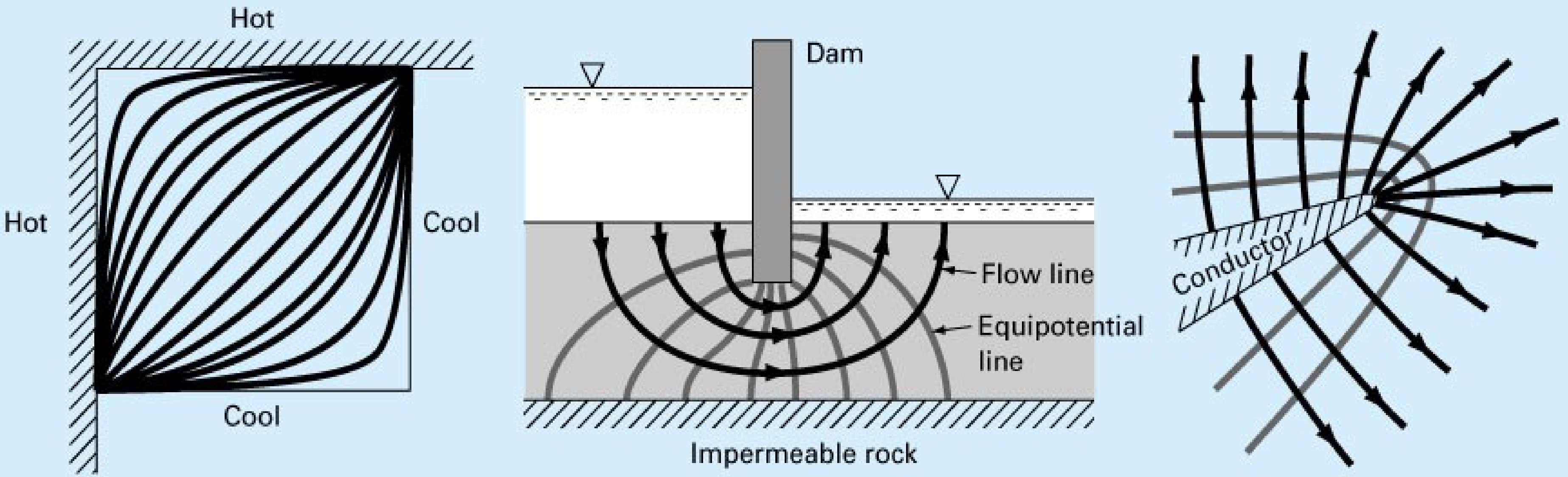
Partial Differential Equations

Table PT8.1



(a) (b) (c)

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Finite Difference: Elliptic Equations
Chapter 29

Solution Technique

- Elliptic equations in engineering are typically used to characterize steady-state, boundary value problems.
- For numerical solution of elliptic PDEs, the PDE is transformed into an algebraic difference equation.
- Because of its simplicity and general relevance to most areas of engineering, we will use a heated plate as an example for solving elliptic PDEs.

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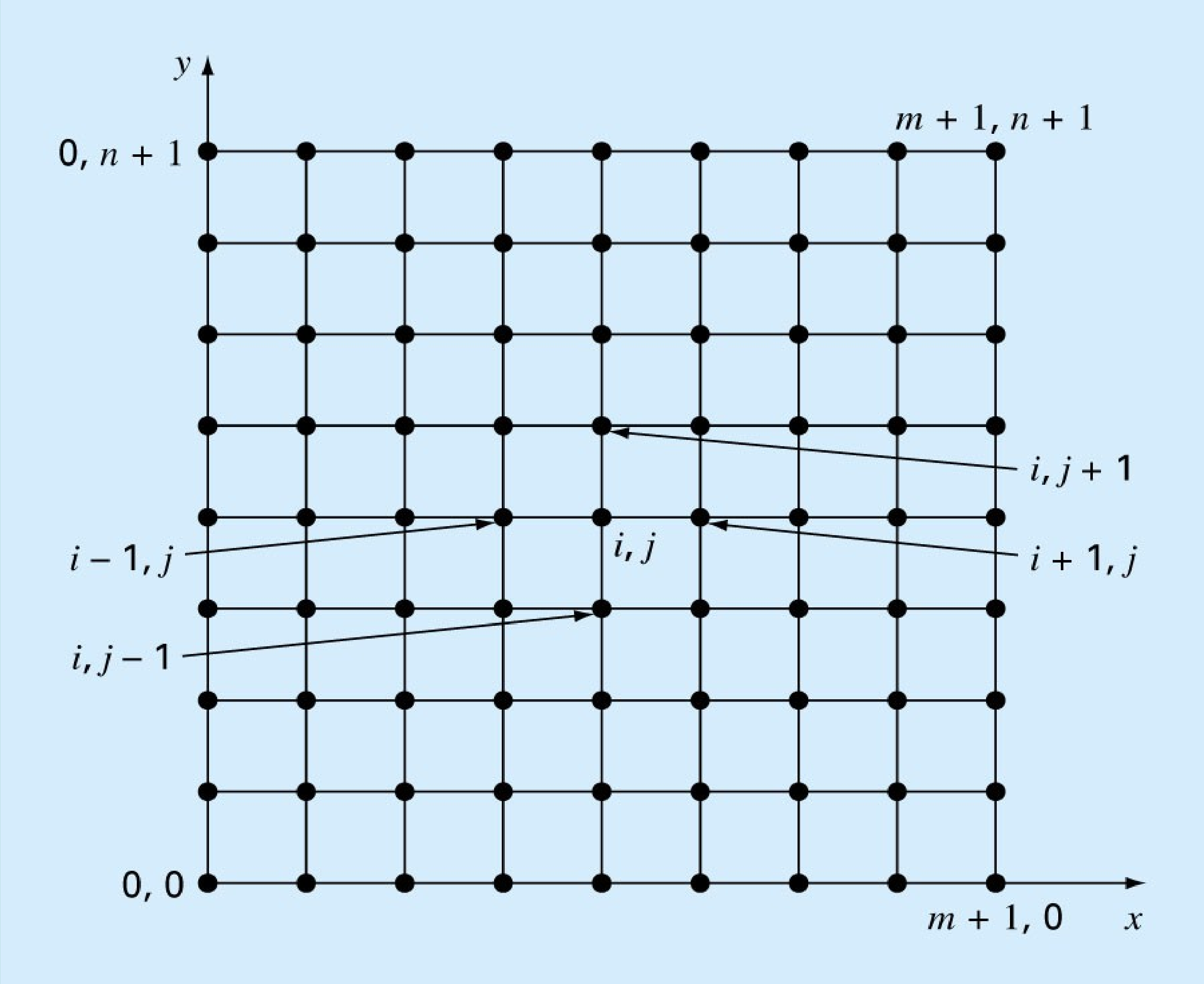
Figure 29.3



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The Laplacian Difference Equations/

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Laplace Equation

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} \qquad O[\Delta(x)^2]$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2} \qquad O[\Delta(y)^2]$$

$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2} = 0$$

$$\Delta x = \Delta y$$

$$T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} - 4T_{i,j} = 0$$

Laplacian difference equation.

Holds for all interior points

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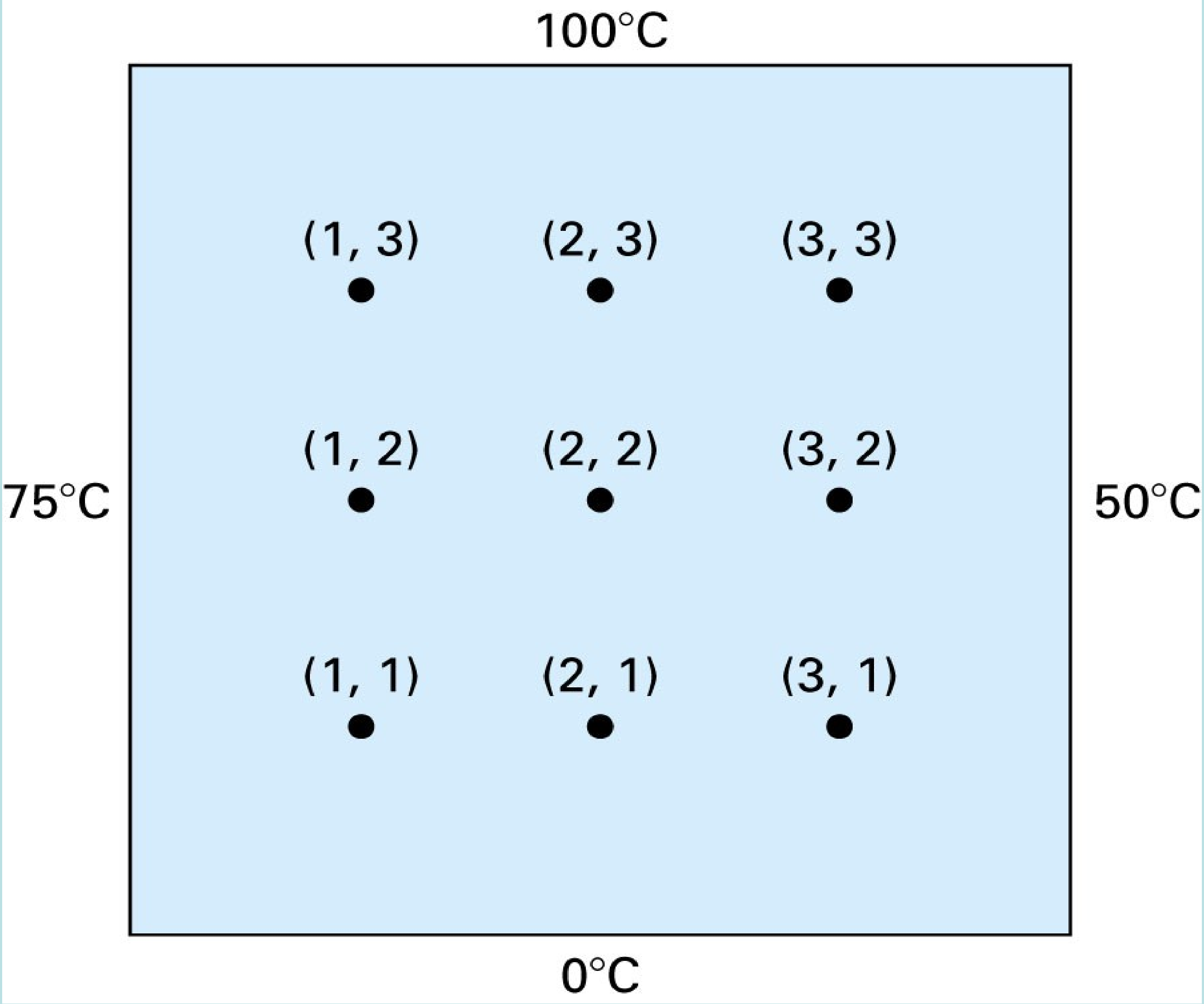
Figure 29.4



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- In addition, boundary conditions along the edges must be specified to obtain a unique solution.
- The simplest case is where the temperature at the boundary is set at a fixed value, *Dirichlet boundary condition*.
- A balance for node (1,1) is:

$$T_{21} + T_{01} + T_{12} + T_{10} - 4T_{11} = 0$$

$$T_{01} = 75$$

$$T_{10} = 0$$

$$-4T_{11} + T_{12} + T_{21} = 0$$

- Similar equations can be developed for other interior points to result a set of simultaneous equations.

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• The result is a set of nine simultaneous equations with nine unknowns:

$$
\begin{array}{llllllllll}
4T_{11} & -T_{21} & & -T_{12} & & & & & & =75 \\
-T_{11} & +4T_{21} & -T_{13} & & -T_{22} & & & & & =0 \\
 & -T_{21} & +4T_{31} & & & -T_{32} & & & & =50 \\
-T_{11} & & & +4T_{12} & -T_{22} & & -T_{13} & & & =75 \\
 & -T_{21} & & -T_{12} & +4T_{22} & -T_{32} & & -T_{23} & & =0 \\
 & & -T_{31} & & -T_{22} & +4T_{32} & & & -T_{33} & =50 \\
 & & & -T_{12} & & & +4T_{13} & -T_{23} & & =175 \\
 & & & & -T_{22} & & -T_{13} & +4T_{23} & -T_{33} & =100 \\
 & & & & & -T_{32} & & -T_{23} & +4T_{33} & =150
\end{array}
$$

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