Directional Derivative

Directional derivative of ϕ in the direction of $\underset{\sim}{a}$ is

$$\frac{d\phi}{ds} = \underset{\sim}{a} \, . grad\phi$$

where $\underset{\sim}{a} = \dfrac{d\underset{\sim}{r}}{\left|d\underset{\sim}{r}\right|}$,

which is a unit vector in the direction of $d\underset{\sim}{r}$.

Unit Normal Vector

Equation $\phi(x, y, z) =$ constant is a surface equation. Since $\phi(x, y, z) =$ constant, the derivative of ϕ is zero; i.e.

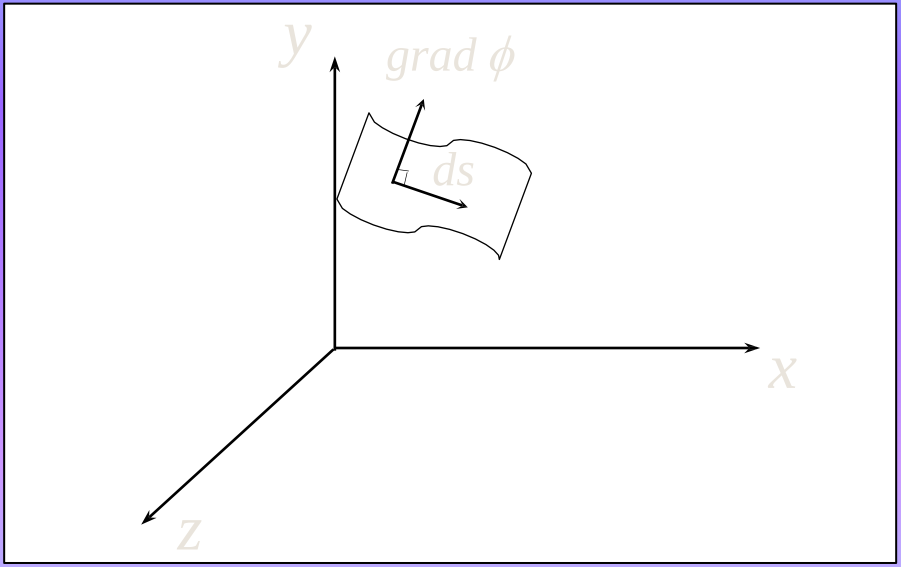
$$d\phi = d\underset{\sim}{r} \cdot \text{grad}\,\phi = 0$$

$$\Rightarrow \left|d\underset{\sim}{r}\right| \left|\text{grad}\,\phi\right| \cos\theta = 0$$

$$\Rightarrow \cos\theta = 0$$

$$\Rightarrow \theta = 90^\circ.$$

- This shows that when $\phi(x, y, z) =$ constant, $grad\,\phi \perp d\underset{\sim}{r}$.



- Vector *grad* $\phi = \nabla\,\phi$ is called normal vector to the surface $\phi(x, y, z) =$ constant

Unit normal vector is denoted by

$$\underset{\sim}{n} = \frac{\nabla \phi}{|\nabla \phi|}.$$

Example:

Calculate the unit normal vector at (-1,1,1) for $2yz + xz + xy = 0$.

Given $2yz + xz + xy = 0$. Thus

Solution

$$\nabla \phi = (z+y)\underset{\sim}{i} + (2z+x)\underset{\sim}{j} + (2y+x)\underset{\sim}{k}.$$

$$\text{At } (-1,1,1), \quad \nabla \phi = (1+1)\underset{\sim}{i} + (2-1)\underset{\sim}{j} + (2-1)\underset{\sim}{k}$$

$$= 2\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$$

$$\text{and } |\nabla \phi| = \sqrt{4+1+1} = \sqrt{6}.$$

$\therefore$ The unit normal vector is

$$\underset{\sim}{n} = \frac{\nabla \phi}{|\nabla \phi|} = \frac{2\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}(2\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k})$$