MATTER WAVE

B.TECH I-SEM

MATTER WAVES

De Broglie postulated that every particles has an associated wave of wavelength:

$$\lambda = h / p$$

Wave nature of matter confirmed by electron diffraction studies *etc* (see earlier)**.**

If matter has wave-like properties then there must be a mathematical function that is the solution to a differential equation that describes electrons, atoms and molecules.

The differential equation is called the *Schrödinger equation* and its solution is called the *wavefunction,* Ψ.

What is the form of the *Schrödinger equation* ?

THE CLASSICAL WAVE EQUATION

We have seen previously that the wave equation in 1–d is:

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}$$

Where v is the speed of the wave. Can this be used for **matter** waves in free space?

Try a solution: e.g.

$$\Psi(x,t) = e^{i(kx-\omega t)}$$

Not correct! For a free particle we know that $E=p^2/2$m.

AN ALTERNATIVE….

Try a modified wave equation of the following type:

(α is a constant)

$$\frac{\partial^2 \Psi}{\partial x^2} = \alpha \frac{\partial \Psi}{\partial t}$$

Now try same solution as before: e.g.

$$\Psi(x,t) = e^{i(kx-\omega t)}$$

Hence, the equation for matter waves in *free space* is:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} = i\hbar \frac{\partial \Psi}{\partial t}$$

For $\Psi(x,t) = e^{i(kx-\omega t)}$ then we have $\dfrac{k^2\hbar^2}{2m}\Psi(x,t) = \hbar\omega\,\Psi(x,t)$

which has the form: (KE) × wavefunction = (Total energy) × wavefunction