Change of order of integration

In a double integral, if the limits of the Integration are constant, then the order of integration is immaterial, provided the limits of integration are changed accordingly. Thus

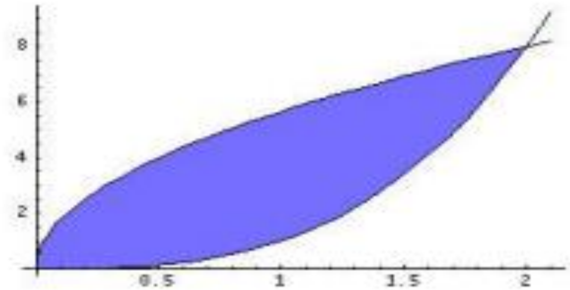
$$\int_c^d \int_a^b f(x, y)dx\, dy = \int_a^b \int_c^d f(x, y)dy\, dx$$

But if the limits of integration are variable, a change in the order of integration necessitates change in the limits of integration. A rough sketch of the region of integration help in fixing the new limit of integration.

1. Change the order of integration, you don't need to compute the resulting integral

$$\int_0^8 \int_{y^2/32}^{y^{1/3}} xy\,dx\,dy$$

To begin, we will look at the region we are integrating



We want to setup an integral $dydx$ so we begin by looking at the x-axis and see that x is at least 0 and at most 2. We can get the same numbers algebraically, by computing $0^(1/3) = 0$ and $8^(1/3) = 2$.

For the inner bounds we look at the curves $x = y^{1/3}$ and $x = y^2/32$. The lower curve is $x = y^{1/3}$ which we want to solve in terms of x. We get $y = x^3$, which is our lower bound. The upper curve is $x = y^2/32$, which we also solve in terms of x and have $y = \sqrt{32x}$. This gives the integral

$$\int_0^2 \int_{x^3}^{\sqrt{32x}} xy\,dy\,dx$$

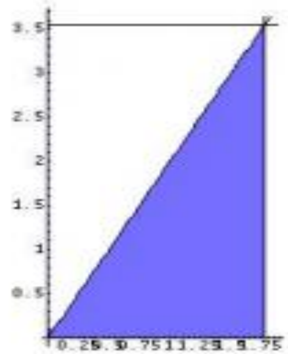
The evaluation of this integral is not required, but we can compute

$$\int_0^2 \int_{x^3}^{\sqrt{32x}} xy\,dy\,dx = \frac{80}{3}$$

2. Change the order of integration and evaluate the double integral

$$\int_0^{2\sqrt{\pi}} \int_{y/2}^{\sqrt{\pi}} \sin(x^2)\,dx\,dy$$

As usual, we begin by sketching the region of integration.



To reverse the order of integration, we must setup the integral $dydx$ we can tell in the picture that x is bounded by 0 and $\sqrt{\pi}$, and y is bounded by 0 and $2x$. This allows us to reverse the order of integration

$$\int_0^{\sqrt{\pi}} \int_0^{2x} \sin(x^2)\,dy\,dx$$

We can now evaluate this integral

$$\int_0^{\sqrt{\pi}} \int_0^{2x} \sin(x^2)\,dy\,dx = \int_0^{\sqrt{\pi}} \left(\sin(x^2) y \Big|_0^{2x} \right) dx =$$

$$= \int_0^{\sqrt{\pi}} \sin(x^2) 2x\,dx$$

$$= -\cos(x^2) \Big|_0^{\sqrt{\pi}} = 2$$

Notice that we could not have made the u substitution before changing the order of integration.