2.$$

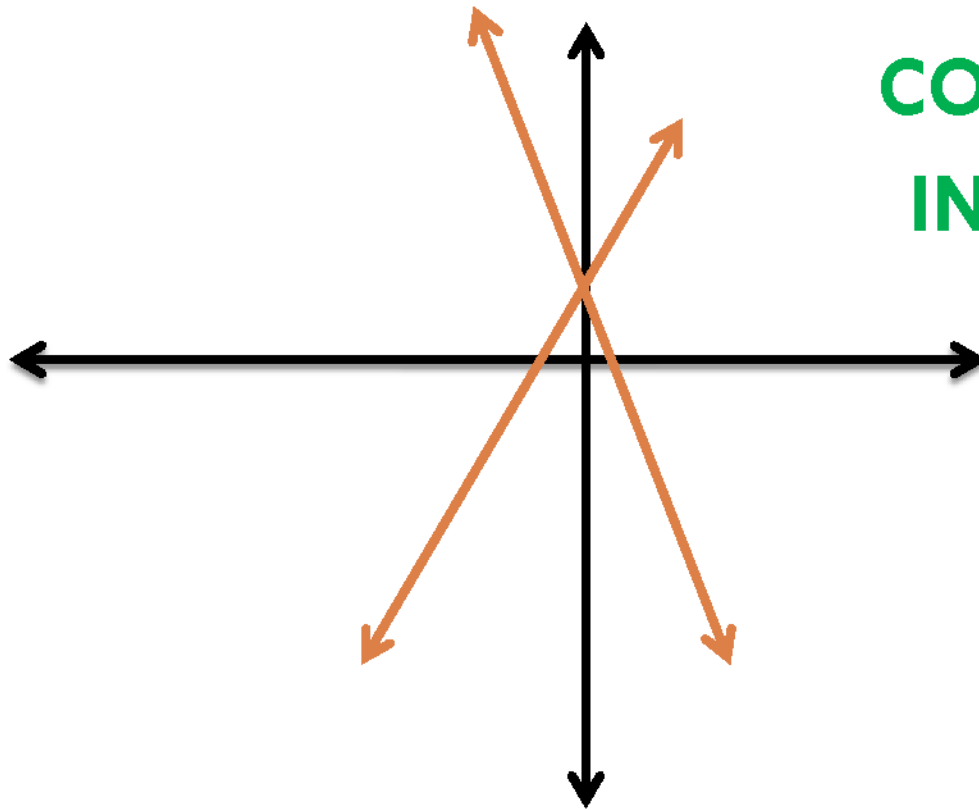
DESCRIPTION	Slopes and y- intercepts	Graphs	Solutions
CONSISTENT	$m_1 \neq m_2$	Intersect in one point	One
DEPENDENT	$m_1 = m_2$ and $b_1 = b_2$	Coincide	Infinite number
INCONSISTENT	$m_1 = m_2$ and $b_1 \neq b_2$	Parallel	None

Use the graph of each system to classify it as
INCONSISTENT, CONSISTENT, DEPENDENT.



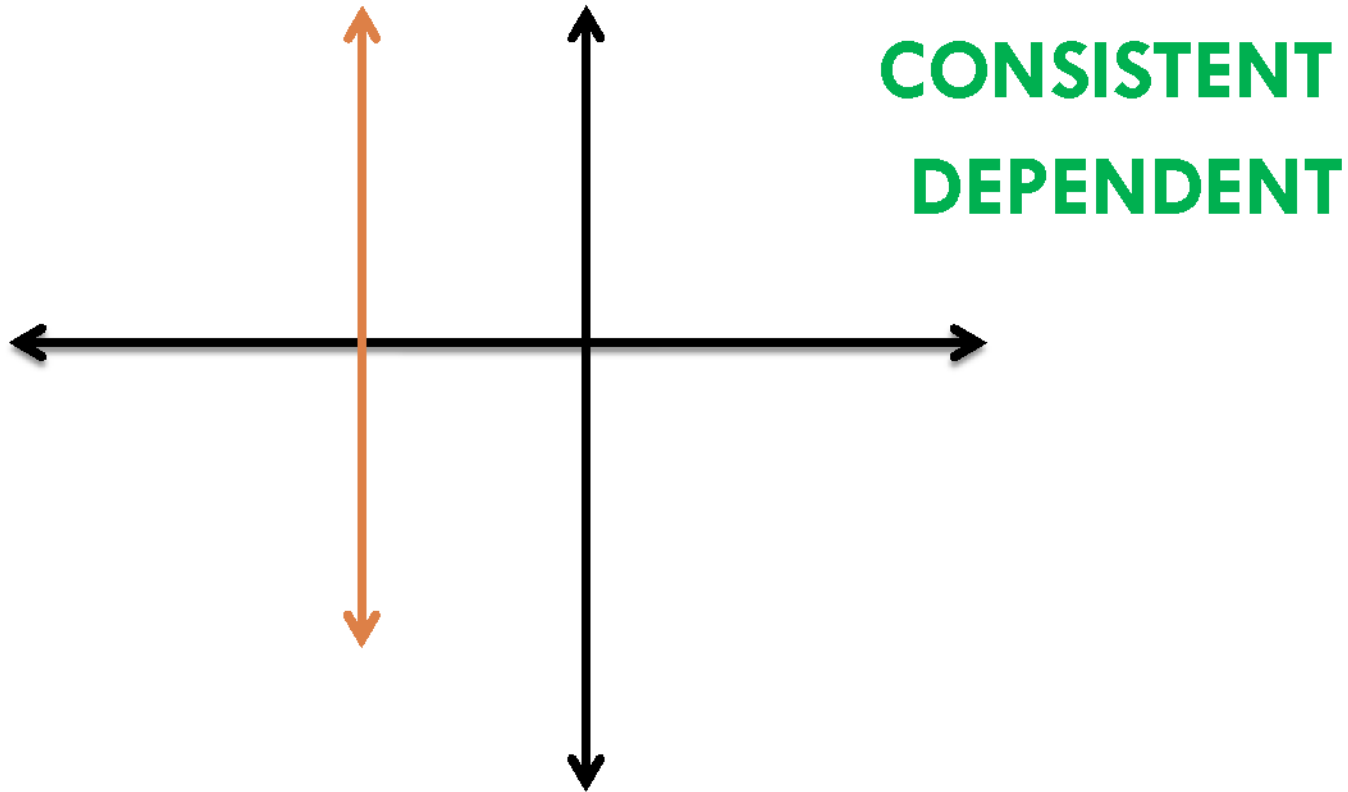
INCONSISTENT

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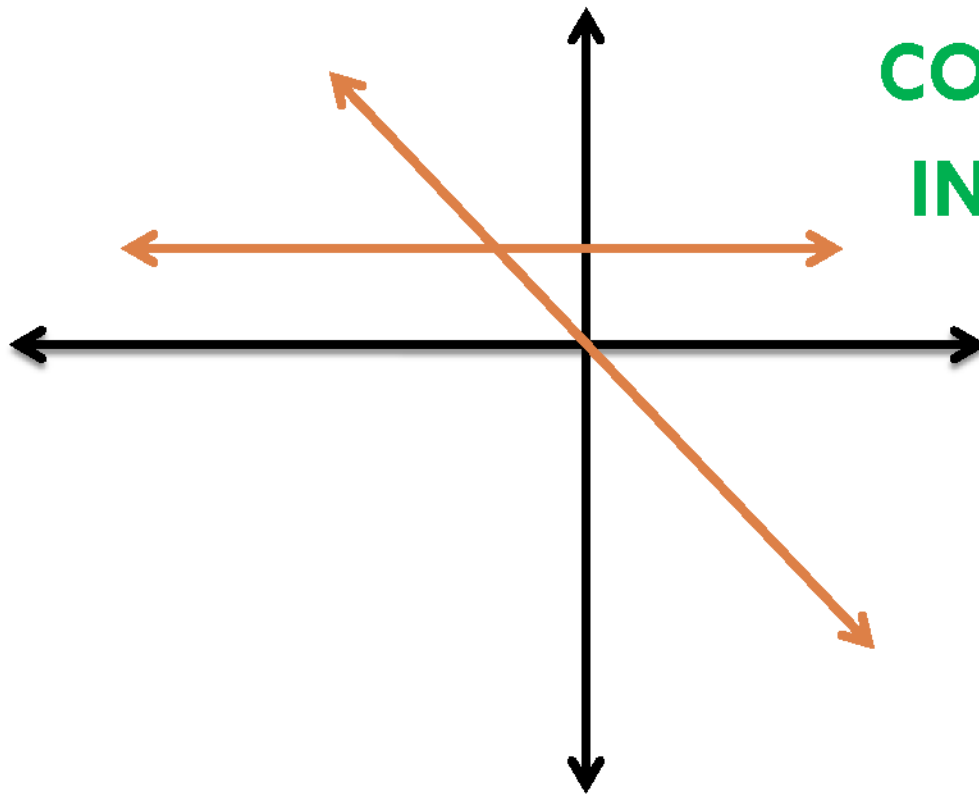


**CONSISTENT
INDEPENDENT**

Use the graph of each system to classify it as
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INCONSISTENT, CONSISTENT, DEPENDENT.



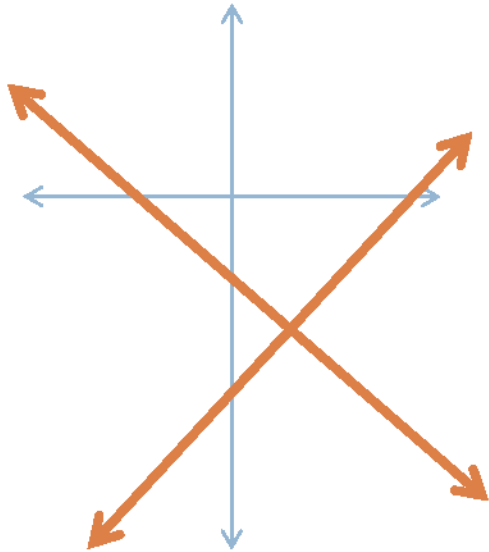
**CONSISTENT
INDEPENDENT**

OTHER WAY OF DETERMINING WHETHER THE SYSTEMS ARE CONSISTENT, INCONSISTENT, or DEPENDENT.

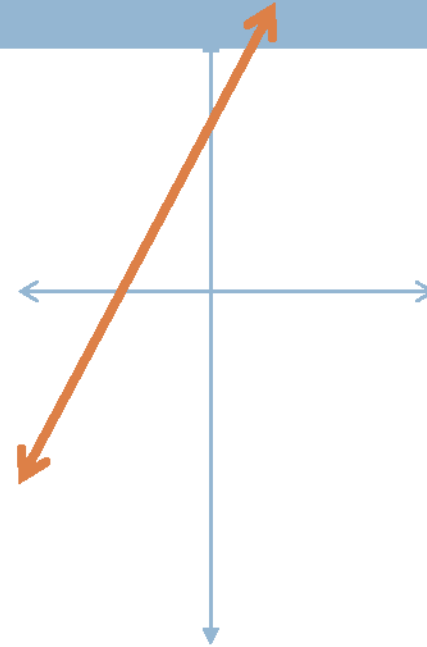
- Given $a_1 x + b_1 y = c_1$ and $a_2 x + b_2 y = c_2$.
- The system is **DEPENDENT** if
- $a_1 : a_2 = b_1 : b_2 = c_1 : c_2$
- One equation is a multiple to the other.
- Graphically, the lines coincide.

CONSISTENT & DEPENDENT

CONSISTENT & INDEPENDENT

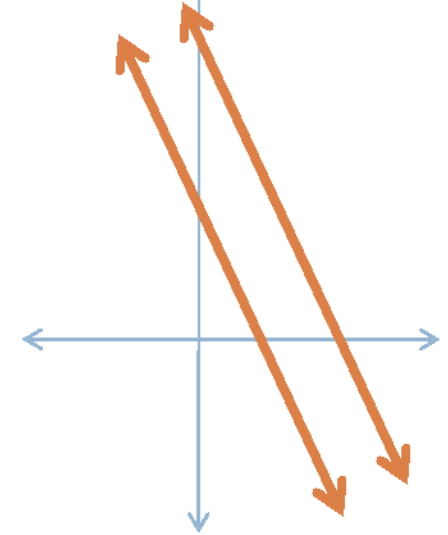


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INCONSISTENT



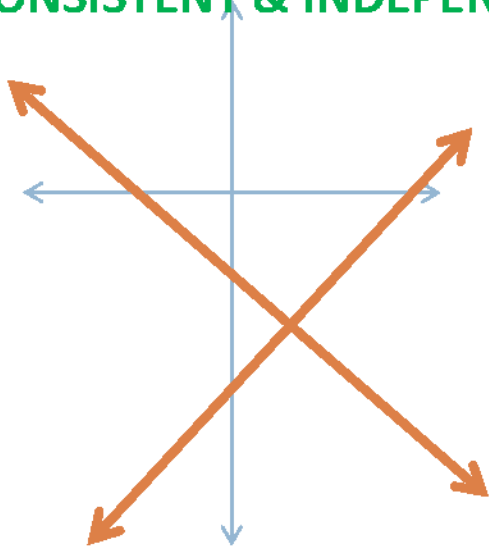
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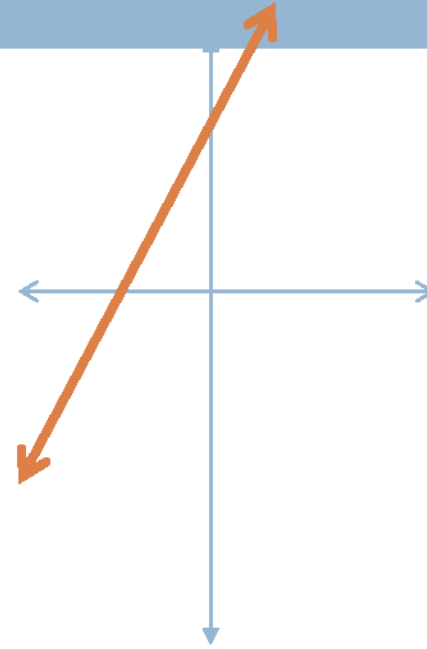
- Given $a_1 x + b_1 y = c_1$ and $a_2 x + b_2 y = c_2$.
- The system is **INCONSISTENT** if
- $a_1 : a_2 = b_1 : b_2 \neq c_1 : c_2$
- **Graphically, the lines are parallel.**

CONSISTENT & DEPENDENT

CONSISTENT & INDEPENDENT

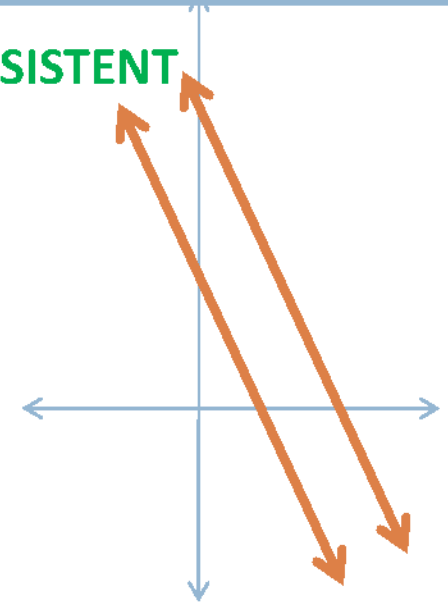


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INCONSISTENT



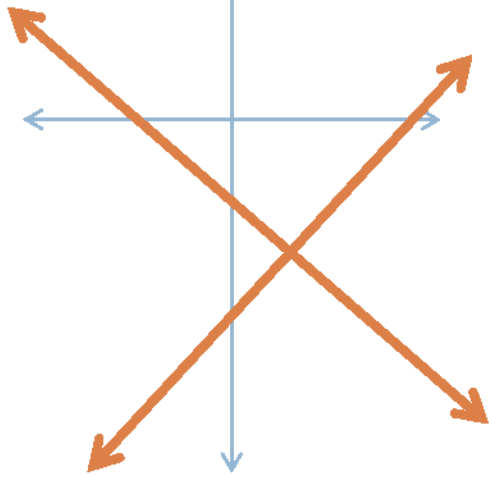
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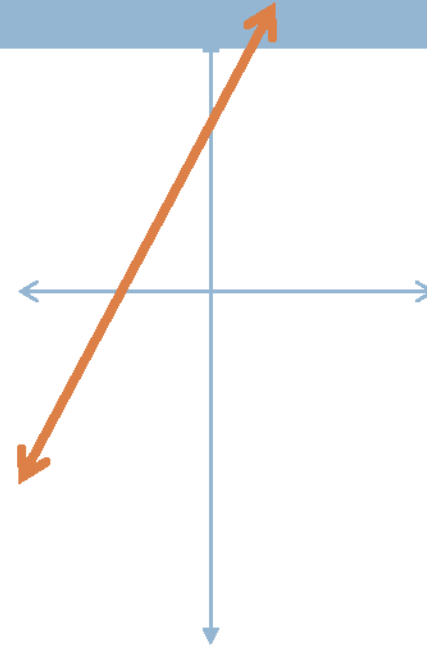
- Given $a_1 x + b_1 y = c_1$ and $a_2 x + b_2 y = c_2$.
- The system is **CONSISTENT** if neither holds.
- $a_1 : a_2 = b_1 : b_2 = c_1 : c_2$
- $a_1 : a_2 \neq b_1 : b_2$
- **Graphically, the lines are intersect.**

CONSISTENT & DEPENDENT

CONSISTENT & INDEPENDENT

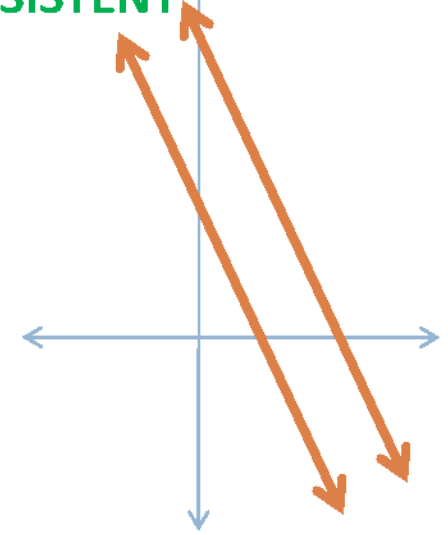


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INCONSISTENT



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