Gradient

- If $\phi\,(x,y,z)$ is a scalar function of three variables and ϕ is differentiable, the gradient of ϕ is defined as

$$\text{grad}\ \phi = \nabla\phi = \frac{\partial\phi}{\partial x}\underset{\sim}{i} + \frac{\partial\phi}{\partial y}\underset{\sim}{j} + \frac{\partial\phi}{\partial z}\underset{\sim}{k}.$$

* ϕ is a scalar function

* $\nabla\phi$ is a vector function

Example:

If $\phi = x^2 yz^3 + xy^2 z^2$, determine grad ϕ at P = (1,3,2).

Solution

Given $\phi = x^2 yz^3 + xy^2 z^2$, hence

$$\frac{\partial \phi}{\partial \mathrm{x}} = 2xyz^3 + y^2 z^2$$

$$\frac{\partial \phi}{\partial \mathrm{y}} = x^2 z^3 + 2xyz^2$$

$$\frac{\partial \phi}{\partial \mathrm{z}} = 3x^2 yz^2 + 2xy^2 z$$

Therefore,

$$\nabla\phi = \frac{\partial\phi}{\partial x}\underset{\sim}{i} + \frac{\partial\phi}{\partial y}\underset{\sim}{j} + \frac{\partial\phi}{\partial z}\underset{\sim}{k}$$

$$= (2xyz^3 + y^2z^2)\underset{\sim}{i} + (x^2z^3 + 2xyz^2)\underset{\sim}{j}$$

$$+ (3x^2yz^2 + 2xy^2z)\underset{\sim}{k}.$$

At P = (1,3,2), we have

$$\nabla\phi = (2(1)(3)(2)^3 + (3)^2(2)^2)\underset{\sim}{i} + ((1)^2(2)^3 + 2(1)(3)(2)^2)\underset{\sim}{j}$$

$$+ (3(1)^2(3)(2)^2 + 2(1)(3)^2(2))\underset{\sim}{k}.$$

$$= 84\underset{\sim}{i} + 32\underset{\sim}{j} + 72\underset{\sim}{k}.$$

Exercise :

If $\phi = x^3 yz + xy^2 z^3$,

determine grad ϕ at point $P = (1,2,3)$.

Solution

Given $\phi = x^3yz + xy^2z^3$, then

$$\frac{\partial\phi}{\partial x} = \ldots$$

$$\frac{\partial\phi}{\partial y} = \ldots$$

$$\frac{\partial\phi}{\partial z} = \ldots$$

$\therefore$ Grad $\phi = \nabla\phi = \ldots$

At $P = (1,2,3)$, $\quad \nabla\phi = 126\underset{\sim}{i} + 111\underset{\sim}{j} + 110\underset{\sim}{k}$.

Grad Properties

If A and B are two scalars, then

1) $\nabla(A+B)=\nabla A+\nabla B$
2) $\nabla(AB)=A(\nabla B)+B(\nabla A)$