Introduction to Trees

Objectives

Upon completion you will be able to:

- Understand and use basic tree terminology and concepts
- Recognize and define the basic attributes of a binary tree
- Process trees using depth-first and breadth-first traversals
- Parse expressions using a binary tree
- Design and implement Huffman trees
- Understand the basic use and processing of general trees

Basic Tree Concepts

- A tree consists of finite set of elements, called nodes, and a finite set of directed lines called branches, that connect the nodes.
- The number of branches associated with a node is the degree of the node.

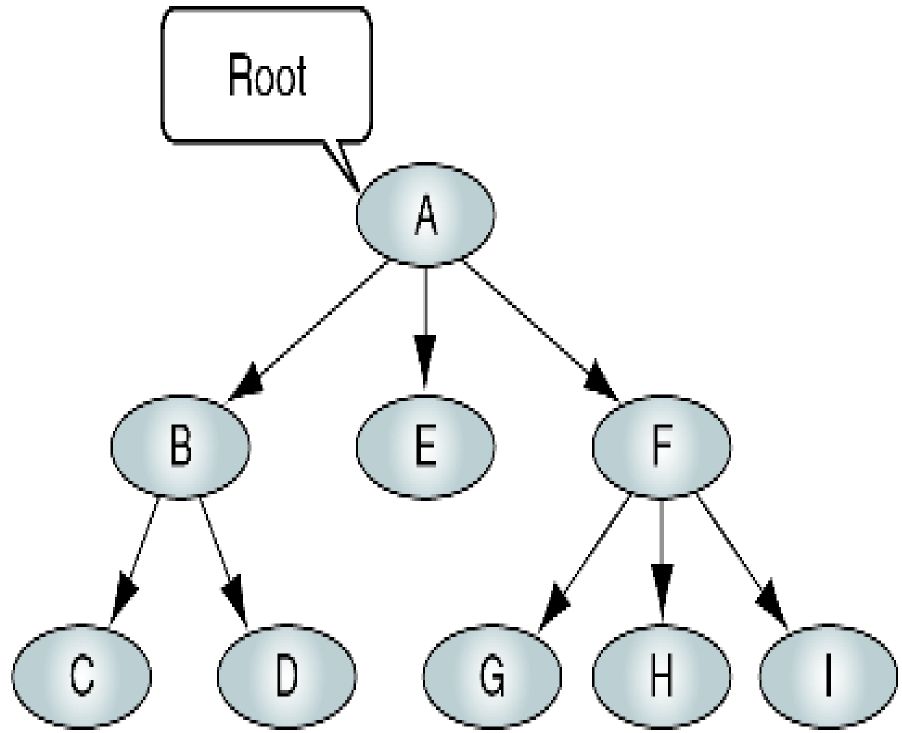


FIGURE 6-1 Tree

Basic Tree Concepts

- When the branch is directed toward the node, it is indegree branch.
- When the branch is directed away from the node, it is an outdegree branch.
- The sum of the indegree and outdegree branches is the degree of the node.
- If the tree is not empty, the first node is called the root.

Basic Tree Concepts

- The indegree of the root is, by definition, zero.
- With the exception of the root, all of the nodes in a tree must have an indegree of exactly one; that is, they may have only one predecessor.
- All nodes in the tree can have zero, one, or more branches leaving them; that is, they may have outdegree of zero, one, or more.

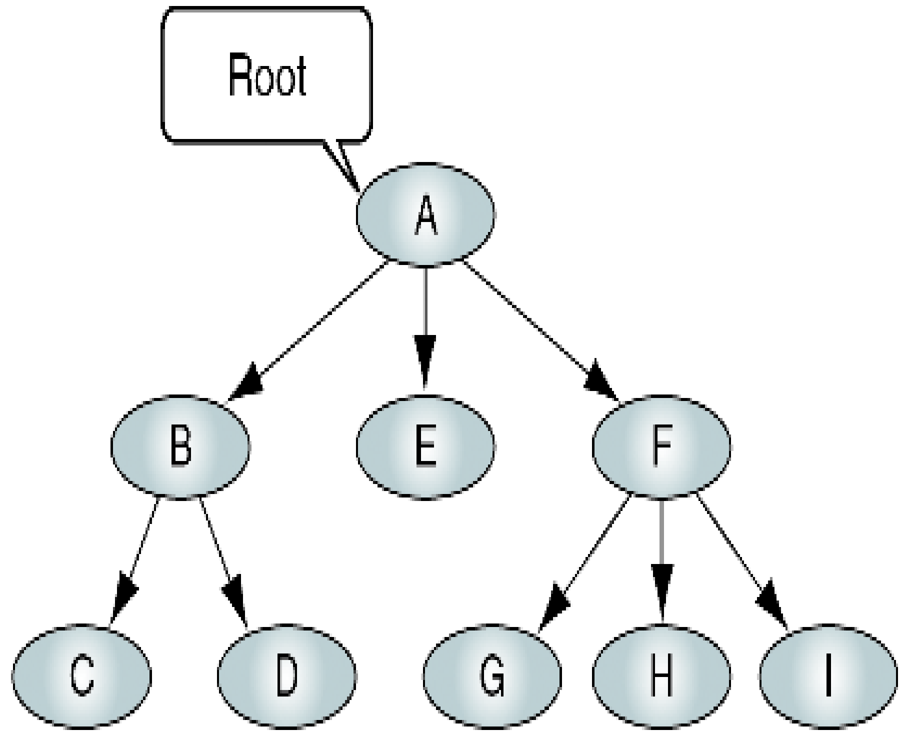


FIGURE 6-1 Tree

Basic Tree Concepts

- A leaf is any node with an outdegree of zero, that is, a node with no successors.
- A node that is not a root or a leaf is known as an internal node.
- A node is a parent if it has successor nodes; that is, if it has outdegree greater than zero.
- A node with a predecessor is called a child.

Basic Tree Concepts

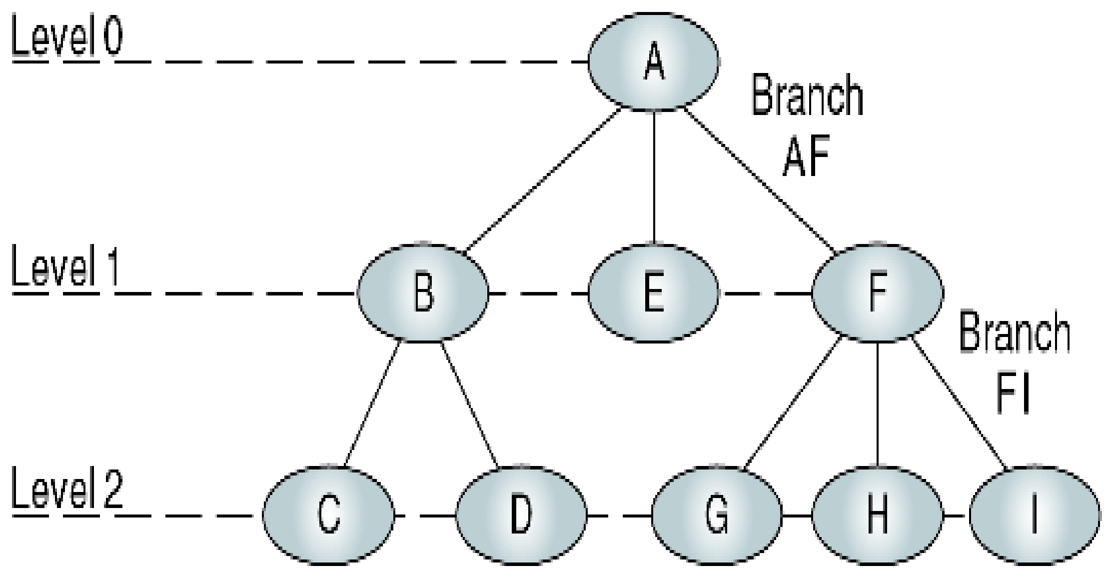
- Two or more nodes with the same parents are called siblings.
- An ancestor is any node in the path from the root to the node.
- A descendant is any node in the path below the parent node; that is, all nodes in the paths from a given node to a leaf are descendants of that node.

Basic Tree Concepts

- A path is a sequence of nodes in which each node is adjacent to the next node.
- The level of a node is its distance from the root. The root is at level 0, its children are at level 1, etc. …

Basic Tree Concepts

- The height of the tree is the level of the leaf in the longest path from the root plus 1. By definition the height of any empty tree is -1.
- A subtree is any connected structure below the root. The first node in the subtree is known is the root of the subtree.



Root: A	Siblings: {B,E,F}, {C,D}, {G,H,I}
Parents: A, B, F	Leaves: C,D,E,G,H,I
Children: B, E, F, C, D, G, H, I	Internal nodes: B,F

FIGURE 6-2 Tree Nomenclature

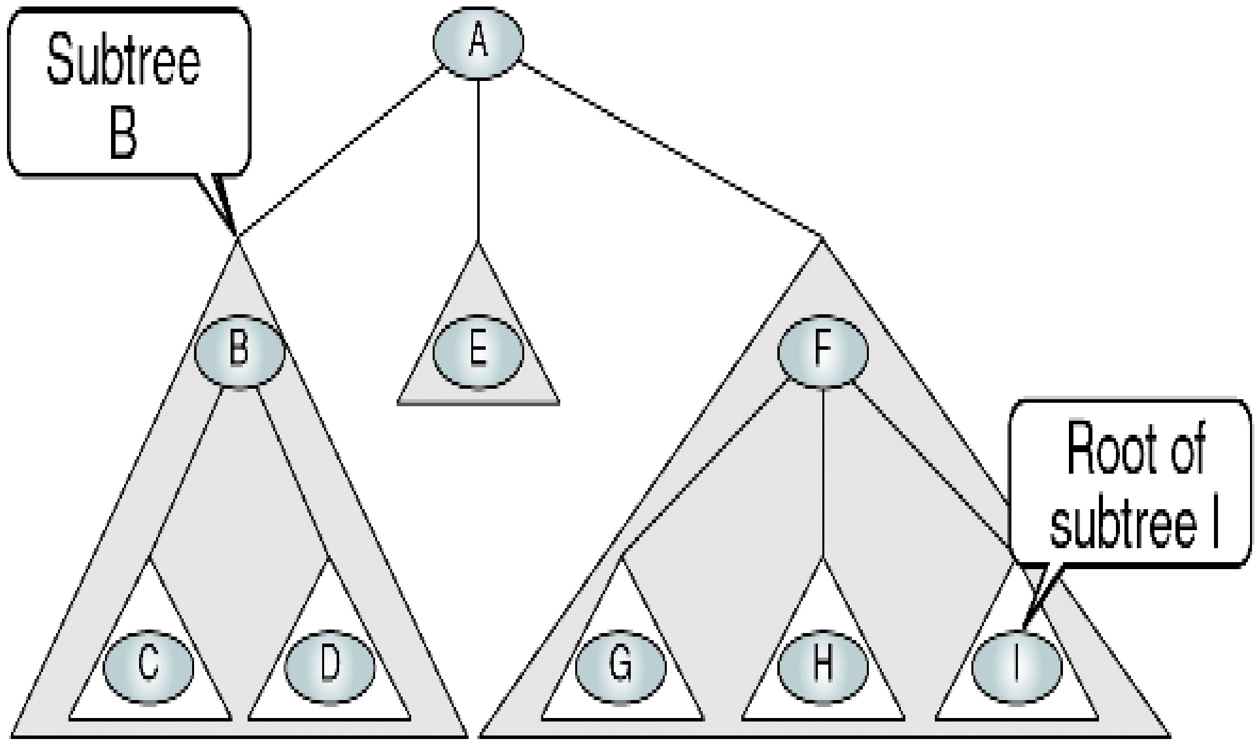


FIGURE 6-3 Subtrees

Recursive definition of a tree

- A tree is a set of nodes that either:
- is empty or
- has a designated node, called the root, from which hierarchically descend zero or more subtrees, which are also trees.

Tree Representation

- General Tree – organization chart format
- Indented list – bill-of-materials system in which a parts list represents the assembly structure of an item

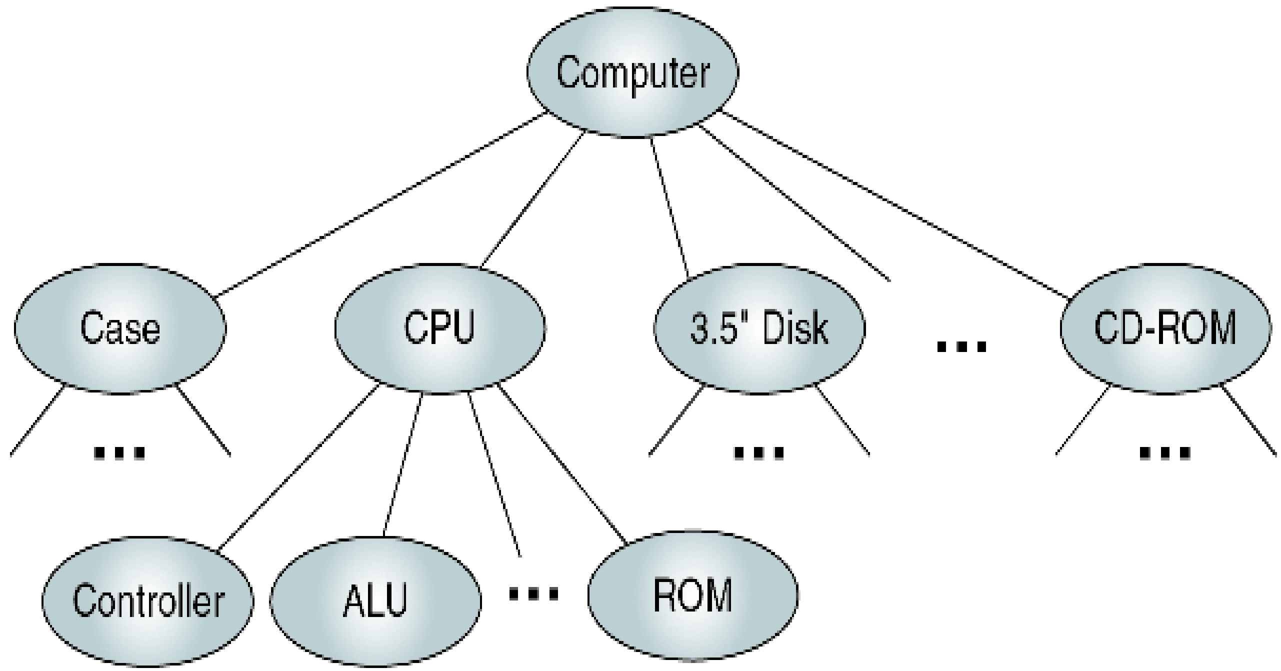


FIGURE 6-4 Computer Parts List as a General Tree

Part number	Description
301	Computer
301-1	Case
...	...
301-2	CPU
301-2-1	Controller
301-2-2	ALU
...	...
301-2-9	ROM
301-3	3.5" Disk
...	...
301-9	CD-ROM
...	...

TABLE 6-1 Computer Bill of Materials

Parenthetical Listing

- Parenthetical Listing – the algebraic expression, where each open parenthesis indicates the start of a new level and each closing parenthesis completes the current level and moves up one level in the tree.

Parenthetical Listing

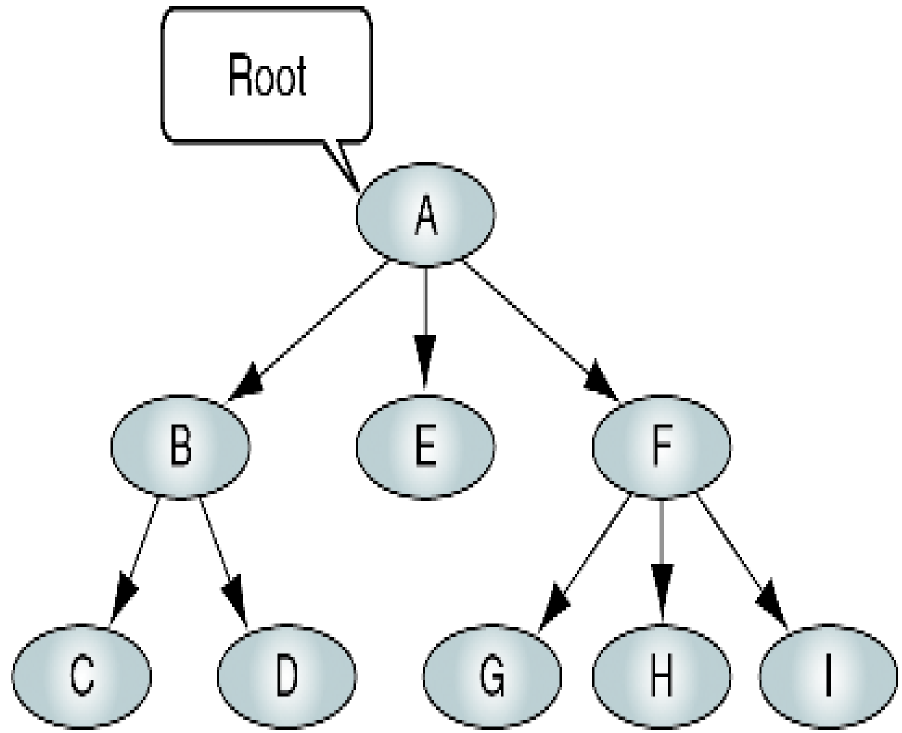


FIGURE 6-1 Tree

A (B (C D) E F (G H I))

ALGORITHM 6-1 Convert General Tree to Parenthetical Notation

```
Algorithm ConvertToParen (root, output)
Convert a general tree to parenthetical notation.
   Pre  root is a pointer to a tree node
   Post output contains parenthetical notation
1 Place root in output
2 if (root is a parent)
   1  Place an open parenthesis in the output
   2  ConvertToParen (root's first child)
   3  loop (more siblings)
      1  ConvertToParen (root's next child)
```

continued

ALGORITHM 6-1 Convert General Tree to Parenthetical Notation *(continued)*

```
   4  end loop
   5  Place close parenthesis in the output
3 end if
4 return
end ConvertToParen
```

6-2 Binary Trees

A binary tree can have no more than two descendents. In this section we discuss the properties of binary trees, four different binary tree traversals

- **Properties**
- **Binary Tree Traversals**
- **Expression Trees**
- **Huffman Code**

Binary Trees

- A **binary tree** is a tree in which no node can have more than two subtrees; the maximum outdegree for a node is two.
- In other words, a node can have zero, one, or two subtrees.
- These subtrees are designated as the left subtree and the right subtree.

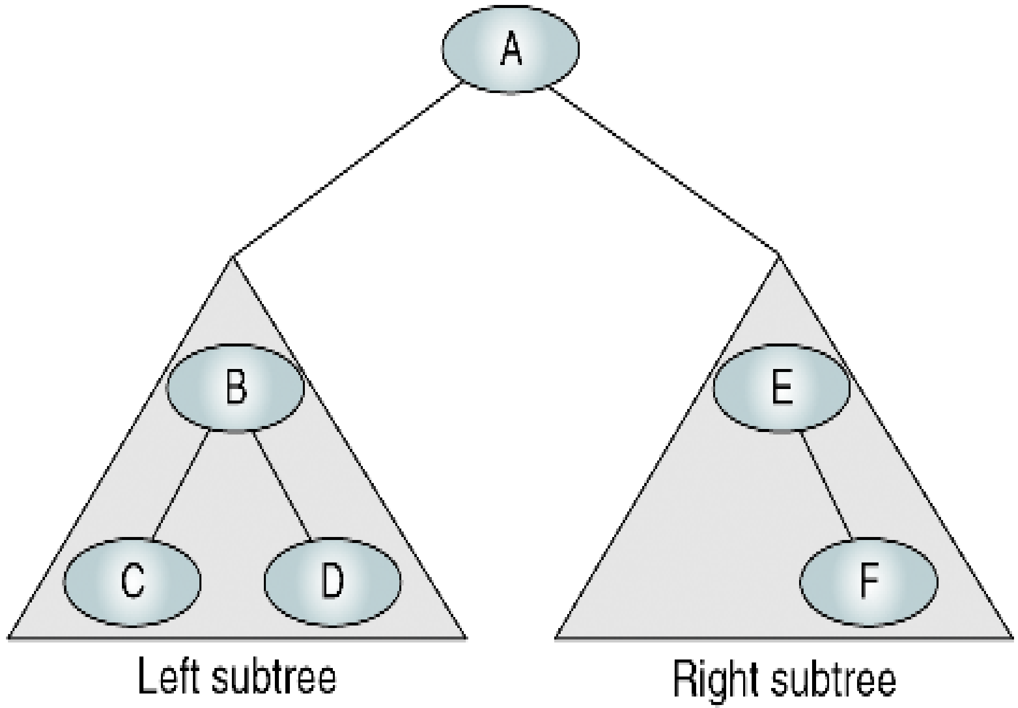


FIGURE 6-5 Binary Tree

A null tree is a tree with no nodes

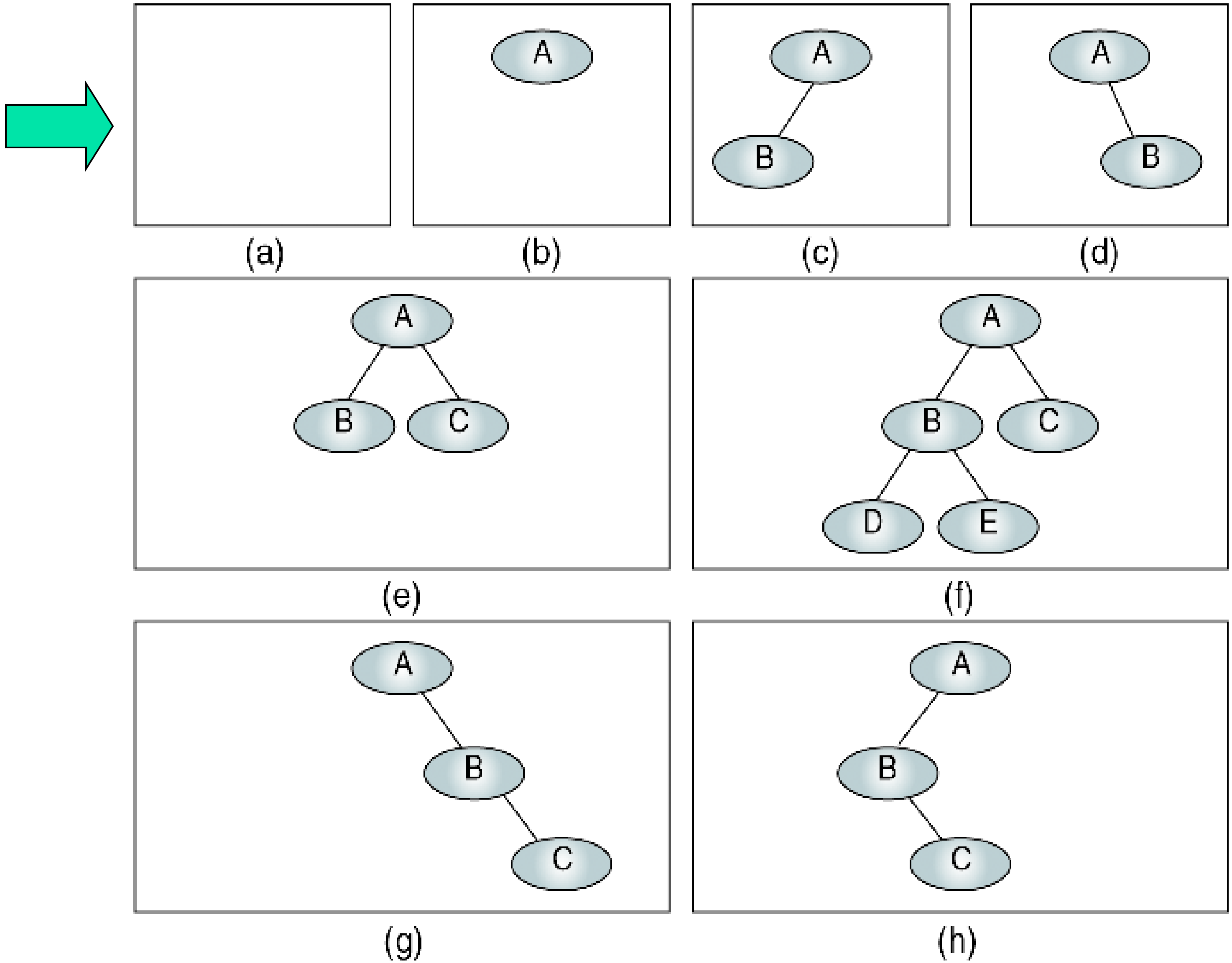


FIGURE 6-6 Collection of Binary Trees

Some Properties of Binary Trees

- The height of binary trees can be mathematically predicted
- Given that we need to store N nodes in a binary tree, the maximum height is

$$H_{\max} = N$$

A tree with a maximum height is rare. It occurs when all of the nodes in the entire tree have only one successor.

Some Properties of Binary Trees

- The minimum height of a binary tree is determined as follows:

$$H_{\min} = \left[\log_2 N\right] + 1$$

For instance, if there are three nodes to be stored in the binary tree (*N*=3) then $H_{\min}$=2.

Some Properties of Binary Trees

- Given a height of the binary tree, H, the minimum number of nodes in the tree is given as follows:

$$N_{\min} = H$$

Some Properties of Binary Trees

- The formula for the maximum number of nodes is derived from the fact that each node can have only two descendents. Given a height of the binary tree, H, the maximum number of nodes in the tree is given as follows:

$$N_{\max} = 2^{H} - 1$$

Some Properties of Binary Trees

- The children of any node in a tree can be accessed by following only one branch path, the one that leads to the desired node.
- The nodes at level 1, which are children of the root, can be accessed by following only one branch; the nodes of level 2 of a tree can be accessed by following only two branches from the root, etc.
- The balance factor of a binary tree is the difference in height between its left and right subtrees:

$$B = H_L - H_R$$

Balance of the tree

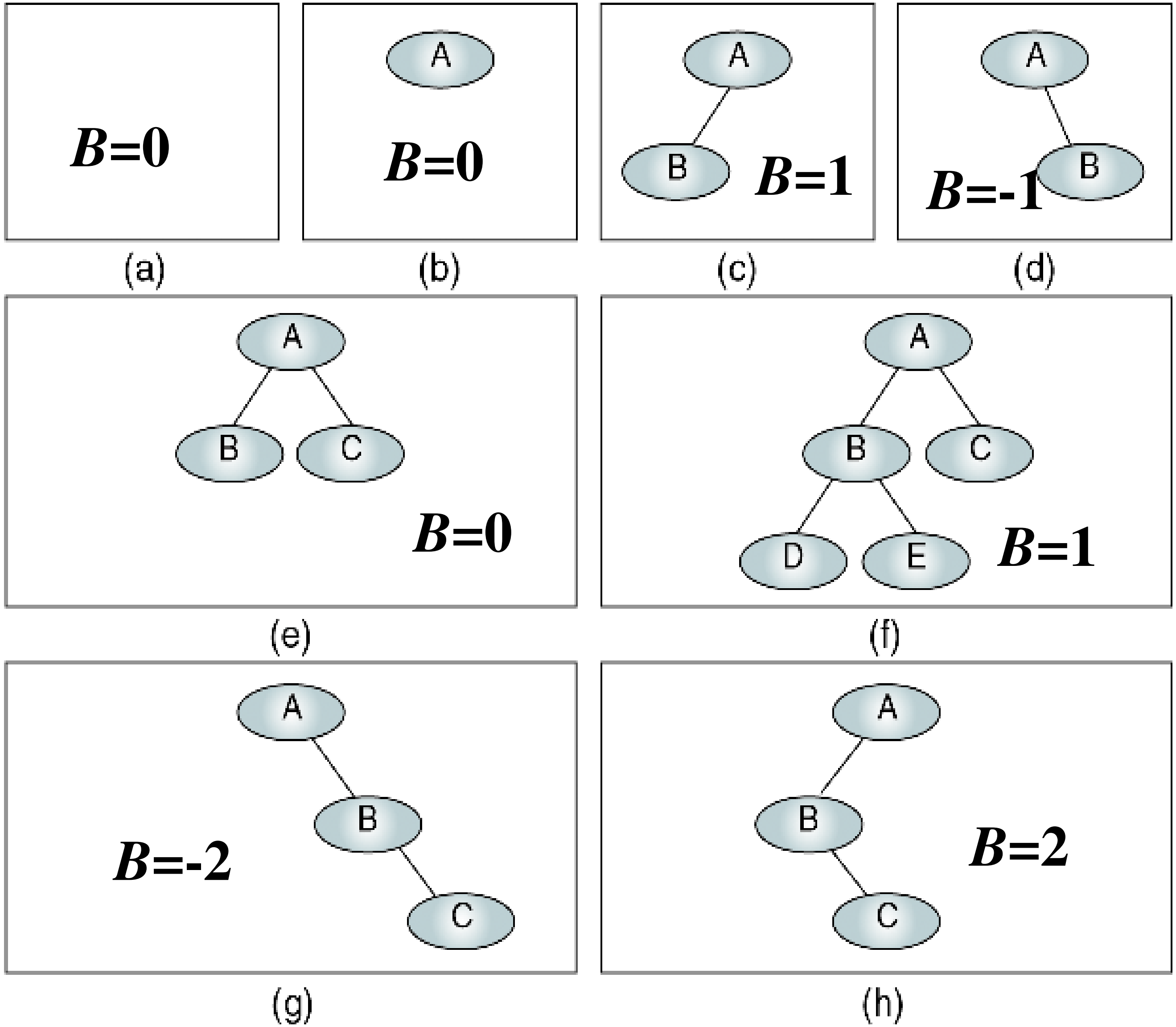


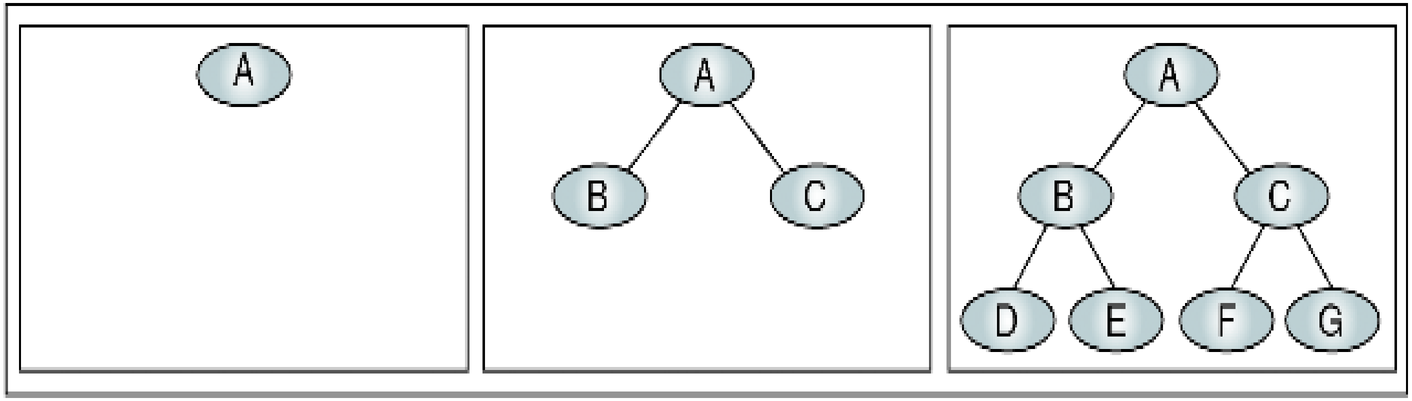
FIGURE 6-6 Collection of Binary Trees

Some Properties of Binary Trees

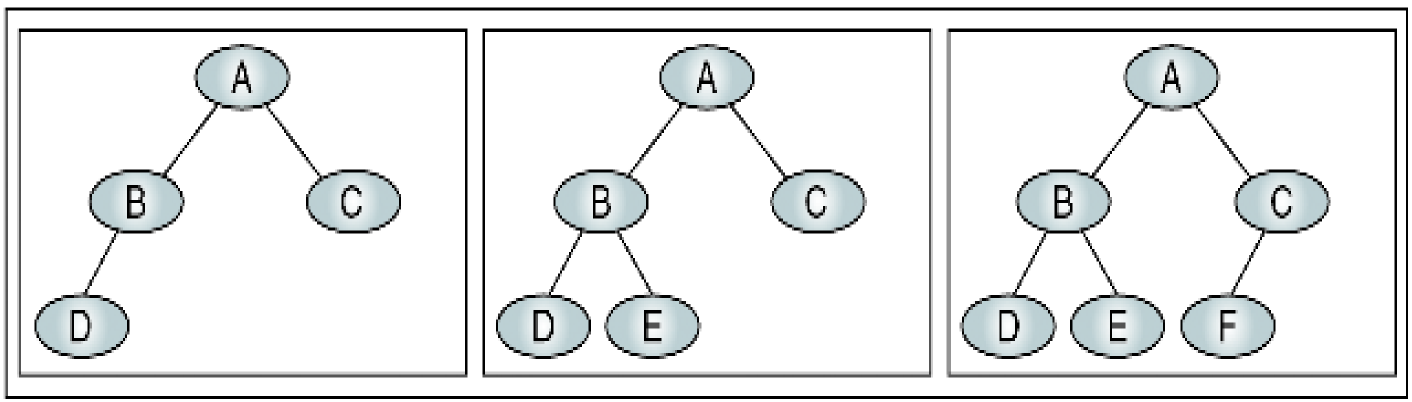
- In the balanced binary tree (definition of Russian mathematicians Adelson-Velskii and Landis) the height of its subtrees differs by no more than one (its balance factor is -1, 0, or 1), and its subtrees are also balanced.

Complete and nearly complete binary trees

- A complete tree has the maximum number of entries for its height. The maximum number is reached when the last level is full.
- A tree is considered nearly complete if it has the minimum height for its nodes and all nodes in the last level are found on the left



(a) Complete trees (at levels 0, 1, and 2)



(b) Nearly complete trees (at level 2)

FIGURE 6-7 Complete and Nearly Complete Trees

Binary Tree Traversal

- A binary tree traversal requires that each node of the tree be processed once and only once in a predetermined sequence.
- In the depth-first traversal processing process along a path from the root through one child to the most distant descendant of that first child before processing a second child.

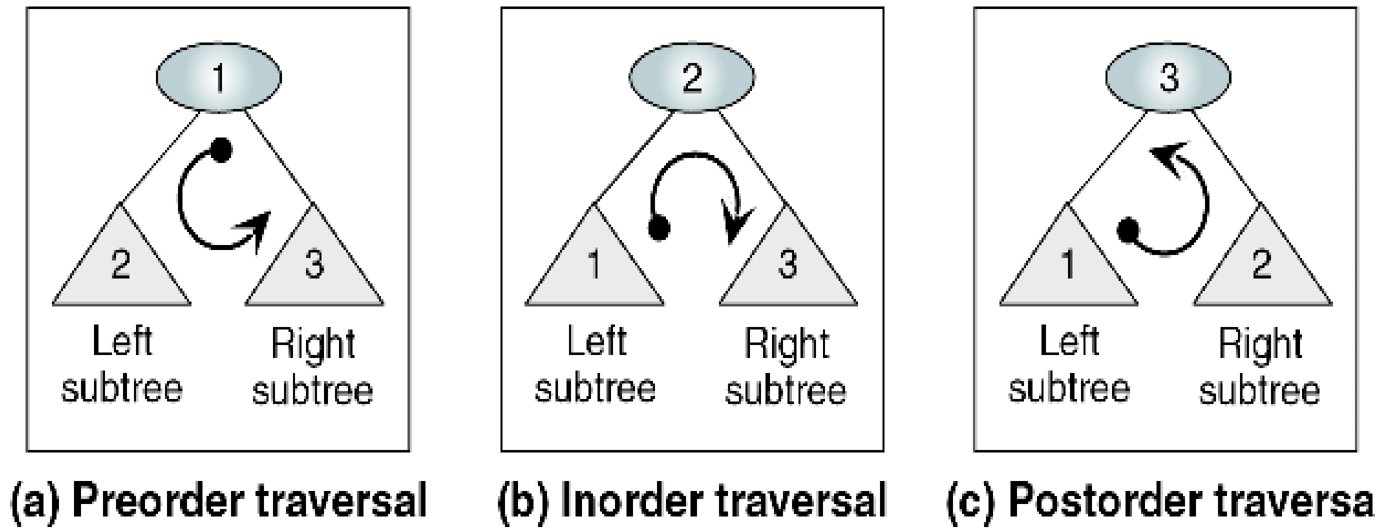


FIGURE 6-8 Binary Tree Traversals

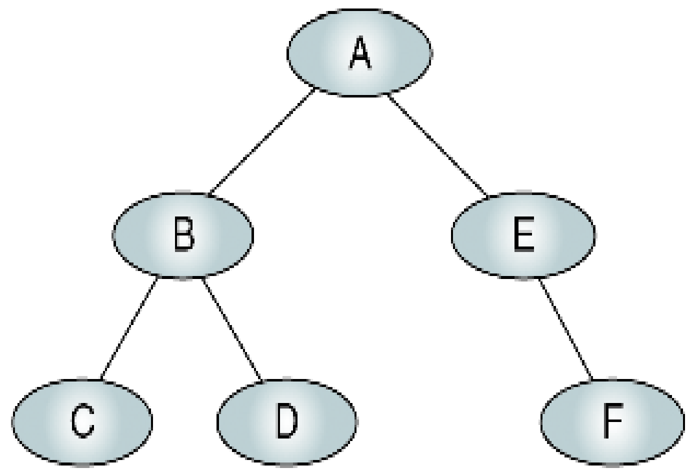
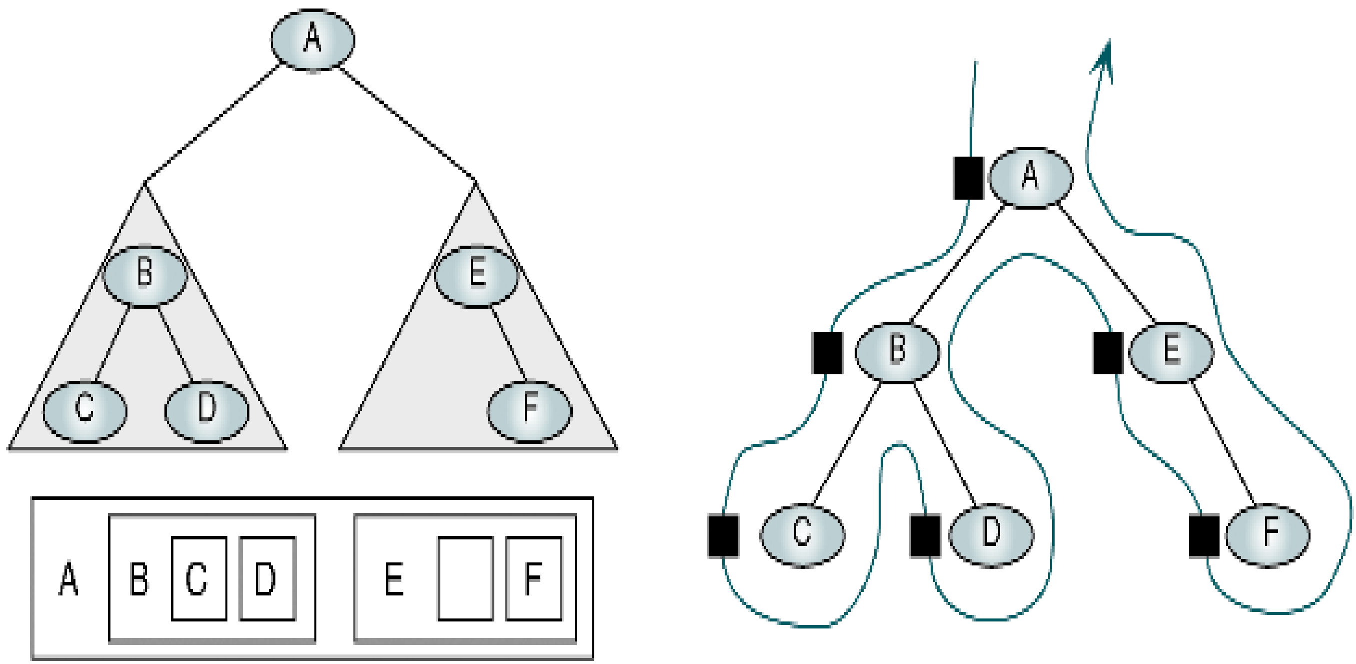


FIGURE 6-9 Binary Tree for Traversals

ALGORITHM 6-2 Preorder Traversal of a Binary Tree

```
Algorithm preOrder (root)
Traverse a binary tree in node-left-right sequence.
   Pre  root is the entry node of a tree or subtree
   Post each node has been processed in order
1 if (root is not null)
   1 process (root)
   2 preOrder (leftSubtree)
   3 preOrder (rightSubtree)
2 end if
end preOrder
```



(a) Processing order

(b) “Walking” order

FIGURE 6-10 Preorder Traversal—A B C D E F

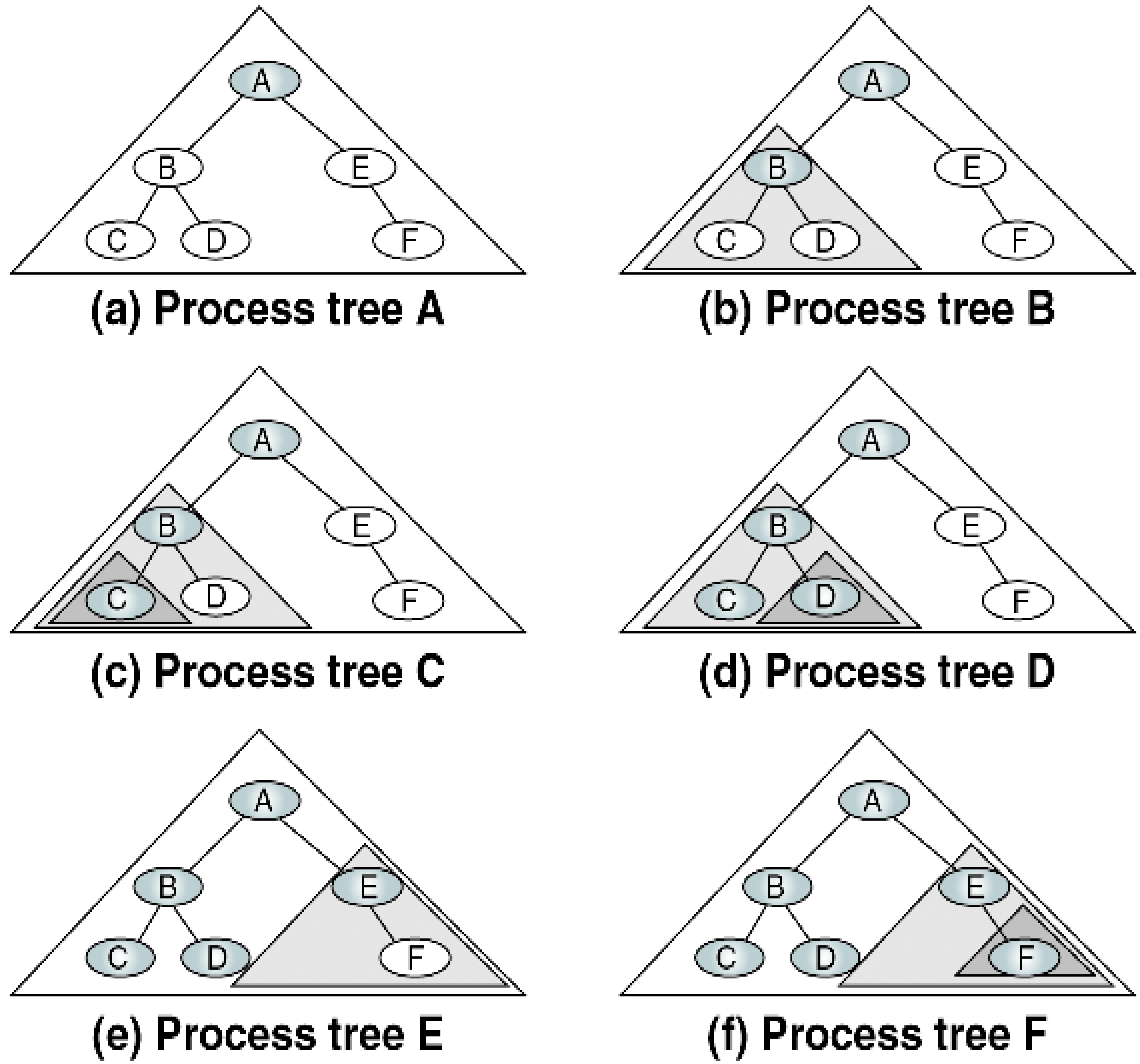


FIGURE 6-11 Algorithmic Traversal of Binary Tree

ALGORITHM 6-3 Inorder Traversal of a Binary Tree

```
Algorithm inOrder (root)
Traverse a binary tree in left-node-right sequence.
   Pre  root is the entry node of a tree or subtree
   Post each node has been processed in order
1 if (root is not null)
   1  inOrder (leftSubTree)
   2  process (root)
   3  inOrder (rightSubTree)
2 end if
end inOrder
```

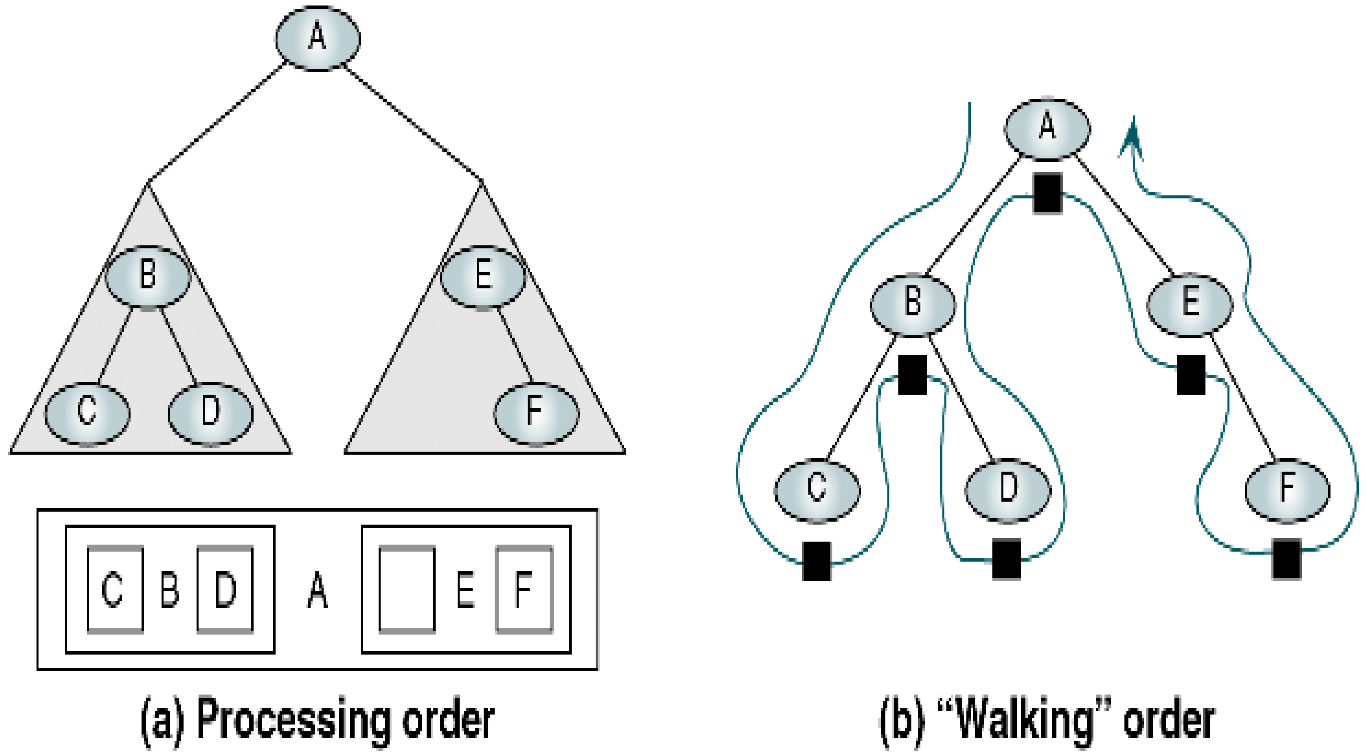


FIGURE 6-12 Inorder Traversal—C B D A E F

ALGORITHM 6-4 Postorder Traversal of a Binary Tree

```
Algorithm postOrder (root)
Traverse a binary tree in left-right-node sequence.
   Pre  root is the entry node of a tree or subtree
   Post each node has been processed in order
1 if (root is not null)
   1  postOrder (left subtree)
   2  postOrder (right subtree)
   3  process (root)
2 end if
end postOrder
```

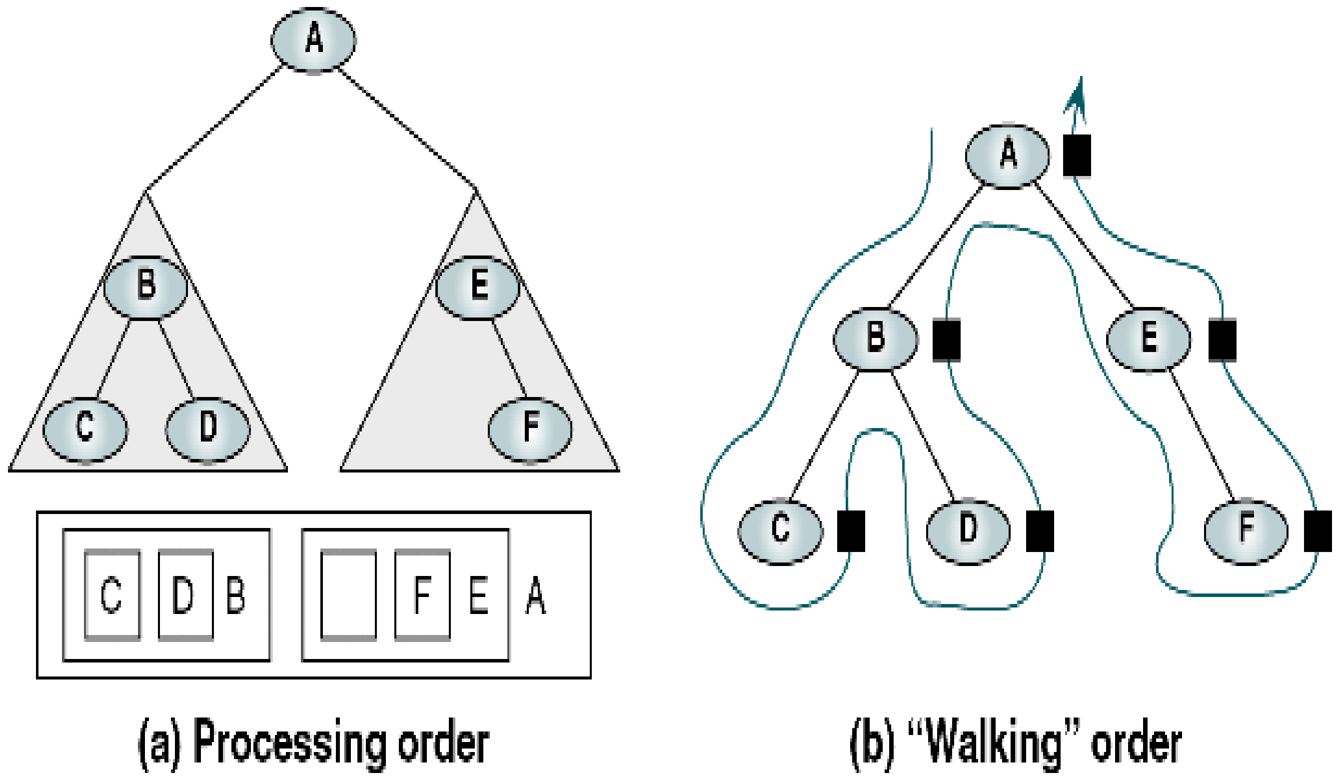
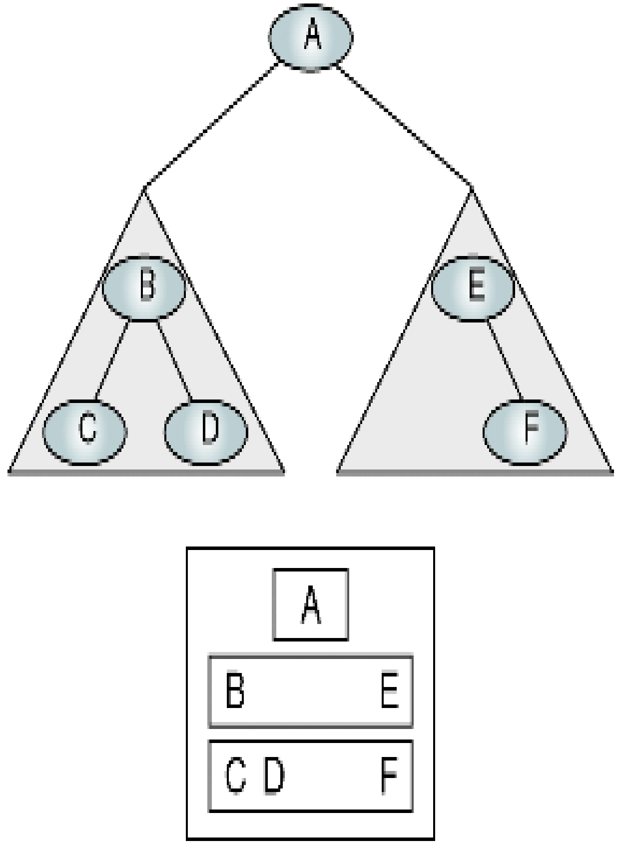


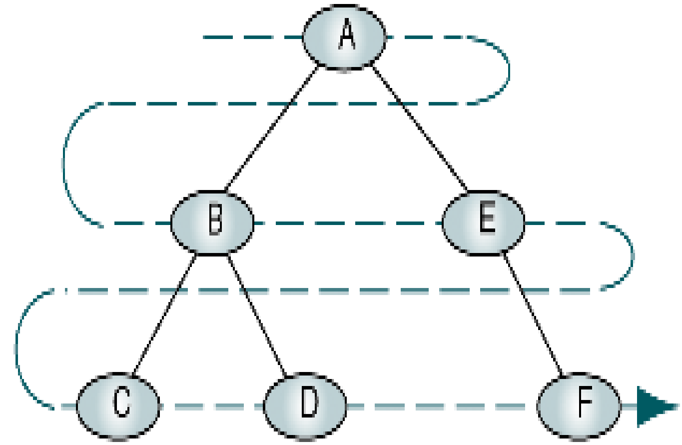
FIGURE 6-13 Postorder Traversal—C D B F E A

ALGORITHM 6-5 Breadth-first Tree Traversal

```
Algorithm breadthFirst (root)
Process tree using breadth-first traversal.
   Pre     root is node to be processed
   Post    tree has been processed
1  set currentNode to root
2  createQueue (bfQueue)
3  loop (currentNode not null)
   1  process (currentNode)
   2  if (left subtree not null)
      1  enqueue (bfQueue, left subtree)
   3  end if
   4  if (right subtree not null)
      1  enqueue (bfQueue, right subtree)
   5  end if
   6  if (not emptyQueue(bfQueue))
      1  set currentNode to dequeue (bfQueue)
   7  else
      1  set currentNode to null
   8  end if
4  end loop
5  destroyQueue (bfQueue)
end breadthFirst
```



(a) Processing order



(b) "Walking" order

FIGURE 6-14 Breadth-first Traversal

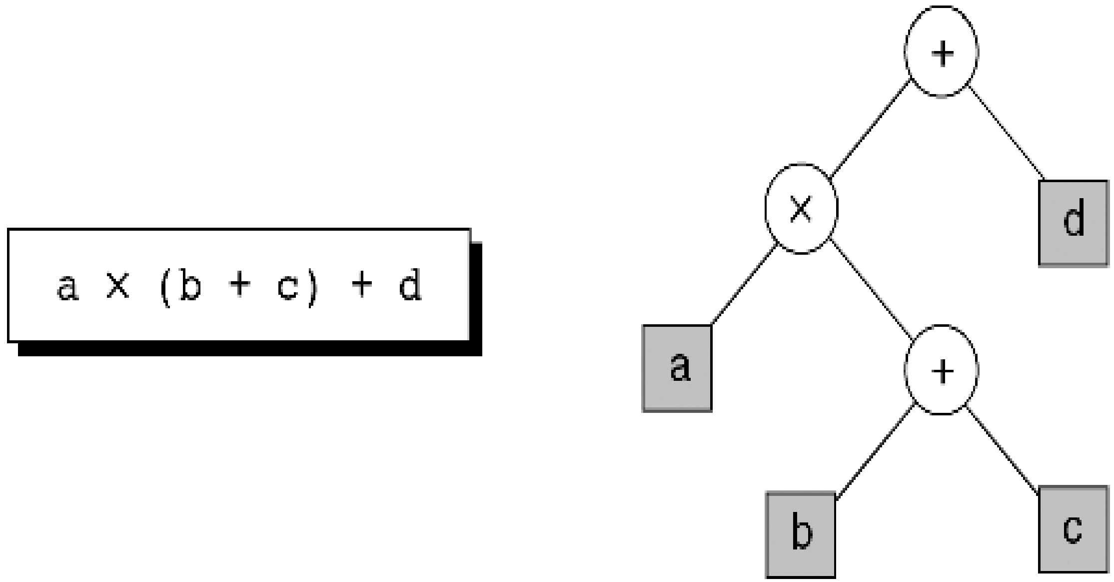


FIGURE 6-15 Infix Expression and Its Expression Tree

$((a \times (b + c)) + d)$

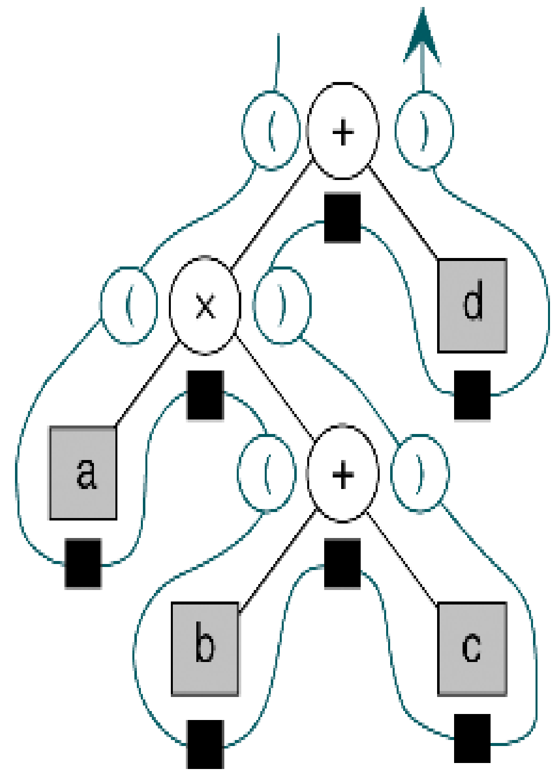


FIGURE 6-16 Infix Traversal of an Expression Tree

ALGORITHM 6-6 Infix Expression Tree Traversal

```
Algorithm infix (tree)
Print the infix expression for an expression tree.
   Pre  tree is a pointer to an expression tree
   Post the infix expression has been printed
1 if (tree not empty)
   1 if (tree token is an operand)
      1 print (tree-token)
   2 else
      1 print (open parenthesis)
      2 infix (tree left subtree)
      3 print (tree token)
      4 infix (tree right subtree)
      5 print (close parenthesis)
   3 end if
2 end if
end infix
```

ALGORITHM 6-7 Postfix Traversal of an Expression Tree

```
Algorithm postfix (tree)
Print the postfix expression for an expression tree.
   Pre  tree is a pointer to an expression tree
   Post the postfix expression has been printed
1 if (tree not empty)
```

continued

ALGORITHM 6-7 Postfix Traversal of an Expression Tree *(continued)*

```
   1  postfix (tree left subtree)
   2  postfix (tree right subtree)
   3  print   (tree token)
2 end if
end postfix
```

ALGORITHM 6-8 Prefix Traversal of an Expression Tree

```
Algorithm prefix (tree)
Print the prefix expression for an expression tree.
   Pre  tree is a pointer to an expression tree
   Post the prefix expression has been printed
1 if (tree not empty)
   1  print  (tree token)
   2  prefix (tree left subtree)
   3  prefix (tree right subtree)
2 end if
end prefix
```