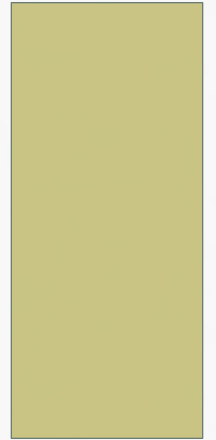


MATTER WAVE

B.TECH I-SEM



MATTER WAVES

De Broglie postulated that every particles has an associated wave of wavelength:

$$\lambda = h / p$$

Wave nature of matter confirmed by electron diffraction studies *etc* (see earlier).

If matter has wave-like properties then there must be a mathematical function that is the solution to a differential equation that describes electrons, atoms and molecules.

The differential equation is called the *Schrödinger equation* and its solution is called the *wavefunction*, Ψ .

What is the form of the *Schrödinger equation* ?

THE CLASSICAL WAVE EQUATION

We have seen previously that the wave equation in 1-d is:

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2}$$

Where v is the speed of the wave. Can this be used for **matter** waves in free space?

Try a solution: e.g.

$$\Psi(x, t) = e^{i(kx - \omega t)}$$

Not correct! For a free particle we know that $E = p^2/2m$.

AN ALTERNATIVE....

Try a modified wave equation of the following type:

(α is a constant)

$$\frac{\partial^2 \Psi}{\partial x^2} = \alpha \frac{\partial \Psi}{\partial t}$$

Now try same solution as before: e.g.

$$\Psi(x, t) = e^{i(kx - \omega t)}$$

Hence, the equation for matter waves in *free space* is:

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} = i\hbar \frac{\partial \Psi}{\partial t}$$

For $\Psi(x, t) = e^{i(kx - \omega t)}$ then we have
$$\frac{k^2 \hbar^2}{2m} \Psi(x, t) = \hbar \omega \Psi(x, t)$$

which has the form:

$$(\text{KE}) \times \text{wavefunction} = (\text{Total energy}) \times \text{wavefunction}$$