Inertial and Non-Inertial Frames of Reference & Galilean transformations

B.Tech-I

Recall: Newton's first law of motion (law of inertia):

An object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force.

We've already talked about frames of reference in this course when we talked about relative velocity. You know that a person sitting on a bus watching a ball roll across the ground will have a very different measurement of the ball's velocity than a person standing outside the bus would measure the ball's velocity.

The person inside the bus and the person outside the bus are measuring from different <u>frames of reference</u>.

The laws of physics seem to momentarily break down for you sitting on the bus. In reality, what has happened is that your frame of reference has been <u>compromised</u>.

An <u>inertial frame of reference</u> is a frame of reference in which the law of inertia and other physics laws <u>are valid</u>. Any frame moving at a constant velocity relative to another frame is also an inertial frame of reference.

- When the breaks are applied to the bus, the bus undergoes a negative acceleration. At this moment, it becomes a non-inertial frame of reference.
- A <u>non-inertial frame</u> of reference is a reference frame in which <u>the law of inertia does not</u> <u>hold</u>.

- Although the ball accelerates toward the front of the bus, there is <u>no net force</u> causing the acceleration.
- But if you are sitting on the bus, you observe the ball accelerating forward. That would imply to you as you sit on the bus that there is a <u>net</u> <u>force forward</u> on the ball.
- The reason there appears to be a net force on the ball is that you are observing the motion of the ball in the non-inertial reference frame.

- If you were observing the motion from the road (which is an inertial frame of reference) the ball just continues to move forward at the speed it was already going, and it's motion is easily explained by the law of inertia.
- To an observer in the inertial frame of reference (the ground) the *bus* experiences a net force causing it to decelerate. The *ball* just continues it's forward velocity with no net force.

To explain the ball's motion if you are sitting on the bus, you need to invent a force that acts on the ball toward the front of the bus. This is called the <u>fictitious force</u>. It is an invented force that we can use to explain the observed motion in the accelerated frame of reference.

Galilean Transformation

Suppose there are two reference frames (systems) designated by S and S' such that the co-ordinate axes are parallel. In S, we have the co-ordinates {x, y, z, t} and in S' we have the co-ordinates {x', y', z', t'}. S' is moving with respect to S with velocity (as measured in S) in the direction. The clocks in both systems were synchronised at time and they run at the same rate.



We have the intuitive relationships

$$\begin{array}{rcl} x' & = & x - vt \\ y' & = & y \\ z' & = & z \\ t' & = & t \end{array}$$

This set of equations is known as the Galilean Transformation. They enable us to relate a measurement in one inertial reference frame to another. For example, suppose we measure the velocity of a vehicle moving in the in -direction in system S, and we want to know what would be the velocity of the vehicle in S'

$$v'_x = \frac{dx'}{dt'} = \frac{d(x-vt)}{dt} = v_x - v$$

- We have stated the we would like the laws of physics to be the same in all inertial reference frames, as this is indeed our experience of nature. Physically, we should be able to perform the same experiments in different reference frames, and find always the same physical laws. Mathematically, these laws are expressed by equations. So, we should be able to ``transform" our equations from one inertial reference frame to the other inertial reference frame, and always find the same answer.
- Suppose we wanted to check that Newton's Second Law is the same in two different reference frames. We put one observer in the un-primed frame, and the other in the primed frame, moving with velocity relative to the un-primed frame. Consider the vehicle of the previous case undergoing a constant acceleration in the -direction,

$$f' = m'a' = m'\frac{d^2x'}{dt'^2}$$
$$= m'\frac{d}{dt'}\left(\frac{dx'}{dt'}\right)$$
$$= m\frac{d}{dt}\left(\frac{d(x-vt)}{dt}\right)$$
$$= m\frac{d(v_x-v)}{dt}$$
$$= m\frac{dv_x}{dt}$$
$$= ma = f$$

Indeed, it does not matter which inertial frame we observe from, we recover the same Second Law of Motion each time. In the parlance of physics, we say the Second Law of Motion is invariant under the

Galilean Transformation.