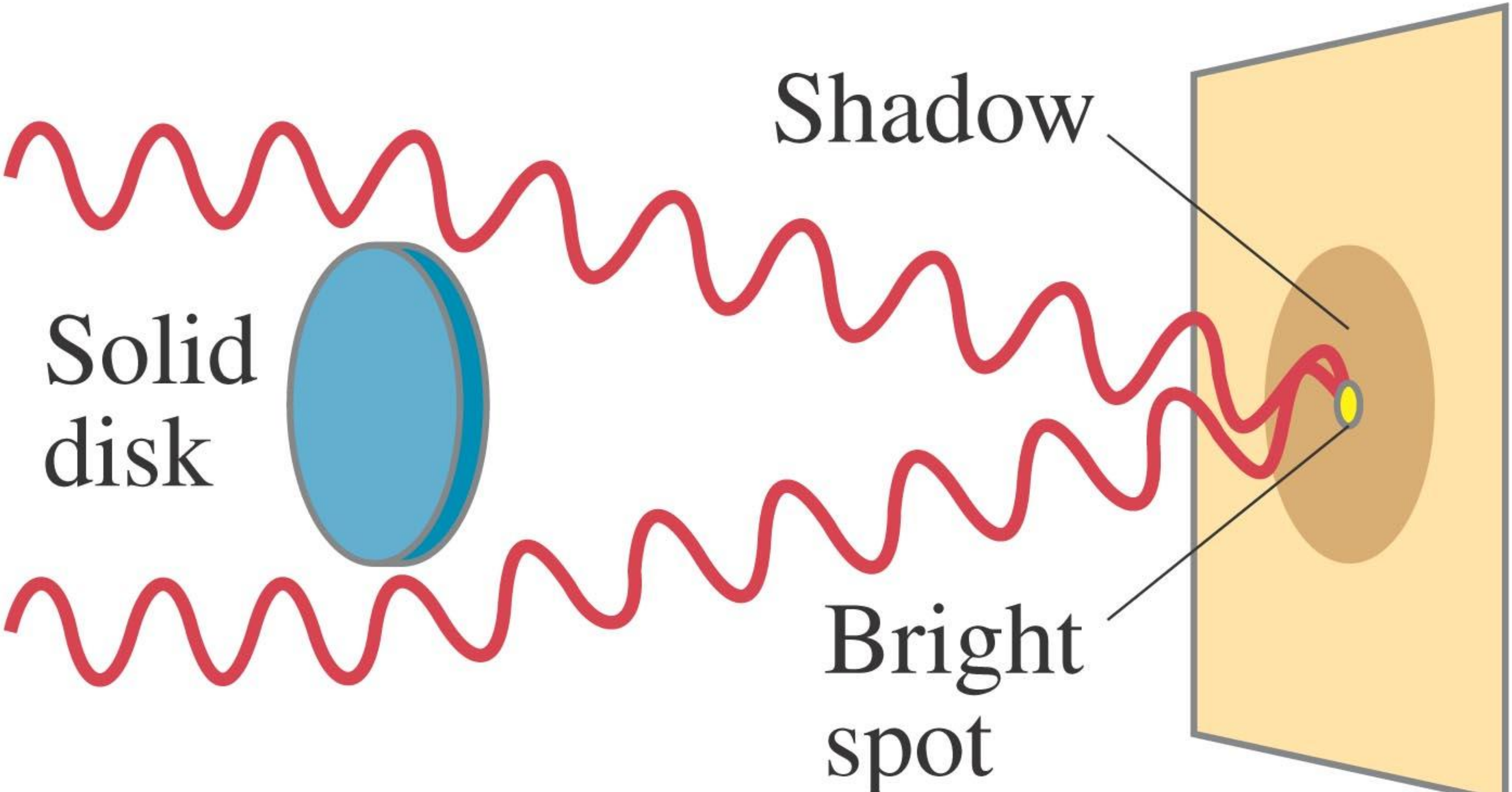
Single, Double And N-Slit Diffraction

B.Tech –I

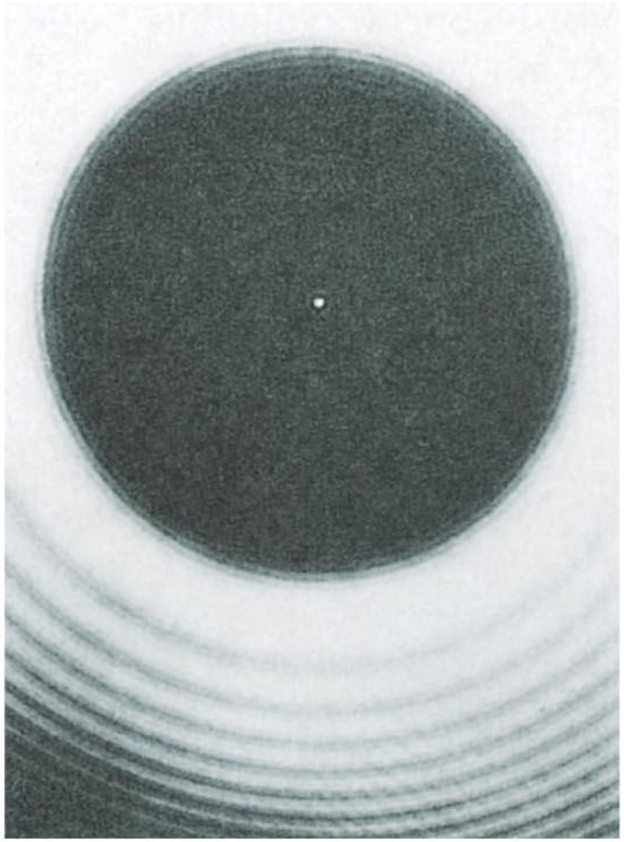
Diffraction by a Single Slit or Disk

If light is a wave, it will diffract around a single slit or obstacle.



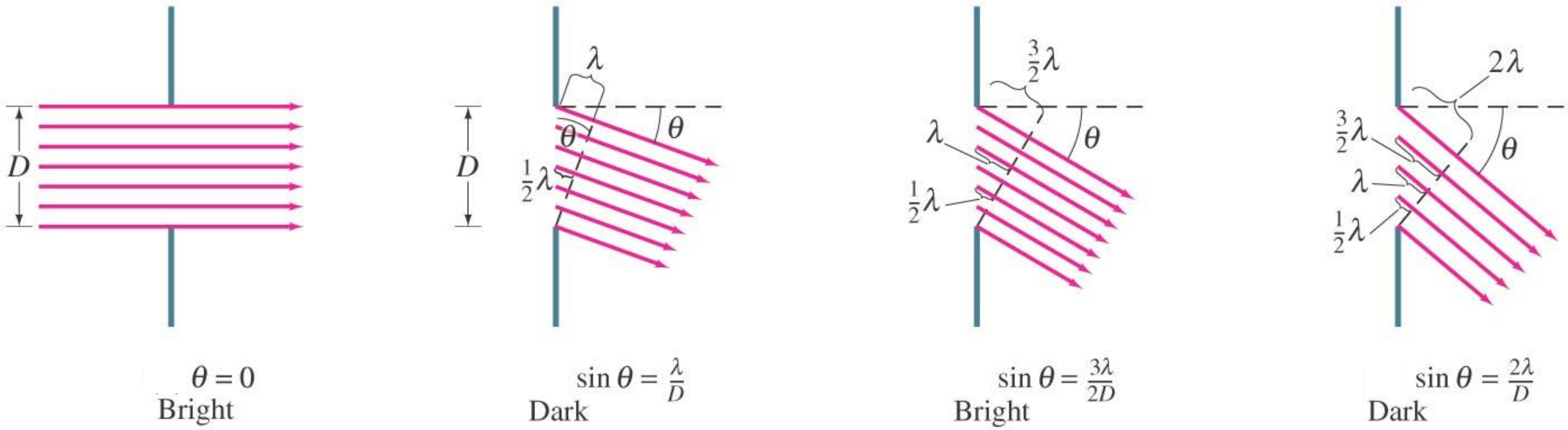
Diffraction by a Single Slit or Disk

The resulting pattern of light and dark stripes is called a diffraction pattern.

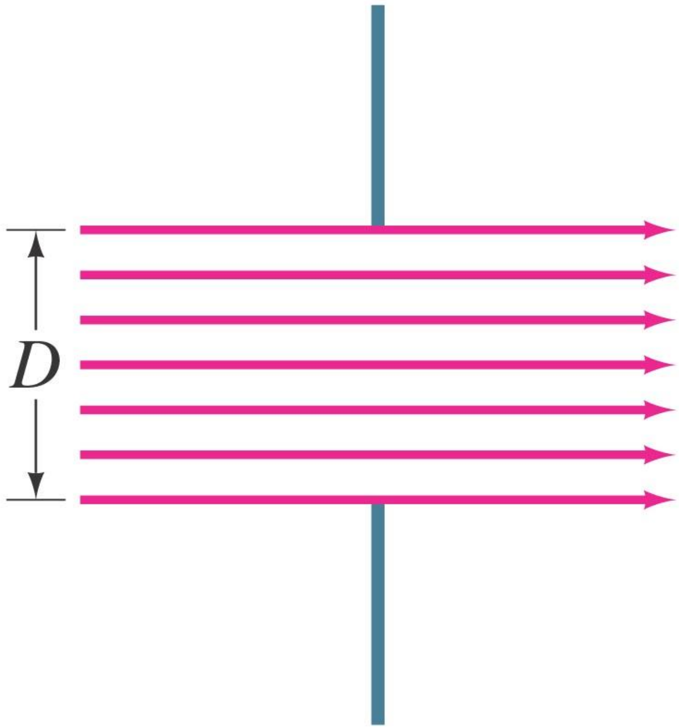


Diffraction by a Single Slit or Disk

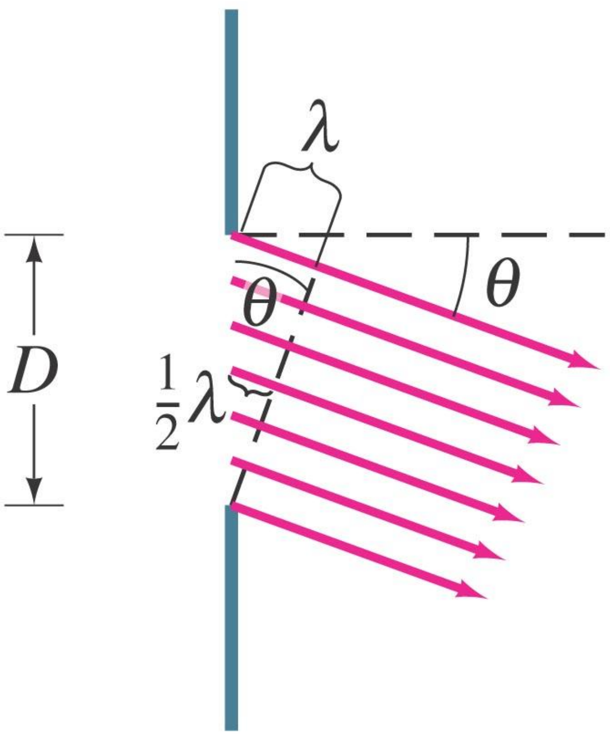
This pattern arises because different points along a slit create wavelets that interfere with each other just as a double slit would.



Diffraction by a Single Slit or Disk



(a) $\theta = 0$
Bright

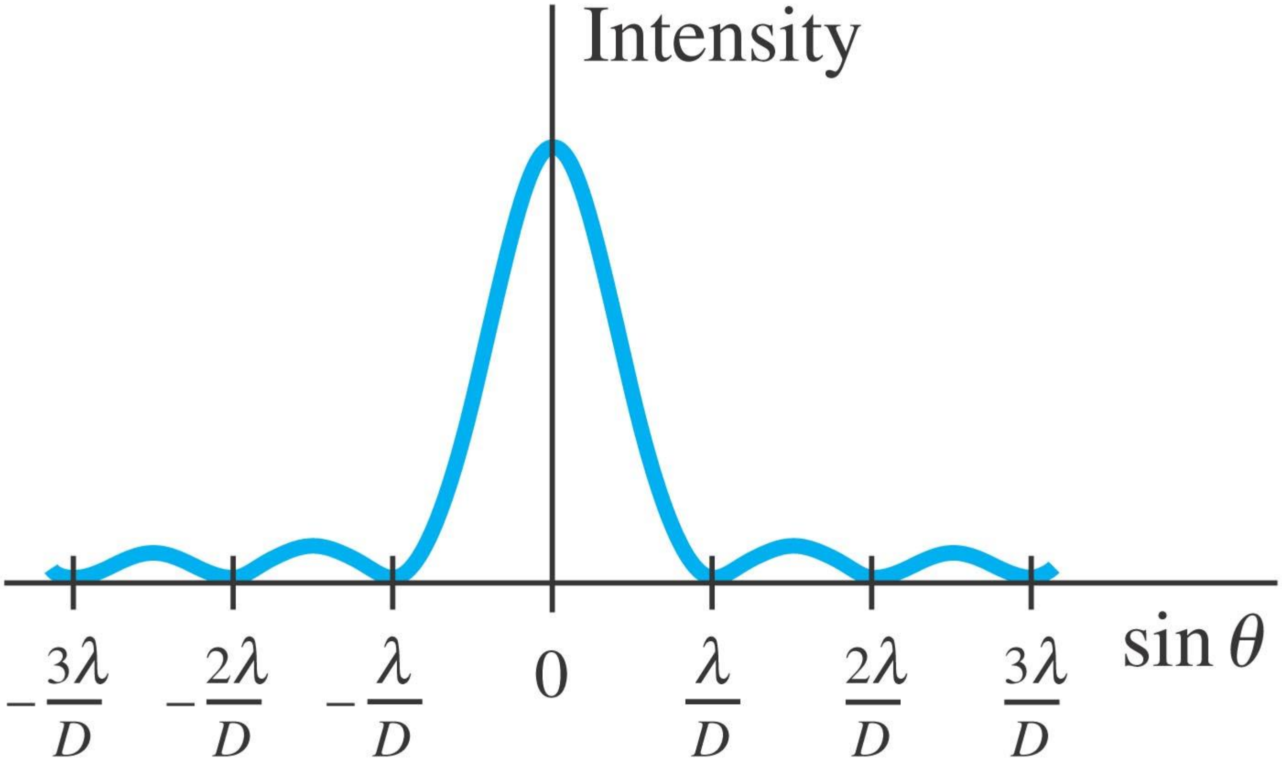


(b) $\sin\theta = \frac{\lambda}{D}$
Dark

Diffraction by a Single Slit or Disk

The minima of the single-slit diffraction pattern occur when

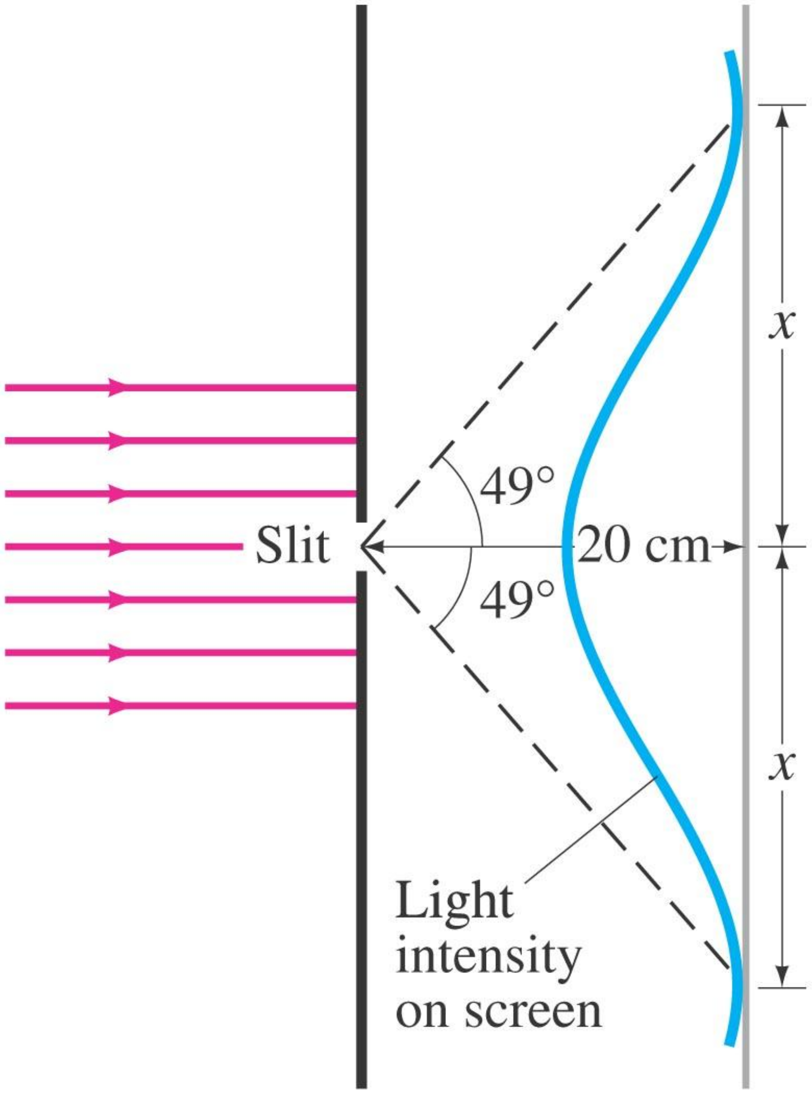
$$D\sin\theta = m\lambda, \qquad m = \pm 1, \pm 2, \pm 3, \cdots. \qquad \text{[minima]}$$



Diffraction by a Single Slit or Disk

Single-slit diffraction maximum.

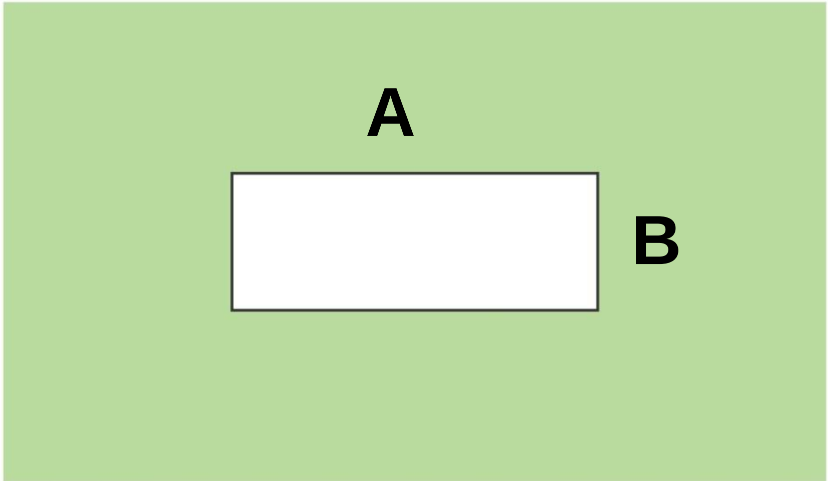
Light of wavelength 750 nm passes through a slit 1.0 x 10^{-3} mm wide. How wide is the central maximum (a) in degrees, and (b) in centimeters, on a screen 20 cm away?



Diffraction by a Single Slit or Disk

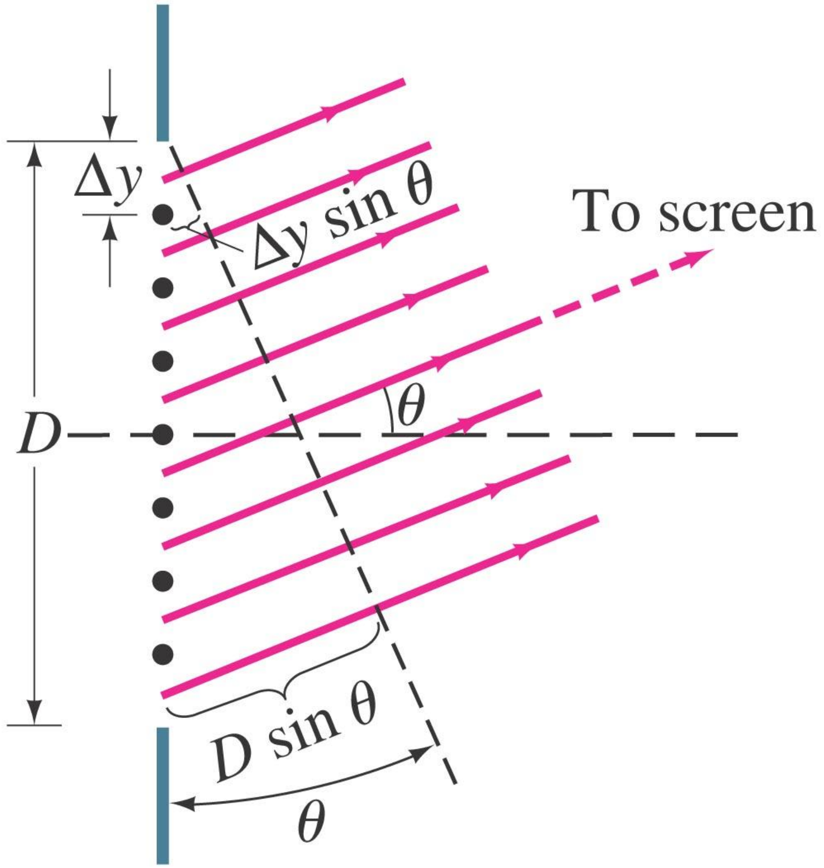
Diffraction spreads.

Light shines through a rectangular hole that is narrower in the vertical direction than the horizontal. (a) Would you expect the diffraction pattern to be more spread out in the vertical direction or in the horizontal direction? (b) Should a rectangular loudspeaker horn at a stadium be high and narrow, or wide and flat?



Intensity in Single-Slit Diffraction Pattern

Light passing through a single slit can be divided into a series of narrower strips; each contributes the same amplitude to the total intensity on the screen, but the phases differ due to the differing path lengths:



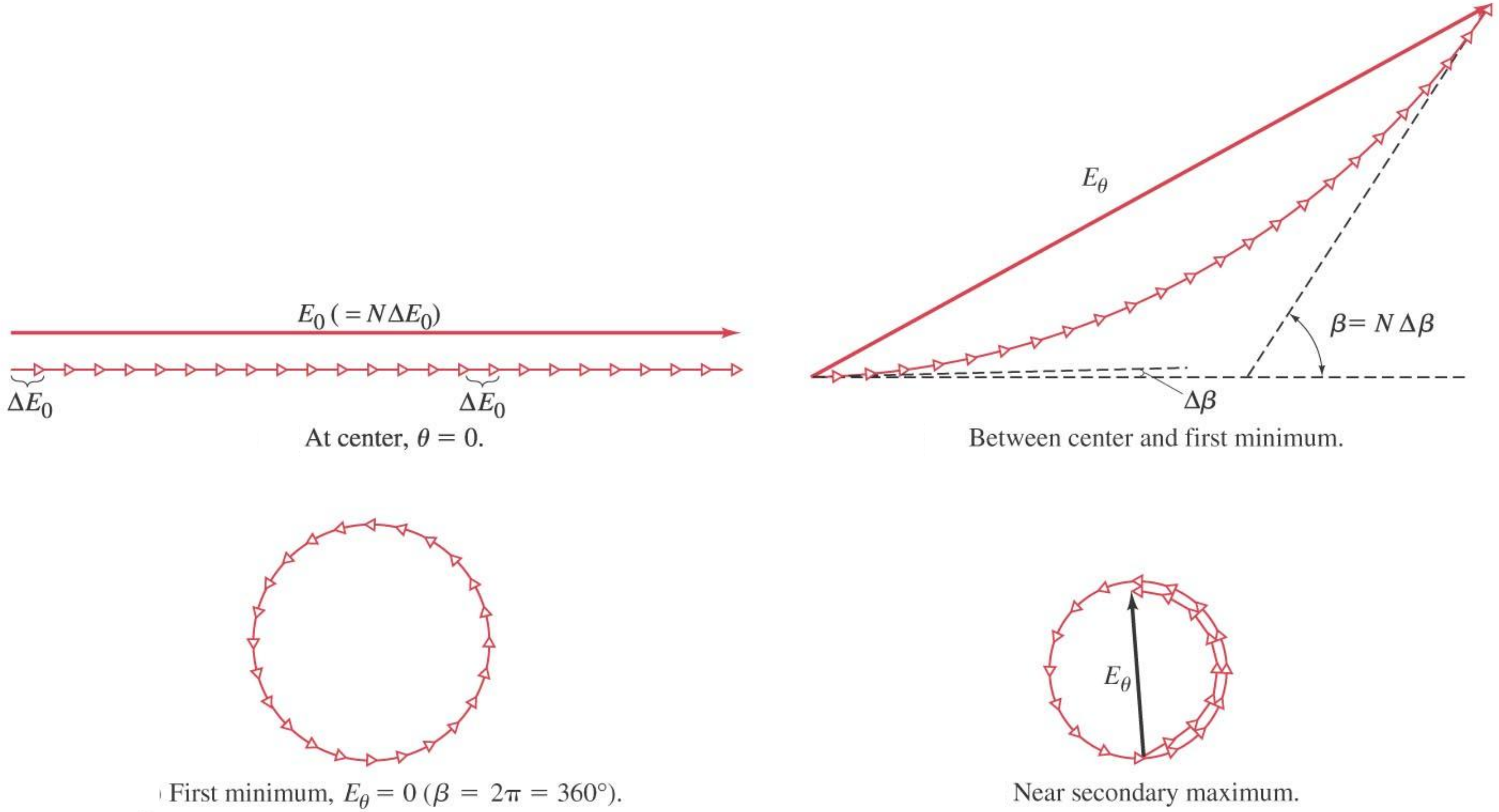
$$\Delta\beta = \frac{2\pi}{\lambda}\Delta y \sin\theta.$$

Slit of width D divided into N strips of width Δy. Each strip is a wave with intensity of I_0/N. Path difference between two adjacent strips is $\Delta y \sin\theta$ **and the corresponding phase angle difference is** $\Delta\beta = \beta / N$ **. The intensity of the diffraction is, by superposition, the vector sum of the N strips of light with N approaching infinity.**

$$E = \lim_{N\to\infty} \sum_{n=0}^{N-1} \frac{E_0}{N} \sin\left(\omega t + n\Delta\beta\right)$$

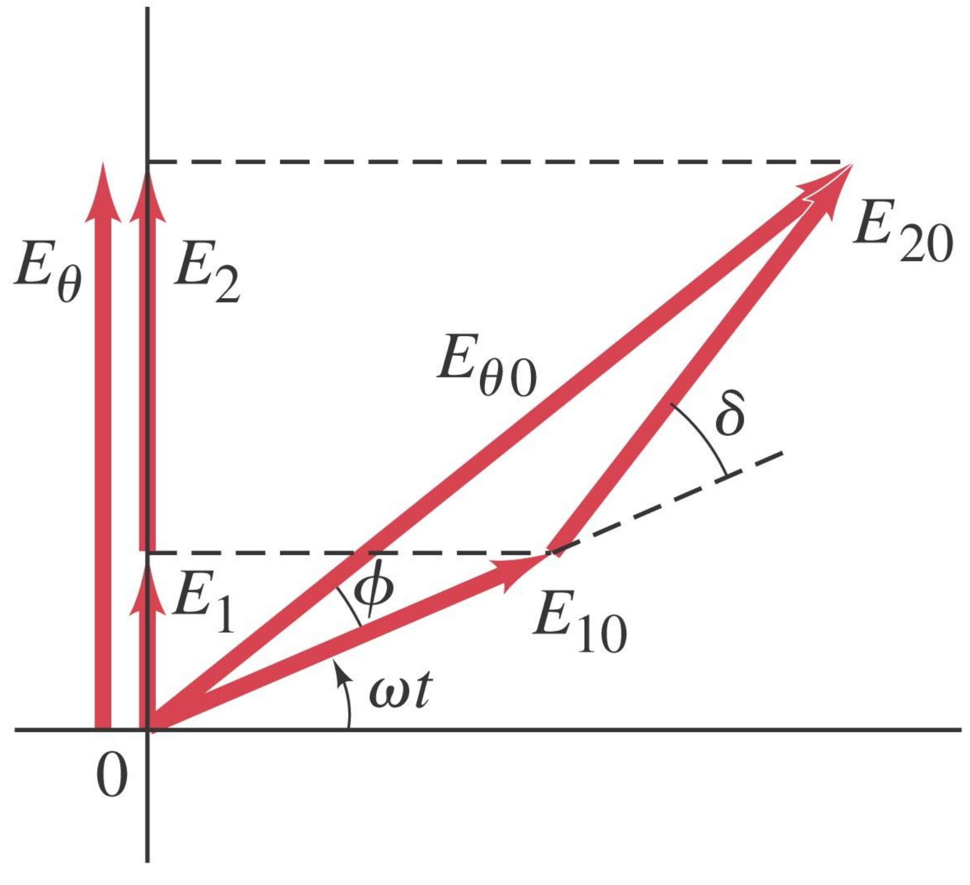
Intensity in Single-Slit Diffraction Pattern

Phasor diagrams give us the intensity as a function of angle.



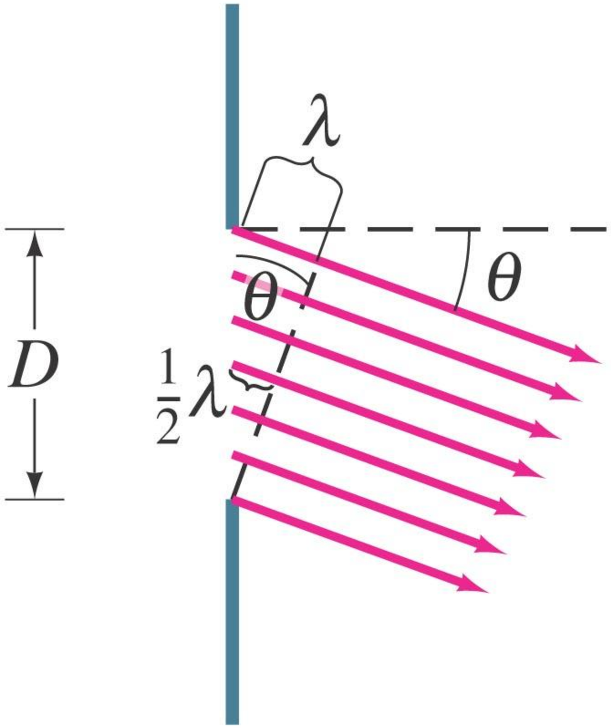
$$E_\theta = E_{\theta 0}\sin(\omega t + \phi)$$

$$= 2E_0 \cos\frac{\delta}{2}\sin\left(\omega t + \frac{\delta}{2}\right)$$

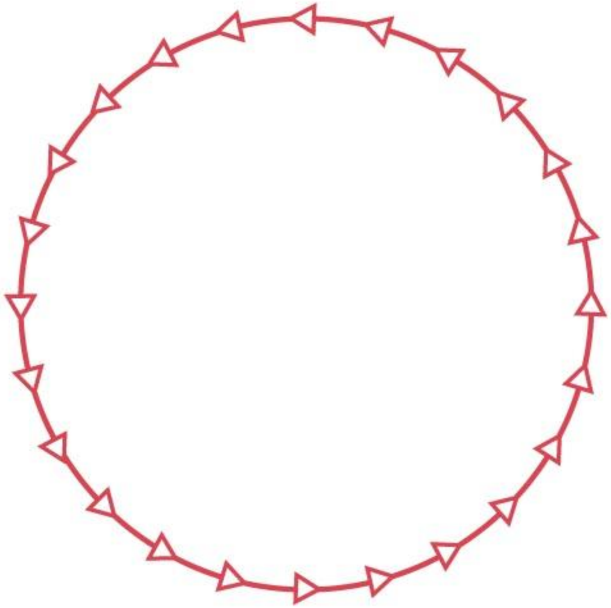


$$m = 1,\ D\sin\theta = \lambda$$

$$\beta = \frac{2\pi}{\lambda} D\sin\theta = 2\pi$$



(b) $\sin\theta = \frac{\lambda}{D}$
Dark

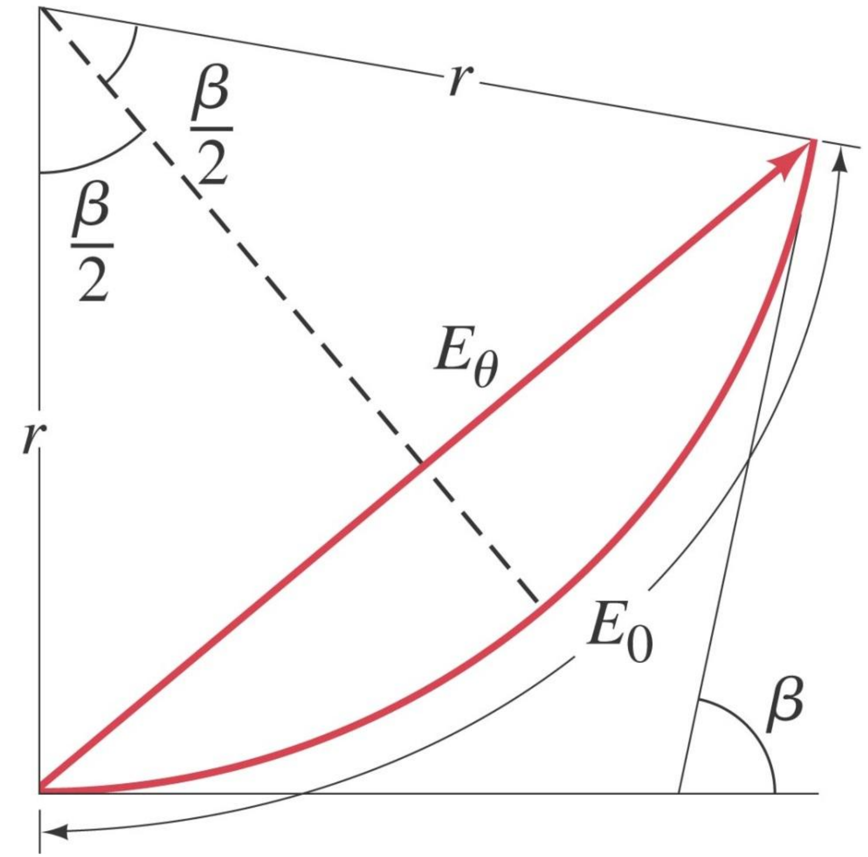


(c) First minimum, $E_\theta = 0$ ($\beta = 2\pi = 360°$).

Intensity in Single-Slit Diffraction Pattern

Taking the limit as the width becomes infinitesimally small gives the field as a function of angle:

$$E_\theta = E_0 \frac{\sin \beta/2}{\beta/2}.$$



Intensity in Single-Slit Diffraction Pattern

Finally, we have the phase difference and the intensity as a function of angle:

$$\beta = \frac{2\pi}{\lambda} D \sin\theta.$$

and

$$I_\theta = I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2 .$$

Intensity in Single-Slit Diffraction Pattern

Intensity at secondary maxima.

Estimate the intensities of the first two secondary maxima to either side of the central maximum.

Diffraction in the Double-Slit Experiment

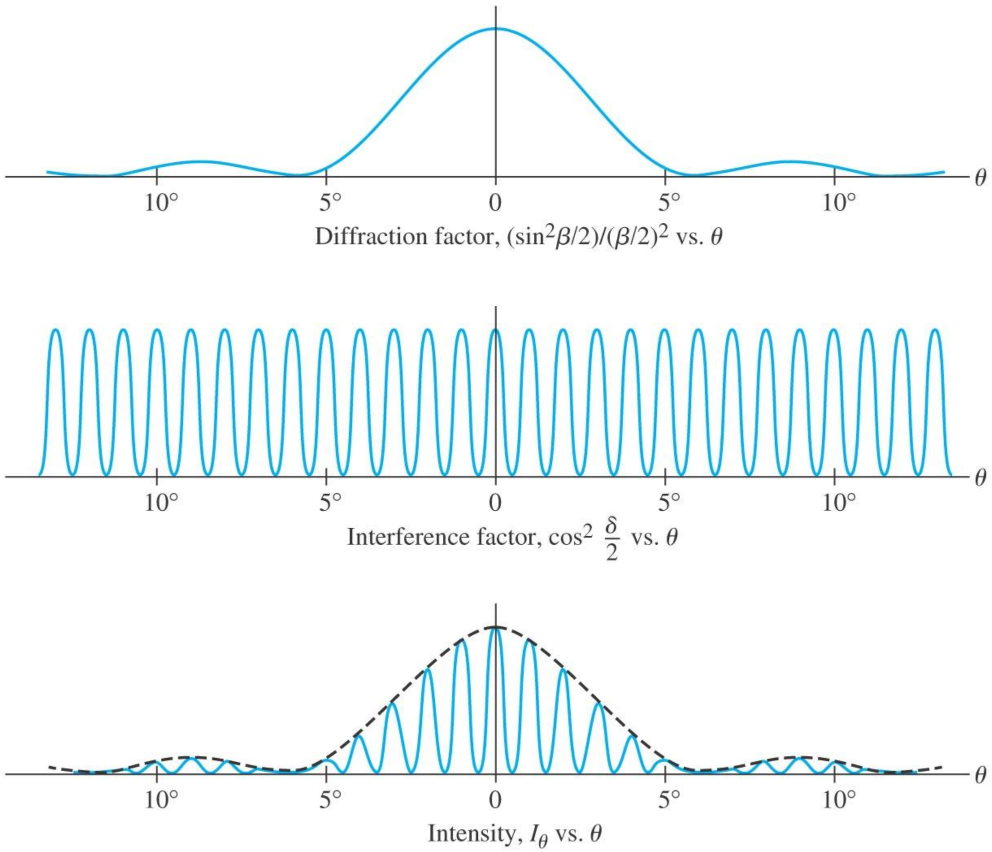
The double-slit experiment also exhibits diffraction effects, as the slits have a finite width. This means the amplitude at an angle θ will be modified by the same factor as in the single-slit experiment:

$$E_{\theta 0} = 2E_0\left(\frac{\sin \beta/2}{\beta/2}\right)\cos\frac{\delta}{2}.$$

The intensity is, as usual, proportional to the square of the field.

Diffraction in the Double-Slit Experiment

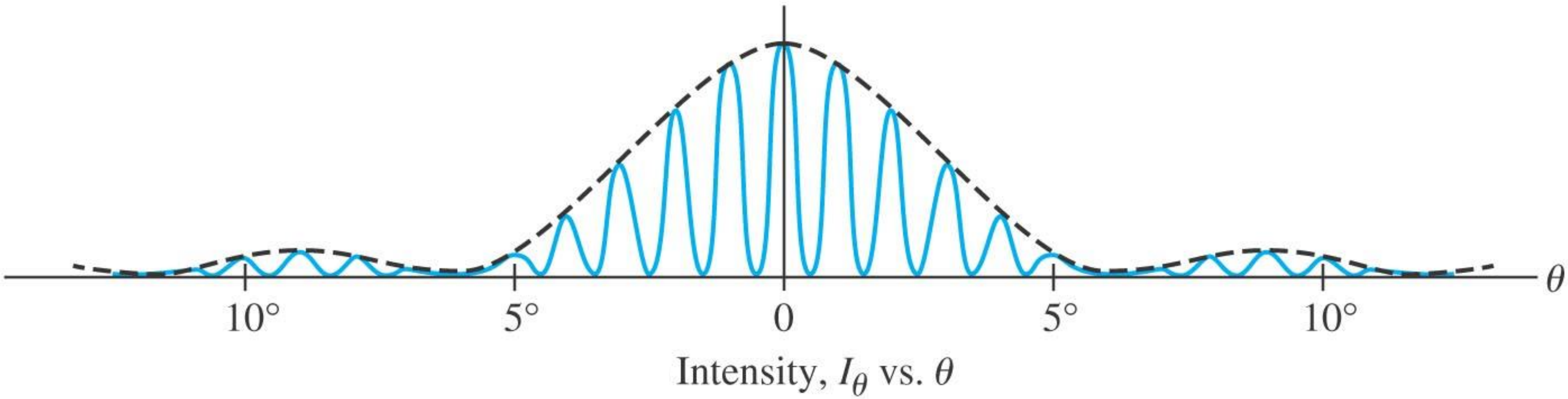
The diffraction factor (depends on *β*) appears as an “envelope” modifying the more rapidly varying interference factor (depends on *δ*).



Diffraction in the Double-Slit Experiment

Diffraction plus interference.

Show why the central diffraction peak shown, plotted for the case where $d = 6D = 60\lambda$, contains 11 interference fringes.



Limits of Resolution; Circular Apertures

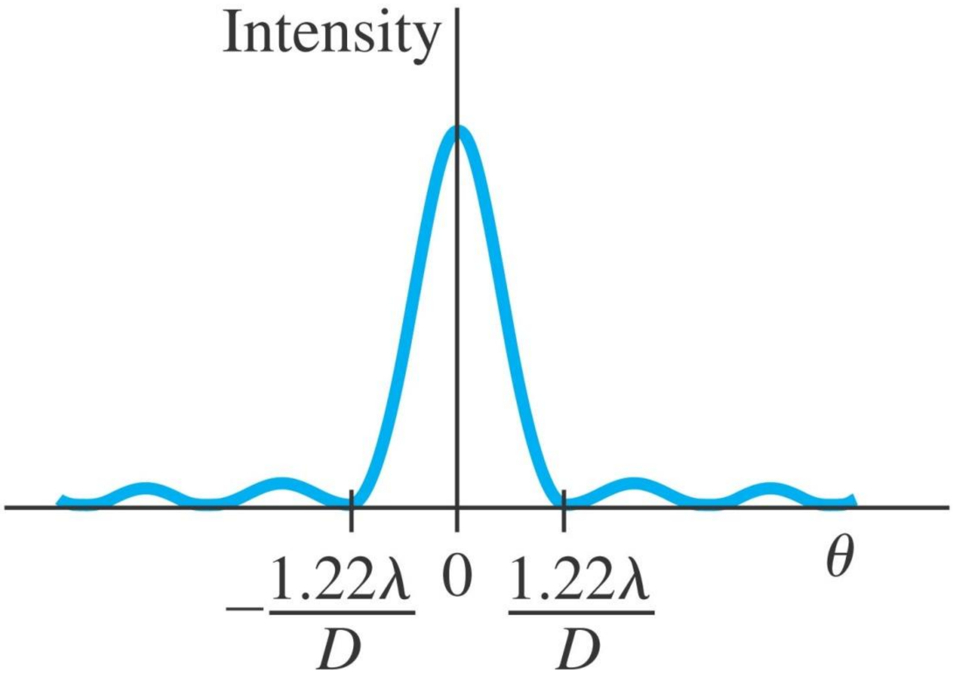
Resolution is the distance at which a lens can **barely** distinguish two separate objects.

Resolution is limited by aberrations and by diffraction. Aberrations can be minimized, but diffraction is unavoidable; it is due to the **size of the lens** compared to the **wavelength of the light.**

Limits of Resolution; Circular Apertures

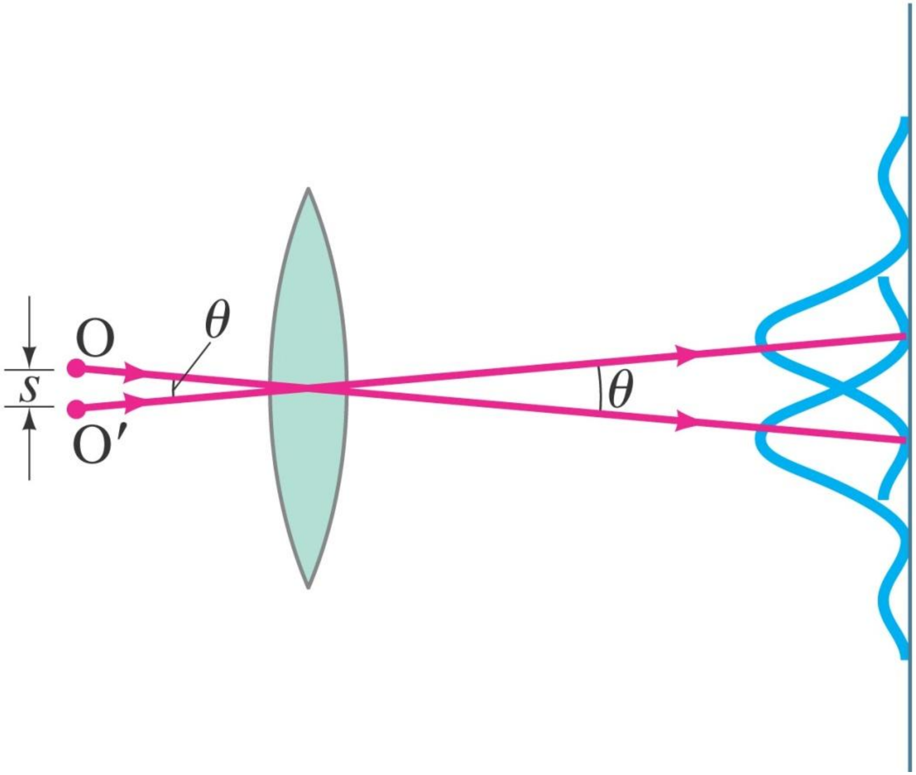
For a circular aperture of diameter *D*, the central maximum has an angular width:

$$\theta = \frac{1.22\lambda}{D}, \quad \theta \text{ in radians.}$$



Limits of Resolution; Circular Apertures

The Rayleigh criterion states that two images are just resolvable when the center of one peak is over the first minimum of the other.



Limits of Resolution; Circular Apertures

Hubble Space Telescope.

The Hubble Space Telescope (HST) is a reflecting telescope that was placed in orbit above the Earth's atmosphere, so its resolution would not be limited by turbulence in the atmosphere. Its objective diameter is 2.4 m. For visible light, say λ = 550 nm, estimate the improvement in resolution the Hubble offers over Earth-bound telescopes, which are limited in resolution by movement of the Earth's atmosphere to about half an arc second. (Each degree is divided into 60 minutes each containing 60 seconds, so 1° = 3600 arc seconds.)

Limits of Resolution; Circular Apertures

Eye resolution.

You are in an airplane at an altitude of 10,000 m. If you look down at the ground, estimate the minimum separation s between objects that you could distinguish. Could you count cars in a parking lot? Consider only diffraction, and assume your pupil is about 3.0 mm in diameter and λ = 550 nm.